

## Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples - each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

Copyright © 2021. Texas Education Agency. All Rights Reserved.
Notwithstanding the foregoing, the right to reproduce the copyrighted work is granted to Texas public school districts, Texas charter schools, and Texas education service centers for nonprofit educational use within the state of Texas, and to residents of the state of Texas for their own personal, non-profit educational use, and provided further that no charge is made for such reproduced materials other than to cover the out-of-pocket cost of reproduction and distribution. No other rights, express or implied, are granted hereby.

For more information, please contact Copyrights@tea.texas.gov.

Instructional Routines for Mathematics Intervention

## MODULE 3

## Representing Fractions

## Module 3: Representing Fractions Mathematics Routines

## A. Important Vocabulary with Definitions

| Term | Definition |
| :--- | :--- |
| denominator | The term in a fraction that tells the number of equal parts in a <br> whole. |
| equal sign | The symbol that tells you that two sides of an equation are the <br> same, balanced, or equal. |
| equivalent | Two numbers that have the same value. |
| fraction | A number representing part of a whole or set. |
| improper fraction | Any fraction in which the numerator is greater than or equal to <br> the denominator. |
| mixed number | A whole number and a fraction combined. |
| numerator | The term in a fraction that tells how many parts of a fraction. |
| proper fraction | A fraction where the numerator is less than the denominator. |

## B. Background Information

In this module, we focus on representing fractions. We use three models of fractions: (1) length model, (2) area model, and (3) set model.

When referring to fractions, be sure to use proper vocabulary. Also, present fractions in different ways.


## C. Routines and Examples

## (1) Length Model

## Routine

## Materials:

- Module 3 Problem Sets
- Module 3 Vocabulary Cards
- If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles, Cuisenaire rods, or number lines

ROUTINE WITH FRACTION TILES

| $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |  |
| :--- | :--- | :--- | :--- |

Teacher
Students
Teacher A fraction is a number - just like $\mathbf{2}$ is a number or 13 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?
Students A numerator and denominator.
Teacher The denominator tells us about the equal parts in the whole. What does the denominator tell us?
Students The equal parts in the whole.
Teacher And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
Students How many equal parts we show for a specific fraction.
Teacher So, let's show different fractions. We'll use these fraction tiles first. (Show manipulatives.)
Teacher When we show fractions with the fraction tiles, let's first start by showing the whole. What should we show?
Students The whole.
Teacher I'll place the whole in the middle of my workspace. (Show whole.)
Teacher Let's show this fraction. (Show fraction.)
Teacher What fraction?
Students
Teacher
Students
Teacher
Let's show different fractions. What's a fraction?
A fraction is a number with a numerator and denominator.

|  | a fraction, the number has a numerator and denominator. What does a fraction have? |
| :---: | :---: |
| Students | A numerator and denominator. |
| Teacher | The denominator tells us about the equal parts in the whole. What does the denominator tell us? |
| Students | The equal parts in the whole. |
| Teacher | And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us? |
| Students | How many equal parts we show for a specific fraction. |
| Teacher | So, let's show different fractions. We'll use these fraction tiles first. (Show manipulatives.) |
| Teacher | When we show fractions with the fraction tiles, let's first start by showing the whole. What should we show? |
| Students | The whole. |
| Teacher | I'll place the whole in the middle of my workspace. (Show whole.) |
| Teacher | Let's show this fraction. (Show fraction.) |
| Teacher | What fraction? |
| Students |  |
| Teacher | What's the denominator of the fraction? |
| Students |  |
| Teacher | That means we want to divide this whole into $\qquad$ (denominator) equal parts. How many equal parts? |


| Students <br> Teacher | So, how could we divide this whole into __ (denominator) equal parts? <br> Students <br> Divide the whole into _ equal parts with __ fraction tiles. <br> (Show whole divided into equal parts. Place equal parts over the whole or <br> above the whole.) |
| :--- | :--- |
| Teacher |  |
| We showed the whole divided into __ equal parts. Now, what's the |  |
| numerator of the fraction? |  |

Teacher
Students
Teacher

Students
Teacher

Students
Teacher

Students
Teacher
Teacher When we show fractions with the Cuisenaire rods, let's first start by thinking about the whole. What should we focus on?
Students The whole.
Teacher Let's show this fraction. (Show fraction.)
Teacher What fraction?
Students
Teacher
Students
Teacher

Students
Teacher
Students
So, how could we show a whole with __ (denominator) equal parts?
Show _ (denominator) equal parts. Then, find a Cuisenaire rod with a whole that's the same length as the __ equal parts.
Teacher Let's first find __ (denominator) equal parts. We'll line those up and find a whole that's the same length as the __ (denominator) equal parts.
(Show whole divided into equal parts. Place equal parts over the whole or above the whole.)
Teacher So, this rod (say color) shows the denominator. What does this rod show?
Let's show different fractions. What's a fraction?
A fraction is a number with a numerator and denominator.
A fraction is a number - just like 4 is a number or 65 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?
A numerator and denominator.
The denominator tells us about the equal parts in the whole. What does the denominator tell us?
The equal parts in the whole.
And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
-.
What's the denominator of the fraction?
$\qquad$
That means we want to show a whole with __ (denominator) equal parts. How many equal parts?
$\qquad$
S

| Students | The denominator. |
| :---: | :---: |
| Teacher | Now, what's the numerator of the fraction? |
| Students |  |
| Teacher | That means we want to show $\qquad$ (numerator) of the equal parts in order to show the fraction $\qquad$ . How many equal parts do we want to show? |
| Students |  |
| Teacher | Let's show $\qquad$ of the $\qquad$ equal parts. <br> (Show the equal parts of the numerator. Leave equal parts over the whole or above the whole.) |
| Teacher | What fraction did we show? |
| Students |  |
| Teacher | We showed the fraction $\qquad$ . First, we divided the whole into $\qquad$ equal parts. Then, we showed the $\qquad$ equal parts of the fraction. How did we show this fraction? |
| Students | We divided the whole into equal parts. Then, we showed the equal parts of the fraction. |
| Teacher | Now, let's think about this fraction. We should interpret the left side of the whole as zero - like zero on a number line. What number? |
| Students | Zero. |
| Teacher | And if we have only one whole, we should interpret the right side of the whole as one - like one of a number line. What number? |
| Students | One. |
| Teacher | Remember to think about the whole as a number line from $\mathbf{0}$ to $\mathbf{1}$ (then 1 to 2, then 2 to 3 ...). That helps us learn the value of the fraction $\qquad$ . Where would the fraction $\frac{1}{2}$ be on this whole? |
| Students | In the middle between 0 and 1. |
| Teacher | Think about $\frac{1}{2}$. What do you notice about __ (fraction) compared to $\frac{1}{2}$ ? |
| Students | __is greater/less than $\frac{1}{2}$. |
| Teacher | Yes, I see $\qquad$ is greater/less than $\frac{1}{2}$. What else do you notice about (fraction)? |
| Students | __ is greater/less than __ (benchmark fraction). |
| Teacher | Great work! Using these Cuisenaire rods helps you understand the value of different fractions. Let's review. What's a denominator? |
| Students | The equal parts in the whole. |
| Teacher | What's a numerator? |
| Students | How many equal parts we show for a specific fraction. |
| Teacher | How can you use the Cuisenaire rods to show a fraction? |
| Students | First, you determine the equal parts of the whole and find a whole of the same length. Then, you look at the denominator and divide the whole into equal parts. Then, you show the numerator of the fraction by showing the number of equal parts by the numerator. |

## ROUTINE WITH NUMBER LINE



Teacher Let's show different fractions. What's a fraction?
Students A fraction is a number with a numerator and denominator.
Teacher A fraction is a number - just like 5 is a number or 17 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?
Students A numerator and denominator.
Teacher The denominator tells us about the equal parts in the whole. What does the denominator tell us?
Students The equal parts in the whole.
Teacher And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
Students How many equal parts we show for a specific fraction.
Teacher So, let's show different fractions. Today, let's draw fractions with a number line.
(Draw a number line.)
Teacher When we show fractions with a number line, let's draw a 0 and 1 on the number line.
(Draw 0 and 1.)
Teacher Now, let's start by thinking about the whole. What should we focus on?
Students The whole.
Teacher Let's show this fraction.
(Show fraction.)
Teacher What fraction?
Students
Teacher
Students
What's the denominator of the fraction?

Teacher

Students
That means we want to show a whole with _ (denominator) equal parts. How many equal parts?

## Teacher

Students
-
So, how could we show a whole with $\qquad$ (denominator) equal parts?

Teacher Let's divide this number line into __ (denominator) equal parts.
(Divide number line into equal parts by drawing tick marks.)
Teacher So, our number line shows _ equal parts or the denominator. What does the number line show?
Students The denominator.
Teacher Now, what's the numerator of the fraction?
Students $\qquad$ -.

| Teacher | That means we want to show $\qquad$ (numerator) of the equal parts in order to show the fraction $\qquad$ . How many equal parts do we want to show? |
| :---: | :---: |
| Students |  |
| Teacher | Let's draw where the fraction $\qquad$ is on the number line. I count $\qquad$ equal parts. (Draw the fraction on the number line.) |
| Teacher | What fraction did we show? |
| Students |  |
| Teacher | We showed the fraction $\qquad$ . First, we drew a number line. Then, we divided the whole into $\qquad$ equal parts. Then, we showed the fraction. How did we show this fraction? |
| Students | We divided the number line into equal parts. Then, we determine the numerator - or the number of equal parts - and drew the fraction on the number line. |
| Teacher | Let's think about this fraction on a number line from $\mathbf{0}$ to $\mathbf{1}$ (then 1 to 2 , then 2 to $3 . .$.$) . That helps us learn the value of the fraction$ $\qquad$ . Where would the fraction $\frac{1}{2}$ be on this whole? |
| Students | In the middle between 0 and 1. |
| Teacher | Think about $\frac{1}{2}$. What do you notice about __ (fraction) compared to $\frac{1}{2}$ ? |
| Students | __ is greater/less than $\frac{1}{2}$. |
| Teacher | Yes, I see $\qquad$ is greater/less than $\frac{1}{2}$. What else do you notice about (fraction)? |
| Students | $\ldots$ _ is greater/less than _ (benchmark fraction). |
| Teacher | Great work! Using the number line helps you understand the value of different fractions. Let's review. What's a denominator? |
| Students | The equal parts in the whole. |
| Teacher | What's a numerator? |
| Students | How many equal parts we show for a specific fraction. |
| Teacher | How can you use the number line to show a fraction? |
| Students | First, draw the equal parts of the whole. Then, draw the fraction on the numb line |

## EXAMPLE WITH FRACTION TILES



Teacher Let's show different fractions. What's a fraction?
Students A fraction is a number with a numerator and denominator.
Teacher A fraction is a number, except with a fraction, the number has a numerator and denominator. What does a fraction have?
Students A numerator and denominator.
Teacher The denominator tells us about the equal parts in the whole. What does the denominator tell us?
Students The equal parts in the whole.
Teacher And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
Students How many equal parts we show for a specific fraction.
Teacher So, let's show different fractions. We'll use these fraction tiles first. (Show manipulatives.)
Teacher When we show fractions with the fraction tiles, let's first start by showing the whole. What should we show?
Students The whole.
Teacher I'll place the whole in the middle of my workspace. (Show whole.)
Teacher Let's show this fraction. (Show fraction.)
Teacher What fraction?
Students $\frac{3}{4}$.
Teacher What's the denominator of the fraction?
Students 4.
Teacher That means we want to divide this whole into 4 equal parts. How many equal parts?
Students 4.
Teacher So, how could we divide this whole into 4 equal parts?
Students Divide the whole into 4 equal parts with 4 one-fourth fraction tiles.
(Show whole divided into equal parts. Place equal parts over the whole or above the whole.)
Teacher We showed the whole divided into 4 equal parts. Each equal part is onefourth of the whole. Now, what's the numerator of the fraction?
Students 3.

| Teacher | That means we want to show 3 of the equal parts in order to show the fraction $\frac{3}{4}$. How many equal parts do we want to show? |
| :---: | :---: |
| Students | 3. |
| Teacher | Let's show 3 of the 4 equal one-fourth parts. <br> (Show the equal parts of the numerator. Leave equal parts over the whole or above the whole.) |
| Teacher | What fraction did we show? |
| Students | $\frac{3}{4}$. |
| Teacher | We showed the fraction $\frac{3}{4}$. First, we divided the whole into 4 equal parts. Then, we showed the 3 equal parts of the fraction. How did we show this fraction? |
| Students | We divided the whole into 4 equal parts. Then, we showed the 3 equal parts of the fraction. |
| Teacher | Now, let's think about this fraction. We should interpret the left side of the whole tile as zero - like zero on a number line. What number? |
| Students | Zero. |
| Teacher | And if we have only one whole, we should interpret the right side of the whole tile as one - like one on a number line. What number? |
| Students | One. |
| Teacher | Remember to think about the whole as a number line from 0 to 1 . That helps us learn the value of the fraction $\frac{3}{4}$. Where would the fraction $\frac{1}{2}$ be on this whole? |
| Students | In the middle between 0 and 1. |
| Teacher | Think about $\frac{1}{2}$. What do you notice about $\frac{3}{4}$ compared to $\frac{1}{2}$ ? |
| Students | $\frac{3}{4}$ is greater than $\frac{1}{2}$. |
| Teacher | Yes, I see $\frac{3}{4}$ is greater than $\frac{1}{2}$. What else do you notice about $\frac{3}{4}$ ? |
| Students | $\frac{3}{4}$ is less than 1. |
| Teacher | Excellent! Using these fraction tiles helps you understand the value of different fractions. Let's review. What's a denominator? |
| Students | The equal parts in the whole. |
| Teacher | What's a numerator? |
| Students | How many equal parts we show for a specific fraction. |
| Teacher | How can you use the fraction tiles to show a fraction? |
| Students | First, you show the whole. Then, you look at the denominator and divide the whole into equal parts. Then, you show the numerator of the fraction by showing the equal parts in the numerator. |

## (2) Area Model

## Routine

## Materials:

- Module 3 Problem Sets
- Module 3 Vocabulary Cards
- If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction circles, geoboards, or pattern blocks

ROUTINE WITH FRACTION CIRCLES


Teacher Let's show different fractions. What's a fraction?
Students A fraction is a number with a numerator and denominator.
Teacher A fraction is a number - just like $\mathbf{2}$ is a number or 13 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?
Students A numerator and denominator.
Teacher The denominator tells us about the equal parts in the whole. What does the denominator tell us?
Students The equal parts in the whole.
Teacher And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
Students How many equal parts we show for a specific fraction.
Teacher So, let's show different fractions. We'll use these fraction circles. (Show manipulatives.)
Teacher These fraction circles can help us see fractions by area. Any area - a circle, triangle, rectangle, or any other area - can also be used to show fractions. What's an area that could be used to show fractions?
Students Rectangle or triangle.
Teacher When we show fractions with the fraction circles, let's first start by showing the whole. What should we show?
Students The whole.
Teacher I'll place the whole in the middle of my workspace. (Show whole.)
Teacher Let's show this fraction.
(Show fraction.)
Teacher What fraction?

| Students <br> Teacher | What's the denominator of the fraction? |
| :---: | :---: |
| Students |  |
| Teacher | That means we want to divide this whole into $\qquad$ (denominator) equal parts. How many equal parts? |
| Students |  |
| Teacher | So, how could we divide this whole into _ ( ${ }^{\text {denominator) equal parts? }}$ |
| Students | Divide the whole into $\qquad$ equal parts with $\qquad$ fraction tiles. (Show whole divided into equal parts. Place equal parts over the whole.) |
| Teacher | We showed the whole divided into $\qquad$ equal parts. Now, what's the numerator of the fraction? |
| Students |  |
| Teacher | That means we want to show $\qquad$ (numerator) of the equal parts in order to show the fraction $\qquad$ . How many equal parts do we want to show? |
| Students |  |
| Teacher | Let's show $\qquad$ of the $\qquad$ equal parts. <br> (Show the equal parts of the numerator. Leave equal parts over the whole.) |
| Teacher | What fraction did we show? |
| Students |  |
| Teacher | We showed the fraction $\qquad$ . First, we divided the whole into $\qquad$ equal parts. Then, we showed the $\qquad$ equal parts of the fraction. How did we show this fraction? |
| Students | We divided the whole into equal parts. Then, we showed the equal parts of the fraction. |
| Teacher | Now, let's think about this fraction. We're looking at a fraction by area. What would be $\frac{1}{2}$ of this area |
| Students | Half of the circle. |
| Teacher | What do you notice about _ ( fraction) compared to $\frac{1}{2}$ ? |
| Students | $\qquad$ is greater/less than $\frac{1}{2}$. |
| Teacher | Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about _ ? |
| Students | __ is greater/less than 1. |
| Teacher | Great work! Using these fraction circles helps you understand the area of different fractions. Let's review. What's a denominator? |
| Students | The equal parts in the whole. |
| Teacher | What's a numerator? |
| Students | How many equal parts we show for a specific fraction. |
| Teacher | How can you use the fraction tiles to show a fraction? |
| Students | First, you show the whole area. Then, you look at the denominator and divide the whole into equal parts. Then, you show the numerator of the fraction by placing the equal parts of the numerator on the whole. |

## ROUTINE WITH GEOBOARDS



Teacher Let's show different fractions. What's a fraction?
Students A fraction is a number with a numerator and denominator.
Teacher A fraction is a number - just like 5 is a number or 25 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?
Students
A numerator and denominator.
Teacher The denominator tells us about the equal parts in the whole. What does the denominator tell us?
Students The equal parts in the whole.
Teacher And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
Students How many equal parts we show for a specific fraction.
Teacher So, let's show different fractions. We'll use this geoboard. (Show manipulatives.)
Teacher This geoboard can help us see fractions by area. Any area - a circle, triangle, rectangle, or any other area - can also be used to show fractions. What's an area that could be used to show fractions?
Students Rectangle or triangle or circle or trapezoid.
Teacher When we show fractions with the geoboard, let's first start by showing the whole. What should we show?
Students The whole.
Teacher Let's show this fraction. (Show fraction.)
Teacher What fraction?
Students -.
Teacher
Students

Teacher

Students
Teacher
Students
Teacher

What's the denominator of the fraction?
$\qquad$
That means we want to make an area that can be divided into (denominator) equal parts. How many equal parts?
$\qquad$
So, how could we make an area divided into __ (denominator) equal parts? Ring the rubber band around __ equal parts. (Use rubber band to ring whole divided into equal parts.) We showed an area or whole divided into __ equal parts. Now, what's the numerator of the fraction?

Students
Teacher

Students Teacher

Teacher
Students
Teacher

Students

Teacher

Students
Teacher
Students
Teacher
Students Teacher

Students
Teacher
Students
Teacher
Students

Teacher
Students A fraction is a number with a numerator and denominator.
$\left.\begin{array}{ll}\text { Teacher } & \begin{array}{l}\text { A fraction is a number - just like } 14 \text { is a number or } 8 \text { is a number. Except with } \\ \text { a fraction, the number has a numerator and denominator. What does a } \\ \text { fraction have? }\end{array} \\ \text { Students } & \begin{array}{l}\text { A numerator and denominator. }\end{array} \\ \text { Teacher } \\ \text { The denominator tells us about the equal parts in the whole. What does the } \\ \text { denominator tell us? }\end{array}\right]$

| Teacher | We showed the fraction__. First, we showed an area divided into __ equal <br> parts. Then, we showed the __ equal parts of the fraction. How did we show <br> this fraction? |
| :--- | :--- |
| Students |  |
| We showed the area of the whole. Then, we showed the equal parts of the |  |
| fraction. |  |
| Now, let's think about this fraction. What's the area of this fraction? What |  |
| would be $\frac{1}{2}$ of this area? |  |

Example


Teacher Let's show different fractions. What's a fraction?
Students A fraction is a number with a numerator and denominator.
Teacher A fraction is a number but the number has a numerator and denominator. What does a fraction have?
Students A numerator and denominator.

| Teacher | The denominator tells us about the equal parts in the whole. What does the <br> denominator tell us? |
| :--- | :--- |
| Students | The equal parts in the whole. |
| Teacher | And the numerator tells us how many equal parts should be shown in a <br> specific fraction. What does the numerator tell us? |
| Students | How many equal parts we show for a specific fraction. <br> So, let's show different fractions. We'll use this geoboard. <br> (Show manipulatives.) |
| Teacher |  |

Students $\quad \frac{3}{4}$ is greater than $\frac{1}{2}$.
Teacher Yes, I see $\frac{3}{4}$ is greater than $\frac{1}{2}$. What else do you notice about $\frac{3}{4}$ ?
Students $\quad \frac{3}{4}$ is less than 1.
Teacher Awesome! Using a geoboard helps you understand the area of different fractions. Let's review. What's a denominator?
Students The equal parts in the whole.
Teacher What's a numerator?
Students How many equal parts we show for a specific fraction.
Teacher How can you use the geoboard to show a fraction?
Students First, you show the whole by ringing a rubber band around an area. Then, you show the numerator of the fraction by ringing the area of the numerator.

## (3) Set Model

## Routine

## Materials:

- Module 3 Problem Sets
- Module 3 Vocabulary Cards
- If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like two-color counters or colored cubes

ROUTINE WITH TWO-COLOR COUNTERS


Teacher Let's show different fractions. What's a fraction?
Students A fraction is a number with a numerator and denominator.
Teacher A fraction is a number - just like $\mathbf{3}$ is a number or $\mathbf{3 0 0}$ is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?
Students A numerator and denominator.
Teacher The denominator tells us about the equal parts in the whole. What does the denominator tell us?
Students The equal parts in the whole.
Teacher And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
Students How many equal parts we show for a specific fraction.
Teacher So, let's show different fractions. We'll use these two-color counters. (Show manipulatives.)
Teacher When we show fractions with the two-color counters, let's first start by showing the whole. What should we show?
Students The whole.
Teacher Let's show this fraction.
(Show fraction.)
Teacher What fraction?
Students _.
Teacher What's the denominator of the fraction?
Students __.
Teacher That means we want to show _ (denominator) equal parts. How many equal parts?
Students $\qquad$

| Teacher | So, how could we show a set with _ (denominator) equal parts? |
| :---: | :---: |
| Students | Use __ counters. |
| Teacher | With two-color counters, we can create a set of $\qquad$ (denominator) counters. We'll use the yellow side to show the denominator. (Show yellow counters in a set. Place the counters so they are touching one another.) |
| Teacher | We showed a set divided into $\qquad$ equal parts. Now, what's the numerator of the fraction? |
| Students |  |
| Teacher | That means we want to show $\qquad$ (numerator) of the equal parts in order to show the fraction $\qquad$ . How many equal parts do we want to show? |
| Students |  |
| Teacher | We can show the numerator by turning over the counter to the red side. (Turn counters to red side.) |
| Teacher | What fraction did we show? |
| Students |  |
| Teacher | We showed the fraction $\qquad$ . First, we showed a set divided into $\qquad$ equal parts. Then, we showed the $\qquad$ equal parts of the fraction. How did we show this fraction? |
| Students | We showed a set of yellow counters for the denominator, then we turned over counters to the red side to show the numerator. |
| Teacher | Now, let's think about this fraction. What would be $\frac{1}{2}$ of this set? |
| Students | Half of the counters. |
| Teacher | What do you notice about _ (fraction) compared to $\frac{1}{2}$ ? |
| Students | __ is greater/less than $\frac{1}{2}$. |
| Teacher | Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about _ ? |
| Students | $\ldots$ is greater/less than 1. |
| Teacher | Great work! Using the two-color counters helps you understand how sets can be used to show different fractions. Let's review. What's a denominator? |
| Students | The equal parts in the whole. |
| Teacher | What's a numerator? |
| Students | How many equal parts we show for a specific fraction. |
| Teacher | How can you use the two-color counters to show a fraction? |
| Students | We showed a set of yellow counters to show the denominator, then we turned over the counters to the red side to show the numerator or fraction. |



Teacher Let's show different fractions. What's a fraction?
Students A fraction is a number with a numerator and denominator.
Teacher A fraction is a number - just like 150 is a number or 15 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?
Students A numerator and denominator.
Teacher The denominator tells us about the equal parts in the whole. What does the denominator tell us?
Students The equal parts in the whole.
Teacher And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
Students How many equal parts we show for a specific fraction.
Teacher So, let's show different fractions. We'll use these colored cubes.
(Show manipulatives.)
Teacher When we show fractions with the cubes, let's first start by showing the whole. What should we show?
Students The whole.
Teacher Let's show this fraction.
(Show fraction.)
Teacher What fraction?
Students _.
Teacher What's the denominator of the fraction?
Students _.

Teacher

Students


Students
Teacher

Teacher

Students

That means we want to show _ (denominator) equal parts. How many equal parts?
$\qquad$
So, how could we show a set with $\qquad$ (denominator) equal parts? Use __ cubes. With the cubes, let's use one color to show the denominator. Let's place __ (denominator) _ (color) cubes in a set.
(Show cubes in a set. Place the cubes so they are close to one another - to show a set of objects.)
We showed a set divided into __ equal parts. Now, what's the numerator of the fraction?
$\qquad$
That means we want to show _ (numerator) of the equal parts in order to show the fraction _. How many equal parts do we want to show?
$\qquad$ _.

Teacher With the cubes, let's use another color to show the numerator. We'll replace
$\qquad$ (denominator) cubes with $\qquad$ (new color) cubes.
(Show numerator cubes in a different color by replacing the denominator cubes.)
Teacher
Students
Teacher

Students
Teacher
Students
Teacher
Students
Teacher
Students
Teacher

Students
Teacher
What fraction did we show?
$\qquad$ .
We showed the fraction $\qquad$ . First, we showed a set of cubes divided into $\qquad$ equal parts. Then, we showed the __ equal parts of the fraction. How did we show this fraction?
We showed a set of cubes for the denominator, then we used different colored cubes to show the numerator.

Students
Teacher
Students
Now, let's think about this fraction. What would be $\frac{1}{2}$ of this set? Half of the cubes.
What do you notice about __ (fraction) compared to $\frac{1}{2}$ ?
$\qquad$ is greater/less than $\frac{1}{2}$.
Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about $\qquad$ ?
$\qquad$ is greater/less than 1.
Super! Using the colored cubes helps you understand how sets can be used to show different fractions. Let's review. What's a denominator?
The equal parts in the whole.
What's a numerator?
How many equal parts we show for a specific fraction.
How can you use the colored cubes to show a fraction?
First, you show a set of the whole with cubes. Then, you use a different colored cube to show the numerator within that set.

## Example

Let's show different fractions. What's a fraction?
Students A fraction is a number with a numerator and denominator.

## EXAMPLE WITH TWO-COLOR COUNTERS



| Teacher | A fraction is a number with a numerator and denominator. What does a <br> fraction have? |
| :--- | :--- |
| Students | A numerator and denominator. <br> The denominator tells us about the equal parts in the whole. What does the <br> denominator tell us? |
| Teacher |  |

Students We showed a set of 4 yellow counters for the denominator, then we turned over 3 counters to the red side to show the numerator.
Teacher Now, let's think about this fraction. What would be $\frac{1}{2}$ of this set?
Students Half of the counters - or 2 counters.
Teacher What do you notice about $\frac{3}{4}$ compared to $\frac{1}{2}$ ?
Students $\quad \frac{3}{4}$ is greater than $\frac{1}{2}$.
Teacher Yes, I see $\frac{3}{4}$ is greater than $\frac{1}{2}$. What else do you notice about $\frac{3}{4}$ ?
Students $\quad \frac{3}{4}$ is less than 1.
Teacher Great work! Using the two-color counters helps you understand how sets can be used to show different fractions. Let's review. What's a denominator?
Students The equal parts in the whole.
Teacher
Students What's a numerator?

Teacher How many equal parts we show for a specific fraction.

Students First, you show a set of the whole by showing a set with yellow counters. Then, you turn over counters to the red side for the numerator.

## D. Problems for Use During Instruction

See Module 3 Problem Sets.

## E. Vocabulary Cards for Use During Instruction

See Module 3 Vocabulary Cards.

Developed by:
Sarah R. Powell (srpowell@austin.utexas.edu)
Katherine A. Berry (kberry@austin.utexas.edu)

## Module 3: Representing Fractions

## Problem Sets

A. Proper fractions (20)
B. Improper fractions (20)
C. Mixed numbers (20)

$$
\frac{2}{5}
$$

## $\frac{6}{10}$

$$
\frac{3}{6}
$$

$$
\frac{1}{4}
$$



$$
\frac{2}{5}
$$

## $\frac{3}{8}$

## $\frac{4}{10}$

$$
\frac{2}{12}
$$

$$
\frac{5}{6}
$$



$$
\frac{1}{6}
$$



$$
\frac{3}{10}
$$

$$
\frac{2}{10}
$$

## 3 9



## $\frac{3}{8}$




$$
\frac{9}{8}
$$

$$
\frac{8}{6}
$$

$$
\frac{5}{4}
$$

$$
\frac{13}{10}
$$



$$
\frac{9}{6}
$$

$$
\frac{6}{2}
$$



## $\frac{11}{10}$



$$
\frac{9}{2}
$$




## 11 4



$$
\frac{8}{6}
$$

$$
\frac{12}{8}
$$

$$
\frac{15}{10}
$$



$$
3 \frac{5}{6}
$$

## $4 \frac{1}{5}$

$$
3 \frac{5}{12}
$$

C.

C.


$$
2 \frac{2}{4}
$$

$$
3 \frac{5}{8}
$$

$$
1 \frac{7}{8}
$$



$$
7 \frac{1}{2}
$$

C.
$1 \frac{5}{6}$
c.


## C. <br> $3 \frac{4}{10}$

$$
2 \frac{5}{12}
$$

$$
3 \frac{1}{2}
$$

$$
6 \frac{5}{8}
$$

C.

$$
4 \frac{5}{10}
$$

## Module 3: Representing Fractions

## Vocabulary Cards

denominator equivalent
fraction
improper fraction
mixed number
numerator
proper fraction

## denominator

The term in a fraction that tells the number of equal parts in a whole.

$$
2 / 3 \frac{2}{3} \text { In these fractions, } 3 \text { is the denominator. }
$$

## equivalent

Two numbers that have the same value.

$$
\frac{1}{4}=\frac{2}{8} \quad \frac{2}{3}=\frac{8}{12}
$$

## fraction

A number representing part of a whole or set.

$$
\frac{3}{6} \quad \frac{10}{12} \quad \frac{8}{3}
$$

## improper fraction

Any fraction in which the numerator is greater than or equal to the denominator.

$$
\begin{array}{lll}
9 & \frac{17}{12} & \frac{10}{3}
\end{array}
$$

## mixed number

A whole number and a fraction combined.

$$
1 \frac{1}{6} \quad 4 \frac{5}{12} \quad 12 \frac{4}{3}
$$

## numerator

The term in a fraction that tells how many parts of a fraction.

$$
2 / 3 \frac{2}{3} \quad \text { In these fractions, } 2 \text { is the numerator. }
$$

## proper fraction

A fraction where the numerator is less than the denominator.

$$
\frac{3}{4} \quad \frac{5}{6} \quad \frac{8}{21}
$$

