Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 3-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA Texas Education Agency | Texas SPED Support

Instructional Routines for Mathematics Intervention

User Guide

I. Overview

Welcome to the *User Guide for the Instructional Routines for Mathematics Intervention*. These materials were created for Texas educators in partnership with the Texas Education Agency and Inclusion in Texas Network.

The goal of the *Instructional Routines for Mathematics* Intervention is to provide educators with a set of instructional materials to use when delivering mathematics intervention to students who experience difficulty. Each of the 23 Modules are focused on different mathematics content and are designed for implementation across grades 2 through 8.

The modules provide educators with easy-to-use materials for mathematics intervention. The aim is to provide educators with resources to address the diverse needs of students who experience mathematics difficulty. These materials can be implemented explicitly to improve mathematics outcomes for students.

Highlighted Module features:

- Step-by-step Routines for explicit teaching of the mathematics content.
- Vocabulary Cards with visuals that easily can be displayed in the classroom.
- Problem Sets with greater than 50 ready-to-use problems for each Module.

This User Guide includes the following sections:

These materials are designed to be easily accessed and utilized. We look forward to hearing your success stories and feedback. Enjoy!

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II. Materials

The table below provides a summary of the 23 Modules with the applicable Texas Essential Knowledge and Skills (TEKS) for each grade level.

Module Number	Module Title	Module Description	Texas Essential Knowledge and Skills (TEKS) Applicable Grade Levels						
			2	**3**	**4**	**5**	**6**	**7**	**8**
1	Place Value	Describe place value of whole and rational numbers	2(A) 2(B)	2(A) 2(B) 2(C)	2(A) 2(B) 2(D) 4(G)	2(C)			
2	Comparison	Compare whole and rational numbers with greater than, less than, or equal to	2(D) 2(E)	2(D) 3(H)	2(C) 2(F) 3(D)	2(B)			2(D)
3	Representing Fractions	Show fractions with the length, area, and set models	3(A) 3(B) 3(C) 3(D)	3(A) 3(B) 3(C) 3(D) 3(E) 3(F) 3(G)	3(B) 3(C) 3(D) 3(G)		2(D) 4(E) 4(F) 4(G) 5(C)		
4	Concepts of Addition	Describe addition as (a) combining sets and (b) joining to a set	4(B) 4(C)	4(A) 4(B)					
5	Addition of Whole Numbers	Add multi-digit numbers using (a) standard algorithm and (b) partial sums	4(B) 4(C)	4(A) 4(B) 5(A)	4(A)	3(A)		2	
6	Addition of Rational Numbers	Add rational numbers with like denominators and unlike denominators			3(A) 3(E) 3(F) 4(A)	3(A) 3(H) 3(K)		3(A) 3(B)	
7	Concepts of Subtraction	Describe subtraction as (a) separating from a set and (b) comparing	4(B) 4(C)	4(A) 4(B)					
8	Subtraction of Whole Numbers	Subtract multi-digit numbers using (a) standard algorithm and (b) adding up	4(B) 4(C)	4(A) 4(B) 5(A)	4(A)	3(A)		2	
9	Subtraction of Rational Numbers	Subtract rational numbers with like denominators and unlike denominators			3(E) 3(F) 4(A)	3(A) 3(H) 3(K)		3(A) 3(B)	

Module Number	Module Title	Module Description	Texas Essential Knowledge and Skills (TEKS) Applicable Grade Levels						
			2	**3**	**4**	**5**	**6**	**7**	**8**
10	Concepts of Multiplication	Describe multiplication as (a) equal groups and (b) comparison	6(A)	4(D) 4(E) 4(F) 4(H) 5(B) 5(D)					
11	Multiplication of Whole Numbers	Multiply multi-digit numbers using (a) standard algorithm and (b) partial products/array		4(D) 4(E) 4(F) 4(G) 4(K) 5(C)	4(B) 4(C) 4(D) 4(H)	3(A) 3(B)			
12	Multiplication of Rational Numbers	Multiply fractions with an emphasis on conceptual understanding				3(A) 3(D) 3(E) 3(I)	3(A) 3(B) 3(E)	2 3(A) 3(B)	
13	Concepts of Division	Describe division as (a) partitive and (b) measurement	6(B)	4(H) 4(I) 4(J) 5(D)					
14	Division of Whole Numbers	Divide multi-digit numbers using (a) standard algorithm and (b) partial quotients		4(H) 4(I) 4(J) 4(K)	4(E) 4(F) 4(H)	3(A) 3(C)			
15	Division of Rational Numbers	Divide fractions with an emphasis on conceptual understanding				3(A) 3(F) 3(G) 3(J) 3(L)	3(A) 3(E)	2 3(A) 3(B)	
16	Representing Decimals	Show decimals using proportional and non-proportional materials			2(E) 2(F) 2(G) 2(H) 3(G)	2(A) 2(B)	4(E) 4(F) 4(G) 5(C)		2(C)
17	Integers	Understand positive and negative integers					2(B) 2(C)		
18	Addition and Subtraction of Integers	Add and subtract positive and negative integers					3(C) 3(D)		
19	Multiplication and Division of Integers	Multiply and divide positive and negative integers					3(D)		

Module Number	Module Title	Module Description	Texas Essential Knowledge and Skills (TEKS) Applicable Grade Levels						
			2	3	4	5	6	7	8
20	Functions and Ordered Pairs	Describe functions, ordered pairs, and graphing related to functions		5(E)	5(B)	4(C) 4(D)	4(A) 7	7(A)	5(A) 5(C) 5(G)
21	Ratios, Proportions, Rates, and Percentages	Represent ratios, proportions, rates, and percentages					4(B) 4(C) 4(D) 4(E) 4(F) 4(G) 4(H) 5(A) 5(B) 5(C)	4(A) 4(B) 4(C) 4(D) 4(E)	5(A) 5(B) 5(C) 5(D) 5(E) 5(F) 5(G) 5(H) 5(I)
22	Representing Expressions and Equations	Describe order of operations, representing expressions, and representing equations				4(E) 4(F)	6(A) 6(B) 6(C) 7(B) 7(C) 7(D)	7	
23	Solving Equations	Solve (a) single-step equations with one variable, (b) multi-step equations with one variable, and (c) equations with variables on both sides					9(A) 9(B) 9(C) 10(A) 10(B)	10(A) 10(B) 10(C) 11(A) 11(B) 11(C)	8(A) 8(B) 8(C) 9

Each of the 23 Modules includes the following components:

1. Routines
2. Vocabulary Cards
3. Problem Sets

Section IV describes the Routines, Vocabulary Cards, and Problem Sets in detail.

III. Effective Mathematics Teaching

As with all mathematics teaching, when implementing the Modules, educators need to deliver effective mathematics instruction. Effective mathematics instruction includes (but is not limited to) the following 3 strategies, which are described in detail in this section:

- Explicit Instruction
- Formal Mathematics Language
- Multiple Representations

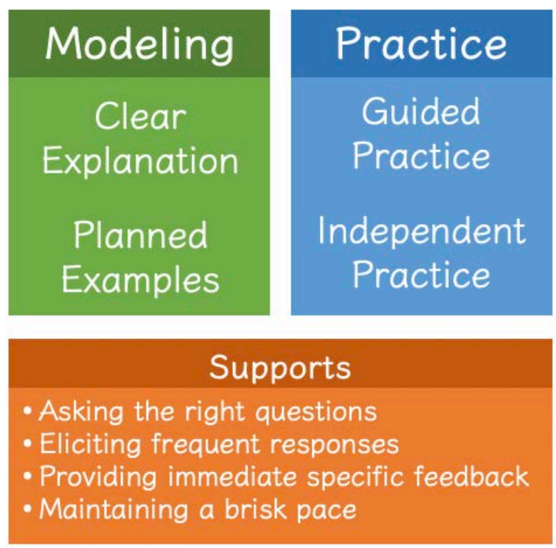
Explicit Instruction

Explicit instruction is defined as "a way of teaching where the educator selects an important objective, specifies the learning outcome, designs structured instructional experiences, explains directly, models the skills being taught, and provides scaffolded practice to help a student achieve mastery" (Kearns, 2018).

Explicit instruction is an evidence-based practice that benefits all students, particularly those identified with disabilities and learning difficulties. For this reason, explicit instruction should be an integral component of any lesson.

The primary components of explicit instruction are:

- **Modeling**: facilitated by the educator.
- **Practice:** involves the student and educator.
- **Supports:** involves an ongoing dialogue between the student and educator. Supports are used <u>during</u> modeling *and* <u>during</u> practice.



Modeling
Modeling prepares students to complete a mathematics skill successfully. Modeling includes two primary components: Clear Explanations and Planned Examples.

Clear Explanations

- Provide a short statement about the lesson's goals and importance.
- Explicitly model the steps for completing the task or solving the problem.
- Incorporate vocabulary and concise mathematics language (see next section).
- Pre-select examples depending on students' exposure to the content.
- Adjust modeling based on students' needs.

Sample Modeling from Module 1: Place Value	
Clear Explanations	**Dialogue**
Goal and importance	*Let's work on composing and decomposing numbers. Composing means to make numbers. What does composing mean?* *Today, we'll compose numbers with these Base-10 blocks.*
Explicitly model steps with concise mathematics language *(Note: **bolded words** represent concise mathematics language)*	*When we read numbers, we read numbers by **period**. A period is each group of digits separated by a comma or the **decimal point**.* *Our common **periods** include the **millions, thousands, ones,** then **thousandths**. What are the common **periods**?* *Let's write __ in **expanded notation**.* *Let's start with the **greatest place value**. What's the **greatest place value** in this number?*

Planned Examples

- Plan examples in a purposeful way prior to the lesson.
- Ask important questions.
- Vary examples: include worked examples of problems solved correctly *and* problems solved incorrectly, non-examples, and open-ended examples.

For planned examples, educators should consider which problems to include from the Problem Sets provided in each Module. Educators also need to create worked examples that are most appropriate for their students. Educators should plan ahead of each lesson and consider a variety of planned examples.

Practice

Practice is intended to provide multiple opportunities for students to *practice* the learned mathematics concepts. Students with disabilities and learning difficulties require additional practice to master new concepts and skills. To ensure students receive sufficient learning opportunities, both Guided Practice and Independent Practice should be included in every lesson.

Guided Practice

- Consists of the educator and students working together to solve problems.
- Can take place at a group table, with the educator and students working together.
- Can take place with the educator at the whiteboard and students at their desks.
- Provides supports to promote understanding and to encourage students' success.
- Includes the use of questioning and mathematics tools (e.g., manipulatives).
- Provides a scaffolded release of responsibility from modeling to independent practice.

Independent Practice

- Consists of students working independently under the guidance of the educator.
- Allows for the educator to provide feedback and answer questions.
- Provides a way to monitor the level of support needed for students to understand.
- Should not be reserved only for homework.

Supports

Students should actively participate in the lessons through supports. During modeling *and* practice, educators should attend to the following four supports, which should be included in every lesson:

Ask High-level and Low-Level Questions

- Ask a combination of high-level and low-level questions to evaluate students' understanding.
- Promote conceptual understanding and reasoning with high-level questions.
- Check for procedural understanding and increase participation with low-level questions.
- Ask a question every 30-60 seconds during modeling to promote active engagement.
- Examples:
  - *How could you explain dividing to a friend?* (high-level)
  - *What is a quotient?* (low-level)

Elicit Frequent Responses

- Engage students frequently by eliciting responses every 30-60 seconds during modeling.
- Provide a variety of ways for students to respond
  - Orally, in writing, chorally, pictures, whiteboard, gesturing, etc.

Provide Immediate Affirmative and Corrective Feedback

- Foster confidence to encourage students with low self-esteem and anxiety.
- Provide feedback immediately and as often as possible.
- Make affirmative feedback specific and related to the mathematics concept.
- Use questioning and encouragement when providing corrective feedback.
- Examples:
  - *I noticed you are using the fraction tiles to demonstrate three-fifths.* (affirmative)
  - *Can you tell your neighbor how you solved the problem?* (corrective)
  - *Can you explain the steps you followed to solve this expression?* (corrective)

Maintain a Brisk Pace

- Plan and organize prior to the lesson.
- Consider any needed materials and technology prior to the lesson.
- Consider any planned examples, including worked examples and non-examples.
- Consider which seating charts and/or student groupings will optimize learning.
- Be knowledgeable about the material and prepared to demonstrate effective modeling.

How Do I Use Explicit Instruction with the Modules?

The components of modeling are provided in the Routines for each Module. When planning for these lessons, you should consider additional examples such as worked examples with problems solved correctly and incorrectly and non-examples. Guided practice can be "modeled" from Routines using the Vocabulary Cards and Problem Sets. Educators should think about the independent practice experiences that will optimize learning for their students. Remember, independent practice should provide an opportunity for students to practice the learned skills independently under the guidance of the educator. Independent practice should not be reserved for homework.

As you plan to incorporate explicit instruction into your teaching of the Modules, consider using the explicit instruction framework from the National Center for Intensive Intervention, displayed below.

MODELING	PRACTICE
Clear explanation	Guided
Planned examples	Independent

Supporting Practices
Asking the right questions
Eliciting frequent responses
Providing immediate specific feedback
Maintaining a brisk pace

Formal Mathematics Language

As educators use explicit instruction, it is important to focus on formal mathematics language. Formal mathematics language refers to the precise mathematics terms used to describe mathematics concepts and procedures.

Examples of formal mathematics vocabulary terms include *product, angle,* and *denominator.* In contrast, informal mathematics language consists of words like *answer, corner,* and *bottom number in the fraction.*

Students are responsible for a tremendous amount of mathematics language at each grade level. At grade 3, students are exposed to over 300 different mathematics terms in their mathematics textbook glossaries. By grade 6, that number grows to over 500 terms. Therefore, it is necessary to have an explicit focus on the language of mathematics.

In addition to the sheer number of terms, the mathematics language often is complex. Mathematics terms are challenging for students, especially those experiencing learning difficulties because of:

- Technical terms that students have never seen (e.g., *perimeter*)
- Multiple meanings in mathematics and English (e.g., *degree*)
- Multiple meanings in mathematics (e.g., *quarter*)
- Multiple meanings across context areas (e.g., *base* in science vs. *base* in math)
- Vocabulary terms with multiple words (e.g., *rectangular prism*)
- Homonyms (e.g., *have* and *half)*
- Similarities to or differences from native language words (e.g. *quarter* vs. *cuarto*)

To promote students' understanding of formal mathematics language, educators should:

1. Use Formal Mathematics Vocabulary Terms During Instruction
   - Use formal mathematics vocabulary terms over informal phrases during every lesson.
   - Align the terms used with those presented in textbooks, videos, and on assessments.
   - Frequently expose students to formal terms in preparation for activities and tests.
   - Explicitly teach mathematics vocabulary terms to ensure students understand.
   - Examples:
     - **Module 2: Comparison**: say *greater than* instead of *bigger.*
     - **Module 6: Addition of Rational Numbers**: say *sum* instead of *answer.*
     - **Module 22: Representing Expressions and Equations**: say *variable* instead of *x.*

2. Use Similar and Related Terms Correctly and Precisely
   - Be correct, precise, and specific when using closely related mathematics terms.
   - Reflect on which formal vocabulary terms to explicitly teach to students.
   - Select terms that directly align with students' language skills, knowledge, and familiarity with the mathematics content.
   - Explicitly teach vocabulary words with a specific mathematics meaning.

3. Plan for Language Use Prior to Instruction
   - Consider language use (as well as students' language use) prior to instruction.
   - Avoid using limited or informal language that does not prepare students for success.
   - Present formal mathematics language from textbooks, assessments, and videos during instruction to support students' long-term learning.

4. Include Explicit Vocabulary Activities in Instruction
   - Directly teach the Vocabulary Cards from each Module to students.
   - Provide meaningful practice opportunities for students to use the Vocabulary Cards.
   - Include vocabulary activities to ensure students actively practice using terms.
   - Consider concept maps, word walls, and student dictionaries of mathematics terms.
   - Use mnemonic devices to access students' prior knowledge.
   - Offer multiple exposures of mathematics terms to build fluency.
   - Consider games to increase students' motivation.

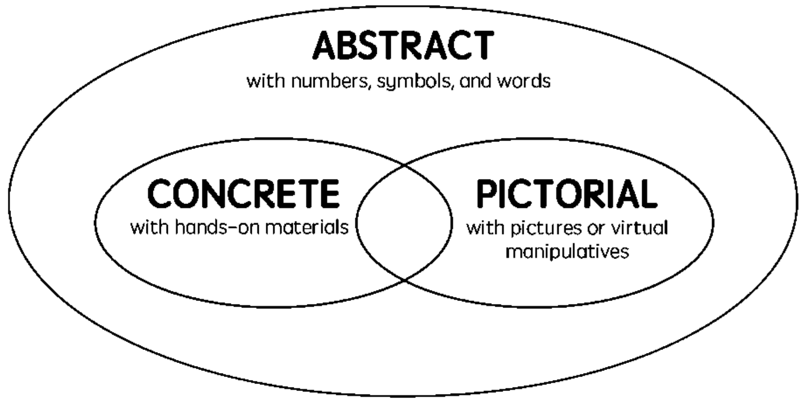
5. Hold Students Accountable
   - Provide opportunities for students to listen to and read formal mathematics language.
   - Create experiences for students to speak and write using formal mathematics language.
   - Focus on using formal language to describe mathematics concepts and procedures.

How Do I Use Formal Mathematics Language with the Modules?
The table below offers a few examples of how educators can translate their informal mathematics language to formal and precise mathematics language when teaching the Modules related to fractions. "*Say this*" refers to the formal mathematics language educators should incorporate into lessons. "*Instead of that*" refers to the informal mathematics language that does not help students to develop a conceptual understanding of the mathematics content. As you teach each Module, use formal mathematics language whenever possible.

“Say this”	**“Instead of that”**
Numerator and denominator	*Top number and bottom number*
Find an equivalent fraction	*Reduce the fraction*
Demonstrate process within Base-10	*Move the decimal point over*
Two-thirds	*2 over 3*
This fraction is a number	*Numbers in the fraction*
Three and four tenths	*Three point four*

Multiple Representations

In addition to using explicit instruction and formal mathematics language, educators should incorporate multiple representations into the mathematics Modules to support students with learning difficulties.



In this User Guide, multiple representations includes the abstract, concrete, and pictorial forms of mathematics. The goal is to utilize the abstract, concrete, and/or pictorial supports as necessary to promote students' deeper understanding of the mathematics concepts and procedures. Some students may need extra practice using the concrete forms; others may require additional pictorial supports to access the abstract. Ultimately, students with and without learning difficulties benefit from using a combination of these three supports. The three primary components of multiple representations include:

Abstract

- Consists of numbers, symbols, and words.
- Reflects the typical view of mathematics (e.g., 42 + 102 = 144).
- Often requires the concrete and pictorial to support students' understanding.

Concrete

- Refers to three-dimensional, hands-on materials and objects that students can touch.
- Includes hands-on formal manipulatives like fraction bars, algebra tiles, geoboards, etc.
- Includes hands-on manipulatives that are less formal (e.g., straws, paper clips).

Pictorial/Virtual

- Includes two-dimensional pictures, images, or virtual manipulatives.
- Often refers to the semi-concrete or representational.
- Includes visuals within textbooks or workbooks, in educator and student drawings, and on high-stakes standardized assessments.
- Includes graphic organizers that help students understand mathematics concepts.
- Includes the use of virtual manipulatives.

How Do I Use Multiple Representations with the Modules?

Abstract and pictorial representations are included throughout the Routines and Vocabulary Cards. The materials section of each Routine provides suggested concrete manipulatives to support students' understanding of the concepts and procedures (see below). Supplement the lessons with additional concrete, pictorial/virtual, and abstract representations based on the specific needs of your students.

Here are examples of multiple representations embedded within the Routines.

(2) Representing Expressions

Routine

Materials:

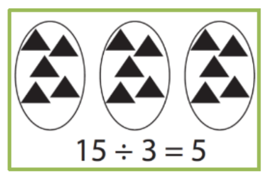
- ☐ Module 22 Problem Sets
- ☐ Module 22 Vocabulary Cards
  - o If necessary, review Vocabulary Cards before teaching
- ☐ A manipulative like algebra tiles

(1) Division as Partitive

Routine

Materials:

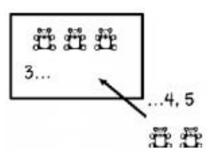
- ☐ Module 13 Problems
- ☐ Module 13 Vocabulary Cards
  - o If necessary, review Vocabulary Cards before teaching
- ☐ Any hands-on tool or manipulative (e.g., cubes, clips) and any container (e.g., plates, cups)



Here are examples of multiple representations embedded within the Vocabulary Cards.

join

To add to an existing set.

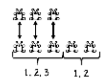


subtract/subtraction

To compare two sets or to take away from a set.

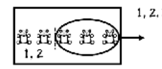
To compare two sets

5 – 3 = 2



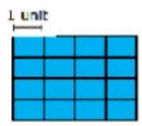
To take away from a set

5 – 3 = 2



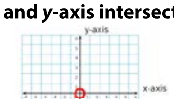
area

The number of square units that covers a closed figure.



origin

A point where the *x*-axis and *y*-axis intersect. The origin has the coordinates (0, 0).



number line

A straight line with numbers placed at equal intervals along its length.

IV. How to use the Routines, Vocabulary Cards, and Problem Sets

Each Module includes:

- Routines
- Problem Sets
- Vocabulary Cards

Routines

At the beginning of each Routine, you will see *Section A: Important Vocabulary with Definitions. Section A* highlights the mathematics vocabulary that will be explicitly used in the Routine and/or the vocabulary to review before using the Routine.

A. Important Vocabulary with Definitions

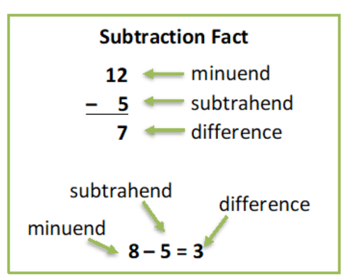
Term	Definition
compare	To find the difference between two sets.
difference	The result of subtracting one number from another number.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
minuend	The number from which another number is subtracted.
minus sign	The symbol that tells you to subtract.
separate	To start with a set and take away from that set.
subtract/subtraction	To compare two sets or to separate from a set.
subtrahend	The number to be subtracted.

After *Section A*, you will see *Section B: Background Information. Section B* provides brief background information on the mathematics content of the Routine. Often important vocabulary or the sequence of learning the mathematics content is described. The information in this section is for educators, but can be shared with students.

B. Background Information

Students need to learn two concepts of subtraction: (1) subtraction as separating from a set and (2) subtraction as comparison for a difference. Typically, students first learn about subtraction as separating from a set. Then, students learn about comparing two sets for a difference.

For learning the concepts of subtraction, we recommend using *mathematics facts*. We define a subtraction mathematics fact as a single- or double-digit minuend less than 19 and a single-digit subtrahend. The subtrahend is subtracted from the minuend for a difference. You may present subtraction facts vertically or horizontally.



The main portion of each Routine is *Section C: Routines and Examples.* In each Module, there are several Routines. As an example, the picture below shows the Routine for *Subtraction as Separating*. This include a list of materials necessary for the Routine. In some cases hands-on tools or manipulatives that can be used along with the Routine. Remember that virtual manipulatives may be used as well.

Following the materials, is a description of how to teach a specific skill. This is the **Routine.** Each Routine outlines educator dialogue (**in bold)** and planned student responses (unbolded). Teachers are not required to read the Routines verbatim. Instead, read the Routine before teaching to become familiar with the content and its delivery.

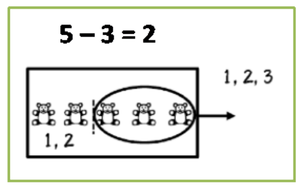
C. Routines and Examples

(1) Subtraction as Separating

Routine

Materials:

- Module 7 Subtraction Problems
- Module 7 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., clips, cubes, dinosaurs)



Teacher	**Let's work on subtraction. Today, let's think about subtraction as separating. What does it mean to separate?**
Students	To take some away.

After most Routines, there is an **Example.** The Example shows how to use the Routine with a specific mathematics problem.

Example

$$\begin{array}{r} 10 \\ -\ 6 \\ \hline 4 \end{array}$$

Teacher	**Let's work on subtraction. Today, let's think about subtraction as separating. What does it mean to separate?**
Students	To take away from a set.
Teacher	**When we separate, we take some away from a set. Let's think about separating numbers. Look at this problem.** (Show problem.)
Teacher	**First, I notice a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.

Following all the Routines and Examples, there is a reference to *Section D: Problems for Use During Instruction* and *Section E: Vocabulary Cards for Use During Instruction.*

D. Problems for Use During Instruction

See Module 7 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 7 Vocabulary Cards.

Several Modules include *Section F: Supplementary Materials.* Below is an example of a Counting Poster used to accompany a subtraction Routine.

COUNTING UP
Subtraction

1. Put the subtrahend in your fist and say it.
2. Count up your fingers to the minuend.
3. The difference is the number of fingers you have up.

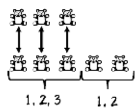
Vocabulary Cards

Vocabulary Cards are available for each of the vocabulary terms listed at the beginning of a Routine. Educators may choose to place these Vocabulary Cards on a mathematics word wall or instruct students to add the Vocabulary Cards to their mathematics glossaries or journals. The example below shows the Vocabulary Cards for the terms *compare* and *difference.*

compare

To find the difference between two sets.

5 – 3 = 2



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difference

The result of subtracting one number from another number.

6 – 4 = 2

2 is the difference

Problem Sets

Problems to accompany each Module's Routine. Problems are purposefully placed one per page. Educators may choose to show these problems on their screen or document camera. Educators also can print out the Problem Sets for use during modeling, guided practice, and independent practice.

$$\begin{array}{r} 9 \\ -\quad 4 \\ \hline \end{array}$$

Many of the Problem Sets have a variety of choices (see example below). The number in parentheses after the description indicates the number of problems for that type. For example, there are 30 problems for Proper fractions. Educators do not need to use all of these Problem Sets. The Problem Sets provide variety and choice during instruction. Educators should view the Problem Sets in advance of the lesson to select the problems that are most appropriate for their students.

Module 15:
Division of
Rational Numbers

Problem Sets

A. Proper fractions (30)
B. Improper fractions (15)
C. Mixed numbers (15)

D. Decimals with tenths; no remainder (20)
E. Decimals with hundredths; no remainder (20)
F. Decimals with tenths and hundredths; no remainder (30)
G. Decimals with tenths and hundredths; remainder (10)

VI. Constructing a Lesson: An Example

This section provides an example of how to use a Module in mathematics intervention.

Module 12: Multiplication of Rational Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A set of steps to solve a problem.
decimal	A number based on powers of ten.
denominator	The term in a fraction that tells the number of equal parts in a whole.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
factor	A number that you multiply with another number to get the product.
fraction	A number representing part of a whole or set.
hundredths	The digit in representing $\frac{1}{100}$.
improper fraction	Any fraction in which the numerator is greater than the denominator.
mixed number	A whole number and a fraction combined.
multiply/multiplication	The process of adding a number to itself a number of times.
multiplication sign	The symbol that tells you to multiply.
numerator	The term in a fraction that tells how many parts of a fraction.
ones	The digit representing 1.
partial products	The product of parts of each factor.
product	The result of multiplying two or more factors.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
tenths	The digit in representing $\frac{1}{10}$.

Module 12 is about multiplication of rational numbers.

This is a list of important mathematics vocabulary for Module 12. Prior to the lesson, review the vocabulary and identify terms students may need to learn or practice.

mixed number

A whole number and a fraction combined.

$1\frac{1}{6}$ $4\frac{5}{12}$ $12\frac{4}{3}$

equivalent

Two numbers that have the same value.

$\frac{1}{4} = \frac{2}{8}$ $\frac{2}{3} = \frac{8}{12}$

Use the Vocabulary Cards to teach or review important vocabulary terms with students.

B. Background Information

Background Information:

In this module, we focus on multiplication with fractions and decimals. As you focus on computation of rational numbers, continue to emphasize multiplication as equal groups and multiplication as comparison because students will see these concepts within word problems.

For multiplication of fractions, we recommend using several models of fractions to help students understand concepts related to multiplication of fractions. We also recommend demonstrating several algorithms for multiplication of decimals. Every student should develop efficiency with strategies for multiplication of fractions and decimals. In the following sections, we provide examples of (1) multiplication of fractions, (2) multiplication of decimals with the traditional algorithm, and (3) multiplication of decimals with the partial products algorithm.

This is background information about the Module.

B. Background Information

Background Information:

In this module, we focus on multiplication with fractions and decimals. As you focus on computation of rational numbers, continue to emphasize multiplication as equal groups and multiplication as comparison because students will see these concepts within word problems.

For multiplication of fractions, we recommend using several models of fractions to help students understand concepts related to multiplication of fractions. We also recommend demonstrating several algorithms for multiplication of decimals. Every student should develop efficiency with strategies for multiplication of fractions and decimals. In the following sections, we provide examples of (1) multiplication of fractions, (2) multiplication of decimals with the traditional algorithm, and (3) multiplication of decimals with the partial products algorithm.

This Module has 3 separate Routines:
(1) Multiplication of Fractions
(2) Multiplication of Decimals with Traditional Algorithm
(3) Multiplication of Decimals with Partial Products

ROUTINE WITH MANIPULATIVES
(Only use manipulatives with simpler problems)

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.

The Routine provides a description of how to use explicit instruction to model a mathematics skill.

Teacher	**Let's do this problem with fraction tiles.** (Move fraction tiles to workspace.)
Teacher	**With multiplication of fractions, we interpret this problem as** __ (first fraction) **of** __ (second fraction). **How do we interpret this problem?**
Students	__ of __.
Teacher	**We want to determine** __ (first fraction) **of** __ (second fraction). **If you wanted to determine half of 8, you would show 8 and then find half of that amount. The same works with fractions. We'll show the second fraction (or factor) and then find the first fraction of the second fraction. Which fraction will we show?**
Students	Second fraction.
Teacher	**So, let's show the second fraction with the fraction tiles.** (Show second fraction with fraction tiles.)
Teacher	**Now, let's find** __ (first fraction) **of** __ (second fraction). **There are several ways to do this, but an easy way is to find** __ (first fraction) **of each one-**__ (second fraction denominator) **part. Let's focus on one-**__ **part at a time. What should we focus on?**
Students	One-__ part.
Teacher	**Let's just think about this one-**__ **part** (second fraction denominator). **What's** __ (first fraction) **of this part?**
Students	__.
Teacher	**If that's hard to answer, think about it this way. What's** __ (first fraction) **times one-**__ (second fraction denominator)**?**
Students	__.
Teacher	__ (first fraction) **of this one-**__ **part** (second fraction denominator) **would be** __**. Let's place that/those fraction tiles on top of the one-**__ **part.** (Place fraction tiles.)
Teacher	**Now, I do that again for each one-**__ **part. I find** __ (first fraction) **of each one-**__ **part.** (Place fraction tiles.)
Teacher	**We're multiplying by finding** __ (first fraction) **of each of the one-**__ **parts. How are we multiplying?**
Students	Finding __ (first fraction) of each of the one-__ parts.
Teacher	**We've determined** __ (first fraction) **of each of the one-**__ **parts with the fraction tiles, these are our partial products. What are these?**
Students	Partial products.
Teacher	**Let's add the partial products to determine the final product. What should we add?**
Students	The partial products.
Teacher	**We have** __ **plus** __ **plus … . That equals** __**. Say that with me.**
Students	__.
Teacher	**So,** __ (first fraction) **of** __ (second fraction) **equals** __**. What's the product?**

The **bolded text** is for the educator. The planned students responses are un-bolded.

As you use the Routine, make sure you are (a) asking the right questions, (b) eliciting student responses frequently, and (c) providing immediate, specific feedback. The Routine includes suggested questions. Teachers may go beyond the written Routine based on intervention with students.

The blank (___) indicates when to fill in information about a specific problem.

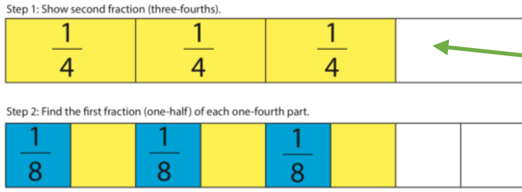
Example

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

This Routine is followed by an Example.

Step 1: Show second fraction (three-fourths).

Step 2: Find the first fraction (one-half) of each one-fourth part.



Example snapshot of the manipulatives used to solve this specific problem.

EXAMPLE WITH MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with fraction tiles.** (Move fraction tiles to workspace.)
Teacher	**With multiplication of fractions, we interpret this problem as $\frac{1}{2}$ of $\frac{3}{4}$. How do we interpret this problem?**
Students	$\frac{1}{2}$ of $\frac{3}{4}$.
Teacher	**Because we want to determine one-half of three-fourths, we show $\frac{3}{4}$. What fraction do we show?** (Show 3 one-fourth parts compared to a whole.)
Students	$\frac{3}{4}$.

A. $\frac{3}{4} \times \frac{2}{3} =$

A. $\frac{1}{2} \times \frac{1}{10} =$

When teaching, choose problems from the Problem Sets.

The Problem Sets can be used during modeling, guided practice, and independent practice by:

- showing the Problem Sets on a document camera or tablet
- printing the problems on paper, or
- using student transcription to a whiteboard.

VII. Glossary of Vocabulary Terms

Vocabulary Term	Definition	In Which Module(s)?
absolute value	The distance of a number from 0 on a number line.	17, 18, 19
add/addition	To put amounts together to find the sum or to increase a set.	4, 5, 6
addend	Any numbers that are added together.	4, 5, 6, 18
algorithm	A procedure or description of steps that can be used to solve a problem.	5, 6, 8, 9, 11, 12, 14, 15
area	The number of square units that covers a closed figure.	10, 11
array	A set of objects, pictures, or numbers arranged in columns and rows.	10, 11
base	A number that is multiplied by an exponent.	22, 23
coefficient	A number that is multiplied by a variable.	21, 22, 23
commutative property (of multiplication)	Two factors can be multiplied in any order.	11
compare (comparison)	To examine differences between numbers, quantities, or values to decide if one quantity is greater than, less than, or equal to another quantity.	2
compare (subtraction)	To find the difference between two sets.	7, 8, 9
compose	To make a number.	1
computation	The action used to solve a problem.	5, 6, 8, 9, 11, 14, 15
constant	A term that does not change; a number on its own.	21, 22, 23
coordinate plane	A two-dimensional plane formed at the intersection of the *x*-axis and *y*-axis.	20
decimal	A number based on powers of ten.	1, 6, 9, 12, 15, 16
decimal point	A dot used to separate ones from tenths in a number or dollars from cents.	1, 16
decompose	To break apart by place value.	1
denominator	The term in a fraction that tells the number of equal parts in a whole.	2, 3, 6, 9, 12, 15, 21
difference	The result of subtracting one number from another number.	7, 8, 9, 18
digit	A symbol used to show numbers.	1, 2
divide/division	To separate into equal groups or among groups.	13, 14, 15, 19
dividend	The number to be divided.	13, 14, 15, 19
division sign	The symbol that tells you to divide.	13, 14, 15
divisor	The number the dividend is divided by.	13, 14, 15, 19
equal	When the number, quantity, or value on the left side of the equal sign is the same as the number, quantity, or value on the right side of the equal sign.	2
equal groups	Groups with the same number of objects or items in each group.	10, 11, 12, 13, 14, 15
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.	2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 21

Vocabulary Term	Definition	In Which Module(s)?
equation	A mathematical statement that two expressions are the same or equal; must have an equal sign.	20, 22, 23
equivalent	Two numbers that have the same value.	2, 3, 6, 9, 12
equivalent fractions	Fractions that have different numerators and denominators that represent the same value or proportion of the whole.	21
equivalent ratios	Ratios that have the same fractional number, value, or measure.	21
estimate	To give an approximate value rather than an exact answer.	1
expanded form	Writing a number to show the place value of each digit.	1
exponent	The power to which a number is raised.	22, 23
expression	A combination of variables, numbers, and/or operations that represents a mathematical relationship; does not have an equal sign.	20, 22, 23
factor	A number you multiply with another number to get the product.	10, 11, 12, 19
fraction	A number representing part of a whole or set.	2, 3, 6, 9, 12, 15, 21
function	A relationship between two quantities in which every input corresponds to exactly one output.	20
function table	A table that displays a set of inputs and outputs in such a way that each input has a unique output.	20
greater than	When the number, quantity, or value on one side of the equal sign is larger than the number, quantity, or value on the other side of the equal sign.	2
grouping	A combination of variables, numbers, and/or operations grouped together in parentheses or brackets.	22, 23
hundred thousands	The digit representing 100,000.	1
hundreds	The digit representing 100.	1, 2, 16
hundreds column	The column with digits in the hundreds place.	5, 8, 11, 14
hundredths	The digit in representing $\frac{1}{100}$.	1, 6, 9, 12, 15, 16
improper fraction	Any fraction in which the numerator is greater than or equal to the denominator.	3, 6, 9, 12, 15, 21
inequality	An algebraic relation showing that a quantity is greater or less than another quantity.	22, 23
input variable	The *x* of an equation; the information put in to find the output.	20
integer	A positive or negative whole number.	17, 18, 19
join	To add to an existing set.	4, 5, 6
least common multiple	The common multiple with the least value.	6, 9, 12, 15, 21
less than	When the number, quantity, or value on one side of the equal sign is smaller than the number, quantity, or value on the other side of the equal sign.	2

Vocabulary Term	Definition	In Which Module(s)?
like fractions	Fractions that have the same denominator.	21
like terms	Terms that have the same variable or constant and can be combined.	22, 23
lowest terms	A fraction is reduced to lowest terms when there is no number other than 1 that will evenly divide into both the numerator and denominator.	21
minuend	The number from which another number is subtracted.	7, 8, 9, 18
minus sign	The symbol that tells you to subtract.	7, 8, 9
mixed number	A whole number and a fraction combined.	3, 6, 9, 12, 15, 21
multiple	The product of a number and any integer.	6, 9, 21
multiplication sign	The symbol that tells you to multiply.	10, 11, 12
multiply/multiplication	The process of adding a number to itself a number of times.	10, 11, 12, 19
negative number	Any number less than 0.	17, 18, 19
number line	A straight line with numbers placed at equal intervals along its length.	2, 17, 18, 19
numerator	The term in a fraction that tells how many parts of a fraction.	2, 3, 6, 9, 12, 15, 21
ones	The digit representing 1.	1, 2, 6, 9, 12, 15, 16
ones column	The column with digits in the ones place.	5, 8, 11, 14
operator	A symbol (+, –, × ÷) that represents a mathematical operation.	22, 23
opposites	Two numbers that are equal distance from 0 on a number line.	17, 18, 19
ordered pair	A pair of numbers used to locate a point on a coordinate plane.	20
origin	A point where the *x*-axis and *y*-axis intersect. The origin has the coordinates (0, 0).	20
output variable	The *y* of an equation; the information gained after the input is plugged into an equation.	20
partial products	The product of parts of each factor.	10, 11, 12
partitive division	To share equally among groups.	13
percentage	A rate of an amount per hundred.	21
period	A group of three digits with each group separated by a comma.	1
place value	The value of a digit depending on its place in a number.	1, 2, 16
plus sign	The symbol that tells you to add.	4, 5, 6
positive number	Any number greater than 0.	17, 18, 19
product	The result of multiplying two or more factors.	10, 11, 12, 19
proper fraction	A fraction where the numerator is less than the denominator.	3, 21
proportion	An equation that states that two ratios are equal.	21

Vocabulary Term	Definition	In Which Module(s)?
quadrant	The *x*- and *y*-axes divide the coordinate plane into four regions called quadrants.	20
quotative division	To measure objects into groups.	13
quotient	The result when one number is divided by another number.	13, 14, 15, 19
rate	A comparison of two quantities that have different units of measure.	21
ratio	A comparison of two quantities that have the same unit of measure.	21
rational number	Any number that can be written as a fraction.	2
reciprocal	The reciprocal of a number is 1 divided by that number.	15
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.	5, 6, 8, 9, 11, 12, 14, 15
remainder	The amount remaining in a division problem.	14, 15
rounding	A process that tells which place value a number is closest to.	1
separate	To start with a set and take away from that set.	7, 8, 9
standard form	A way to write numbers using digits.	1
subtract/subtraction	To compare two sets or to take away from a set.	7, 8, 9
subtrahend	The number to be subtracted.	7, 8, 9, 18
sum	The result of adding two or more numbers or the total number when you combine sets.	4, 5, 6, 18
ten thousands	The digit representing 10,000.	1
tens	The digit representing 10.	1, 2, 16
tens column	The column with digits in the tens place.	5, 8, 11, 14
tenths	The digit in representing $\frac{1}{10}$.	1, 6, 9, 12, 15, 16
term	A single number or a variable, or numbers or variables multiplied together.	22, 23
thousands	The digit representing 1,000.	1, 2, 16
thousandths	The digit in representing $\frac{1}{1000}$.	1, 16
together	To combine sets or numbers.	4, 5, 6
unit rate	A ratio that is written as a number to one.	21
variable	A symbol for an unknown value, which is usually represented by a letter.	22, 23
word form	The form of a number that uses written words.	1
x-axis	The horizontal number line on a coordinate plane.	20
y-axis	The vertical number line on a coordinate plane.	20

Instructional Routines for Mathematics Intervention

MODULE 1

Place Value

Module 1: Place Value

Mathematics Routines

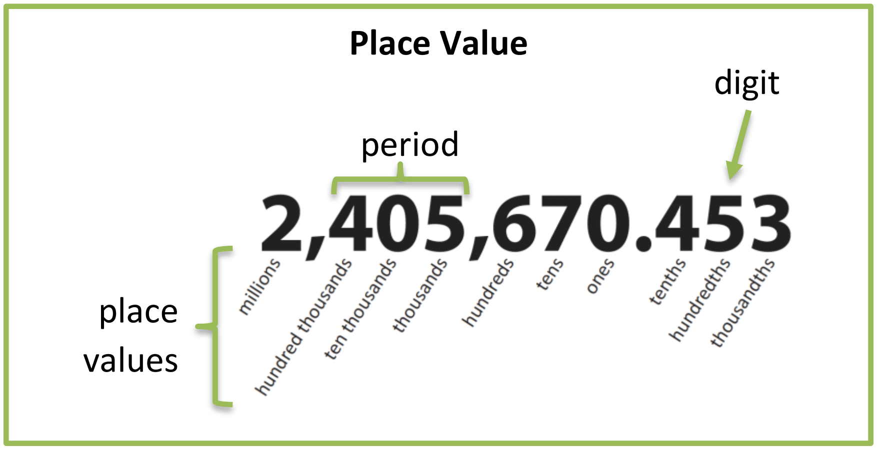
A. Important Vocabulary with Definitions

Term	Definition
compose	To make a number.
decimal	A number based on powers of ten.
decimal point	A dot used to separate ones from tenths in a number or dollars from cents.
decompose	To break apart by place value.
digit	A symbol used to show numbers.
estimate	To give an approximate value rather than an exact answer.
expanded form	Writing a number to show the place value of each digit.
hundreds	The digit representing 100.
hundredths	The digit in representing $\frac{1}{100}$.
hundred thousands	The digit representing 100,000.
ones	The digit representing 1.
period	A group of three digits with each group separated by a comma.
place value	The value of a digit depending on its place in a number.
rounding	A process that tells which place value a number is closest to.
standard form	A way to write numbers using digits.
tens	The digit representing 10.
tenths	The digit in representing $\frac{1}{10}$.
ten thousands	The digit representing 10,000.
thousands	The digit representing 1,000.
thousandths	The digit in representing $\frac{1}{1000}$.
word form	The form of a number that uses written words.

B. Background Information

Place value is essential for understanding numbers. Typically, students first learn about place value with tens and ones by (1) composing and decomposing numbers. Then, students learn about hundreds and thousands and (2) expanded notation. As students learn about rational numbers, they learn about tenths, hundredths, and thousandths. As students work on place value, students learn to (3) round numbers.

When teaching place value, emphasize the names of each place and the digit in each place. Also, practice reading larger numbers by place value.



C. Routines and Examples

(1) Composing and Decomposing Numbers

Routine

Materials:

- Module 1 Problems
- Module 1 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., clips, Base-10 blocks, blank place value mat)

Teacher	**Let's work on composing and decomposing numbers. Composing means to make numbers. What does composing mean?**
Students	To make numbers.
Teacher	**Today, we'll compose numbers with these Base-10 blocks.** (Show Base-10 blocks.)
Teacher	**With Base-10 blocks, one cube represents one thousand. What does a cube represent?**
Students	One thousand.
Teacher	**The flat represents one hundred. What does the flat represent?**
Students	One hundred.
Teacher	**The rod represents one ten. What does the rod represent?**

Students One ten.
Teacher And the unit represents one. What does the unit represent?
Students One.
Teacher Now, let's compose a number. Let's see, first I want __ hundreds. How many hundreds?
Students __.
(Show hundreds flats.)
Teacher And I want __ tens. How many tens?
Students __.
(Show tens rods.)
Teacher And I want __ ones. How many ones?
Students __.
(Show ones units.)
Teacher Now, we compose the number by combining the hundreds, tens, and ones. How do we compose?
Students We combine the hundreds, tens, and ones.
Teacher Let's determine the number we composed. Let's count from the greatest place value to the least place value. What's the greatest place value with our blocks?
Students __.
Teacher So, let's count the hundreds, then tens, then ones. Ready? __, __, __, … How many?
Students __.
(Write number.)
Teacher __ hundreds, __ tens, and __ ones is __. What is the number?
Students __.
Teacher Let's read the number together.
Students __.
Teacher Let's read it again.
Students __.
Teacher Now, let's work on decomposing numbers. That means we'll show a number and figure out how many hundreds, tens, and ones are in that number. We'll break apart the number by place value. What does decomposing mean?
Students Break apart by place value.
Teacher So, here's my number __ with blocks.
(Show blocks and write number.)
Teacher What is the number?
Students __.
Teacher Let's decompose. How many hundreds are in __?
Students __.
Teacher How many tens are in __?
Students __.
Teacher How many ones are in __?
Students __.

Teacher	**So, in __ there are __ hundreds** (point to hundreds digit)**, __ tens** (point to tens digit)**, and __ ones** (point to ones digit). **We just decomposed __. What number did we decompose?**
Students	__.
Teacher	**What does it mean to compose a number?**
Students	To make a number.
Teacher	**What does it mean to decompose a number?**
Students	To break apart by place value.

Example

2.56

Teacher	**Let's work on composing and decomposing numbers. Composing means to make numbers. What does composing mean?**
Students	To make numbers.
Teacher	**Today, we'll compose numbers with these Base-10 blocks.** (Show Base-10 blocks.)
Teacher	**We can use the Base-10 blocks in different ways. Today, with decimals, the one cube represents ten. What does a cube represent?**
Students	Ten.
Teacher	**The flat represents one. What does the flat represent?**
Students	One.
Teacher	**The rod represents one tenth. What does the rod represent?**
Students	Tenths.
Teacher	**And the unit represents hundredths. What does the unit represent?**
Students	Hundredths.
Teacher	**Now, let's compose a number. Let's see, first I want 2 ones. How many ones?**
Students	2. (Show 2 flats.)
Teacher	**And I want 5 tenths. How many tenths?**
Students	5 tenths. (Show 5 rods.)
Teacher	**And I want 6 hundredths. How many hundredths?**
Students	6 hundredths. (Show 6 units.)
Teacher	**Now, we compose the number by combining the ones, tenths, and hundredths. How do we compose?**
Students	We combine the ones, tenths, and hundredths.
Teacher	**Let's determine the number we composed. Let's count from the greatest place value to the least place value. What's the greatest place value with our blocks?**
Students	Ones.

Teacher	**So, let's count the ones, then tenths, then hundredths. Ready? 1, 2: 1 tenth, 2 tenths, 3 tenths, 4 tenths, 5 tenths; 51 hundredths, 52 hundredths, 53 hundredths, 54 hundredths, 55 hundredths, 56 hundredths. How many?**
Students	2 and 56 hundredths.
	(Write 2.56.)
Teacher	**2 ones, 5 tenths, 6 hundredths. What number?**
Students	2 and 56 hundredths.
Teacher	**Excellent. Remember, you say "and" anytime you see the decimal point. When do you say "and?"**
Students	When we see the decimal point.
Teacher	**Let's say that together!**
Students	2 and 56 hundredths.
Teacher	**Great! Now, let's work on decomposing numbers. That means we'll show a number and figure out how many hundreds, tens, and ones are in that number. We'll break apart the number by place value. What does decomposing mean?**
Students	Break apart by place value.
Teacher	**So, here's my number 2.56 with blocks.**
	(Show blocks and write 2.56.)
Teacher	**What number?**
Students	2 and 56 hundredths.
Teacher	**How many ones are in 2 and 56 hundredths?**
Students	2.
Teacher	**How many tenths are in 2 and 56 hundredths?**
Students	5.
Teacher	**How many hundredths are in 2 and 56 hundredths?**
Students	6.
Teacher	**So, in 2.56 there are 2 ones** (point to ones digit)**, 5 tenths** (point to tenths digit)**, and 6 hundredths** (point to hundredths digit). **We just decomposed 2.56. What number did we decompose?**
Students	2.56.
Teacher	**What does it mean to compose a number?**
Students	To make a number.
Teacher	**How does it mean to decompose a number?**
Students	To break apart by place value.

(2) Expanded Notation

Routine

Materials:

- Module 1 Problems
- Module 1 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching

Teacher	**Let's work on writing numbers in expanded notation. When we write a number in expanded notation, we write the number by place value. How do we write the number?**
Students	By place value.
Teacher	**Look at this number.** (Show number.)
Teacher	**When we read numbers, we read numbers by period. A period is each group of digits separated by a comma or the decimal point. What's a period?**
Students	Each group of digits separated by a comma.
Teacher	**Our common periods include the millions, thousands, ones, then thousandths. What are the common periods?**
Students	Million, thousands, ones, thousandths.
Teacher	**Let's read this number together.**
Students	__.
Teacher	**Let's write __ in expanded notation. Let's start with the greatest place value. What's the greatest place value in this number?**
Students	__.
Teacher	**So, what digit is in the thousands place?**
Students	__.
Teacher	**__ is the digit in the thousands place. That means we have __ thousand. How many?**
Students	_,000.
Teacher	**So, let's write __ thousand below our number.** (Write thousands.)
Teacher	**Now, what digit is in the hundreds place?**
Students	__.
Teacher	**__ is the digit in the hundreds place. That means we have __ hundred. How many?**
Students	__ hundred.
Teacher	**How do I write __ hundred?**
Students	_00.
Teacher	**Let's write __ hundred next to __ thousand. Because we're adding the hundreds to the thousands, I like to write a plus sign then the hundreds.** (Write + and hundreds.)
Teacher	**Now, what digit is in the tens place?**

Students	__.
Teacher	**__ is the digit in the tens place. That means we have __. How many?**
Students	__.
Teacher	**How do I write __?**
Students	_0.
Teacher	**Let's write __ next to __ hundred. Because we're adding the tens to the hundreds, I like to write a plus sign then the tens.** (Write + and tens.)
Teacher	**Now, what digit is in the ones place?**
Students	__.
Teacher	**__ is the digit in the ones place. That means we have __. How many?**
Students	__.
Teacher	**How do I write __?**
Students	_.
Teacher	**Let's write __ next to __. Because we're adding the ones to the tens, I like to write a plus sign then the ones.** (Write + and ones.)
Teacher	**We just wrote __ in expanded form. We wrote each digit by place value. So, __ is __ thousand, __ hundred, __, __. Read that with me.**
Students	__ thousand, __ hundred, __, __.
Teacher	**What does it mean to write a number in expanded form?**
Students	Write each digit by place value.

Example

105.7

Teacher	**Let's work on writing numbers in expanded notation. When we write a number in expanded notation, we write the number by place value. How do we write the number?**
Students	By place value.
Teacher	**Look at this number.** (Show number.)
Teacher	**Remember, you read numbers by period. What's a period?**
Students	Each group of digits separated by a comma.
Teacher	**You read numbers by period – millions, thousands, ones, then thousandths. What are our common periods?**
Students	Millions, thousands, ones, and thousandths.
Teacher	**Let's read this number together.**
Students	1 hundred five and 7 tenths.
Teacher	**Let's write 105.7 in expanded notation. Let's start with the greatest place value. What's the greatest place value in this number?**
Students	Hundreds.
Teacher	**So, what digit is in the hundreds place?**

Students	1.
Teacher	**1 is the digit in the hundreds place. That means we have 1 hundred. How many?**
Students	100.
Teacher	**So, let's write 100 below our number.** (Write 100.)
Teacher	**Now, what digit is in the tens place?**
Students	0.
Teacher	**0 is the digit in the tens place. That means we have 0 tens. How many?**
Students	0 tens.
Teacher	**Do I have to write anything if I have 0 tens?**
Students	No!
Teacher	**Now, what digit is in the ones place?**
Students	5.
Teacher	**5 is the digit in the ones place. That means we have 5. How many?**
Students	5.
Teacher	**Let's write 5 next to 100. Because we're adding the ones to the hundreds, I like to write a plus sign then the 5.** (Write + and 5.)
Teacher	**Now, what digit is in the tenths place?**
Students	7.
Teacher	**7 is the digit in the tenths place. That means we have 7 tenths. How many?**
Students	7 tenths.
Teacher	**How do I write 7 tenths?**
Students	0.7.
Teacher	**Let's write 0.7 next to 5. Because we're adding the tenths to the ones, I like to write a plus sign then the tenths.** (Write + and 0.7.)
Teacher	**We just wrote 105.7 in expanded form. We wrote each digit by place value. So, 105.7 is 100 plus 5 plus 0.7. Read that with me.**
Students	100 plus 5 plus 0.7.
Teacher	**What does it mean to write a number in expanded form?**
Students	Write each digit by place value.

(3) Rounding

Routine

Materials:

- Module 1 Problems
- Module 1 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A number line

Teacher	**Let's work on rounding numbers. When we round a number, we estimate the number to a specific place value. What does it mean to round?**
Students	To estimate to a specific place value.
Teacher	**Look at this number.** (Show number.)
Teacher	**When we read numbers, we read numbers by period. A period is each group of digits separated by a comma or the decimal point. What's a period?**
Students	A group of digits separated by a comma.
Teacher	**Our common periods include the millions, thousands, ones, then thousandths. What are the common periods?**
Students	Million, thousands, ones, thousandths.
Teacher	**Let's read this number together.**
Students	__.
Teacher	**Let's round this number to the nearest __. What place value will we round to?**
Students	Nearest __.
Teacher	**So, what digit is in the __ place?**
Students	__.
Teacher	**__ is the digit in the __ place. Let's use the number line to round __** (number) **to the nearest __. Look at this number line.** (Draw open number line.)
Teacher	**In this problem, we'll round to the nearest __. So, I'll write __** (number rounded to lower bound) **on the left side of the number line.** (Write.)
Teacher	**What number?**
Students	__.
Teacher	**Now, what's one more __** (thousand/hundred/ten/one/tenth) **from __** (number rounded to lower bound)?
Students	__.
Teacher	**So, on this side of the number line, I'll write __** (number rounded to upper bound)**.** (Write.)
Teacher	**What number?**
Students	__.

Teacher **Now, what number is halfway between __** (lower bound) **and __** (upper bound)**?**
Let's place that number in the middle of our number line.

Students __.

Teacher **__ is half way between __** (lower bound) **and __** (upper bound)**. Let's write __ in the middle of our number line.**
(Write.)

Teacher **Now, to round, let's determine whether our original number – __ – is closer to __** (lower bound) **or __** (upper bound)**. Look at the number line. What do you think?**

Students Closer to __.

Teacher **Why do you think __ is closer to __?**

Students Because it falls on the number line closer to __ than __.

Teacher **So, what's __ rounded to the nearest __?**

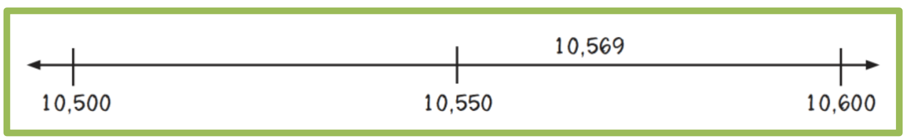
Students __.

Teacher **__ is closer to __ than __. What does it mean to round a number?**

Students To estimate a number to a specific place value.

Example

10,569



Teacher **Let's work on rounding numbers. When we round a number, we estimate the number to a specific place value. What does it mean to round?**

Students To estimate to a specific place value.

Teacher **Look at this number.**
(Show number.)

Teacher **When we read numbers, we read numbers by period. A period is each group of digits separated by a comma or the decimal point. What's a period?**

Students A group of digits separated by a comma.

Teacher **Our common periods include the millions, thousands, ones, then thousandths. What are the common periods?**

Students Million, thousands, ones, thousandths.

Teacher **Let's read this number together.**

Students Ten thousand, five hundred sixty-nine.

Teacher **Let's round this number to the nearest hundred. What place value will we round to?**

Students Nearest hundred.

Teacher **So, what digit is in the hundreds place?**

Students 5.

Teacher	**5 is the digit in the hundreds place. Let's use the number line to round 10,569 to the nearest hundred. Look at this number line.** (Draw open number line.)
Teacher	**We're rounding the nearest hundred. So, I'll write 10,500 on the left side of the number line.** (Write 10,500.)
Teacher	**What number?**
Students	10,500.
Teacher	**Now, what's one more hundred from 500?**
Students	600.
Teacher	**So, on this side of the number line, I'll write 10,600.** (Write 10,600.)
Teacher	**What number?**
Students	10,600.
Teacher	**Now, what number is halfway between 10,500 and 10,600? Let's place that number in the middle of our number line.**
Students	10,550.
Teacher	**10,550 is half way between 10,500 and 10,600. Let's write 10,550 in the middle of our number line.** (Write 10,550.)
Teacher	**Now, to round, let's determine whether our original number – 10,569 – is closer to 10,500 or 10,600. Look at the number line. What do you think?**
Students	Closer to 10,600.
Teacher	**Why do you think 10,569 is closer to 10,600?**
Students	Because it falls on the number line closer to 10,600 than 10,500.
Teacher	**So, what's 10,569 rounded to the nearest hundred?**
Students	10,600.
Teacher	**10,569 is closer to 10,600 than 10,500. What does it mean to round a number?**
Students	To estimate a number to a specific place value.

D. Problems for Use During Instruction

See Module 1 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 1 Vocabulary Cards.

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Module 1:
Place Value

Problem Sets

A. Two-digit numbers (20)
B. Three-digit numbers (20)
C. Four-digit numbers (20)
D. Five-digit numbers (20)
E. Six-digit numbers (20)

F. Decimals with tenths (20)
G. Decimals with hundredths (20)
H. Decimals thousandths (20)

A.

37

A.

42

A.

81

A.

70

44

A.

56

A.

87

A.

10

A.

24

A.

12

A.

28

A.

94

A.

76

A.

30

A.

55

A.

41

A.

60

A.

38

A.

14

A.

53

B.

502

B.

981

B.

487

B.

363

B.

674

B.

720

B.

199

B.

804

B.

347

B.

465

B.

173

B.

209

B.

733

B.

352

B.

166

B.

843

B.

489

B.

707

B.

813

B.

154

c.

5,644

c.

7,761

c.

8,451

c.

9,449

c.

4,758

c.

1,552

c.

6,651

c.

3,138

c.

1,593

c.

9,560

c.

2,577

c.

4,248

c.

3,839

c.

1,128

c.

9,292

c.

3,594

c.

5,653

c.

1,957

c.

8,451

c.

6,260

D.

34,906

D.

98,362

D.

10,785

D.

24,933

D.

80,824

D.

16,328

D.

78,995

D.

46,731

D.

15,673

D.

62,550

D.

29,632

D.

81,555

D.

67,839

D.

33,150

D.

50,107

D.

61,812

D.

75,134

D.

43,192

D.

64,389

D.

91,717

E.

213,593

E.

445,807

E.

145,993

E.

204,235

E.

334,459

E.

728,074

E.

251,401

E.

635,941

E.

431,583

E.

105,802

E.

527,653

E.

229,410

E.

872,543

E.

418,467

E.

103,941

E.

261,338

E.

734,904

E.

654,321

E.

240,920

E.

380,348

F.

0.4

2.7

F.

3.2

F.

0.5

6.8

1.9

F.

4.6

F.

5.1

F.

0.3

F.

511.4

F.

370.3

F.

92.7

F.

36.8

F.

2.5

F.

43.7

2.1

7.6

F.

85.4

F.

123.9

26.1

G.

0.32

G.

6.89

G.

10.41

G.

1.23

G.

4.06

G.

2.45

G.

34.52

G.

6.48

G.

78.07

G.

8.38

G.

54.61

G.

16.49

G.

66.21

G.

80.02

G.

5.48

G.

511.44

G.

370.32

G.

407.49

G.

8.52

G.

11.02

H.

67.213

H.

6.928

H.

4.506

1.748

H.

2.653

H.

7.206

H.

4.564

H.

2.642

H.

158.821

H.

627.011

H.

0.219

H.

6.995

74.153

H.

840.566

H.

2.234

H.

0.506

H.

561.244

5.198

H.

84.354

H.

932.237

Module 1:
Place Value

Vocabulary Cards

compose
decimal
decimal point
decompose
digit
estimate
expanded form
hundreds
hundredths
hundred thousands
ones
period
place value
rounding
standard form
tens
tenths
ten thousands
thousands
thousandths
word form

compose

To make a number.

4,000 + 300 + 80 + 5 = 4,385

---

decimal

A number based on powers of ten.

34.107

tens
ones
tenths
hundredths
thousandths

decimal point

A dot used to separate ones from tenths in a number or dollars from cents.

4.2

. is the decimal point

---

decompose

To break apart by place value.

4, 385 = 4,000 + 300 + 80 + 5

digit

A symbol used to show numbers.

0 1 2 3 4 5 6 7 8 9

---

estimate

To give an approximate value rather than an exact answer.

$$\begin{array}{r} 860 \\ \underline{+\ 120} \end{array} \longrightarrow \begin{array}{r} 900 \\ \underline{+\ 100} \\ 1{,}000 \end{array}$$

expanded form

Writing a number to show the place value of each digit.

9,217

Expanded form: 9,000 + 200 + 10 + 7

---

hundreds

The digit representing 100.

hundredths

The digit in representing $\frac{1}{100}$.

In the number 4.23, 3 is in the hundredths place.

---

hundred thousands

The digit representing 100,000.

ones

The digit representing 1.

---

period

A group of three digits with each group separated by a comma.

882,700

{8 8 2} , {7 0 0}

period period

place value

The value of a digit depending on its place in a number.

thousands	hundreds	tens	ones	.	tenths	hundredths	thousandths
8	7	6	5	.	4	3	2

---

rounding

A process that tells which place value a number is closest to.

Rounded to the nearest ten

73 → 70

76 → 80

standard form

A way to write numbers using digits.

9,217

---

tens

The digit representing 10.

tenths

The digit in representing $\frac{1}{10}$.

In the number 4.23, 2 is in the tenths place.

---

ten thousands

The digit representing 10,000.

thousands

The digit representing 1,000.

---

thousandths

The digit in representing $\frac{1}{1000}$.

In the number 4.238, 8 is in the thousandths place.

word form

The form of a number that uses written words.

9,217

Word form: Nine thousand, two hundred seventeen

Instructional Routines for Mathematics Intervention

MODULE 2

Comparison

Module 2: Comparison
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
compare	To examine differences between numbers, quantities, or values to decide if one quantity is greater than, less than, or equal to another quantity.
denominator	The term in a fraction that tells the number of equal parts in a whole.
digit	A symbol used to show numbers.
equal	When the number, quantity, or value on the left side of the equal sign is the same as the number, quantity, or value on the right side of the equal sign.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
greater than	When the number, quantity, or value on one side of the equal sign is larger than the number, quantity, or value on the other side of the equal sign.
hundreds	The digit representing 100.
less than	When the number, quantity, or value on one side of the equal sign is smaller than the number, quantity, or value on the other side of the equal sign.
number line	A straight line with numbers placed at equal intervals along its length.
numerator	The term in a fraction that tells how many parts in a fraction.
ones	The digit representing 1.
place value	The value of a digit depending on its place in a number.
rational number	Any number that can be written as a fraction.
tens	The digit representing 10.
thousands	The digit representing 1,000.

B. Background Information

Comparison is important for students to understand numbers as greater, less, or equal. Typically, students first learn to compare (1) whole numbers. Then, students learn to compare

(2) fractions and decimals. Decimals can be compared using the same strategy as comparing whole numbers, so we provide an overview of both in section (1).

When teaching about comparison, emphasize place value. Also, emphasize vocabulary related to comparison, such as *greater than, less than*, *equal to*, and *equivalent*, and the symbols representing this vocabulary.

Comparison

16 > 9	1.3 < 1.35	68 = 68
↑	↑	↑
greater than symbol	less than symbol	equal sign

C. Routines and Examples

(1) Comparing Whole Numbers and Decimals

Routine

Materials:

- Module 2 Problems
- Module 2 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., clips, Base-10 blocks)

Teacher	**Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?**
Students	To determine whether a number is greater than, less than, or equal to another number.
Teacher	**Today, we'll compare numbers with these Base-10 blocks.** (Show Base-10 blocks.)
Teacher	**With Base-10 blocks, one cube represents one thousand. What does a cube represent?**
Students	One thousand.
Teacher	**The flat represents one hundred. What does the flat represent?**
Students	One hundred.
Teacher	**The rod represents one ten. What does the rod represent?**

Students One ten.
Teacher And the unit represents one. What does the unit represent?
Students One.
Teacher Now, let's compare numbers. Let's compare __ and __. What numbers are we going to compare?
Students __ and __.
Teacher And for this comparison, we want to determine if __ (first number) **is greater than, less than, or equal to __** (second number). **What do we want to do?**
Students Determine if the first number is greater than, less than, or equal to the second number.
Teacher Now, let's compare numbers. Let's make the first number with the Base-10 blocks. How could I show __?
Students You could use __.
Teacher I'll show __ (first number) **by showing __.**
(Show using Base-10 blocks.)
Teacher Let's make the second number with Base-10 blocks. I'll place my blocks over here (on other side of workspace). **How could I show __?**
Students You could use __.
Teacher I'll show __ (second number) **by showing __.**
(Show using Base-10 blocks.)
Teacher Now, it's time to compare. Look at the greatest place value. What's the greatest place value?
Students __.
Teacher __ is the greatest place of __ (first number) **and __** (second number)**. Look at the first number, how many __** (greatest place value)**?**
Students __.
Teacher Look at the second number, how many __ (greatest place value)**?**
Students __.
Teacher Are the __ (greatest place value) **of the first number the same or different from __** (greatest place value) **of the second number?**
Students *OPTION 1:* The same!
OPTION 2: Different.
Teacher *OPTION 1:* When the greatest place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?
Students __.
Teacher That's right. The next greatest place value is the __ place. Look at the first number, how many __ (place value)**?**
Students __.
Teacher Look at the second number, how many __ (place value)**?**
Students __.
Teacher Are the __ (place value) **of the first number the same or different from __** (greatest place value) **of the second number?**
Students *OPTION 1:* The same!

	OPTION 2:	Different.
Teacher	***OPTION 1:***	**When the place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?**
Students		__.
Teacher		**That's right. The next greatest place value is the __ place. Look at the first number, how many __** (place value)**?**
Students		__.
Teacher		**Look at the second number, how many __** (place value)**?**
Students		__.
Teacher		**Are the __** (place value) **of the first number the same or different from __** (greatest place value) **of the second number?**
Students	*OPTION 1:*	The same!
	OPTION 2:	Different.
Teacher	***OPTION 2:***	**The __** (place value) **of the first number is different from the __** (place value) **of the second number. If the digits are different, then we can compare. What can we do?**
Students		Compare.
Teacher	**Is the __** (place value) **of the first number greater than, less than, or equal to that of the second number?**	
Students	__.	
Teacher	**If it's greater, that means __** (first number) **is greater than __** (second number)**. If it's less, that means __** (first number) **is less than __** (second number)**. If the numbers are the same, they are equal. What's the comparison?**	
Students	__ (greater/less/equal).	
Teacher	**That's right! __** (first number) **is __** (greater than/less that/equal to) **__** (second number). **Let's say that together.**	
Students	__ is greater than/less that/equal to __.	
Teacher	**Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?**	
Students	__.	
Teacher	**Let's write the symbol between the two numbers.** (Write.)	
Teacher	**What does it mean to compare numbers?**	
Students	We determine whether one number is greater than, less than, or equal to another number.	
Teacher	**How did we compare numbers in this example?**	
Students	We compared each digit by place value then determined whether one number was greater than, less than, or equal to the other number.	

Example

105.6 < 106.5

Teacher	**Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?**
Students	To determine whether a number is greater than, less than, or equal to another number.
Teacher	**Now, let's compare numbers. Let's compare 105.6 and 106.5. What numbers are we going to compare?**
Students	105.6 and 106.5.
Teacher	**And for this comparison, we want to determine if 105.6 is greater than, less than, or equal to 106.5. What do we want to do?**
Students	Determine if the first number is greater than, less than, or equal to the second number.
Teacher	**Let's compare. Look at the greatest place value of the numbers. What's the greatest place value?**
Students	Hundreds.
Teacher	**Hundreds is the greatest place value of the numbers 105.6 and 106.5. Look at the first number, how many hundreds?**
Students	1 hundred.
Teacher	**Look at the second number, how many hundreds?**
Students	1 hundred.
Teacher	**Are the hundreds of the first number the same or different compared to the hundreds of the second number?**
Students	Equal or the same.
Teacher	**When the greatest place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?**
Students	Tens.
Teacher	**That's right. The next greatest place value is the tens place. Look at the first number, how many tens?**
Students	0 tens.
Teacher	**Look at the second number, how many tens?**
Students	0 tens.
Teacher	**Are the tens of the first number the same or different compared to the tens of the second number?**
Students	Equal or the same.
Teacher	**When the place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?**
Students	Ones.
Teacher	**That's right. The next greatest place value is the ones place. Look at the first number, how many ones?**

Students 5 ones.
Teacher Look at the second number, how many ones?
Students 6 ones.
Teacher Are the ones of the first number the same or different compared to the ones of the second number?
Students Different.
Teacher The ones of the first number are different from the ones of the second number. If the digits are different, then we can compare. What can we do?
Students Compare.
Teacher Let's compare. Are the ones of the first number greater than, less than, or equal to that of the second number?
Students Less.
Teacher It's less so that means 105.6 is less than 106.5. What's the comparison?
Students Less than.
Teacher That's right! 105.6 is less than 106.5. Let's say that together.
Students 105.6 is less than 106.5.
Teacher Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?
Students Less than symbol.
Teacher Let's write the less than symbol between the two numbers.
(Write.)
Teacher Let's read it together.
Students 105.6 is less than 106.5.
Teacher What does it mean to compare numbers?
Students To determine whether one number is greater than, less than, or equal to another number.

(2) Comparing Fractions*

*For clarity, read **Example** before using **Routines**.

Routine

Materials:

- Module 2 Problems
- Module 2 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., fraction tiles, geoboards)

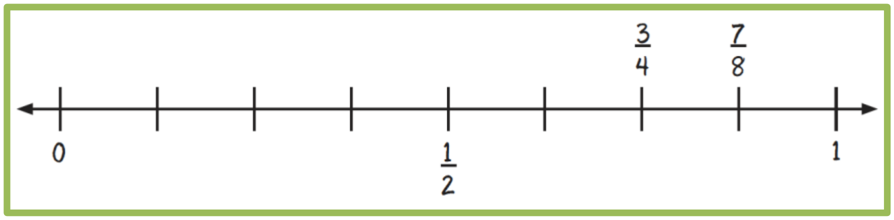
Teacher	**Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?**
Students	To determine whether one number is greater than, less than, or equal to another number.
Teacher	**Today, we'll compare numbers with these fraction tiles.** (Show fraction tiles.)
Teacher	**Now, let's compare numbers. Let's compare __ and __. What numbers are we going to compare?**
Students	__ and __.
Teacher	**And for this comparison, we want to determine if __** (first number) **is greater than, less than, or equal to __** (second number)**. What do we want to do?**
Students	Determine if the first number is greater than, less than, or equal to the second number.
Teacher	**Now, let's compare numbers. Let's make the first number with the fraction tiles. How could I show __?**
Students	You could use __.
Teacher	**I'll show __** (first number) **by showing __. Remember, I want to show the fraction compared to the whole.** (Show using fraction tiles.)
Teacher	**Let's make the second number with fraction tiles. I'll place my fraction tiles over here** (on other side of workspace). **How could I show __?**
Students	You could use __.
Teacher	**I'll show __** (second number) **by showing __. Remember, I want to show the fraction compared to the whole.** (Show using fraction tiles.)
Teacher	**Now, it's time to compare. What are we going to do?**
Students	Compare.
Teacher	**Let's think about the value of each fraction compared to the whole. Let's place both fractions on top of the whole to compare.**

	(Place fractions compared to whole.)	
Teacher	**Look at the first number, is this fraction less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?**	
Students	__.	
Teacher	**The first number is __ than $\frac{1}{2}$. Let's remember that. Look at the second number, is this fraction less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?**	
Students	__.	
Teacher	**The second number is __ than $\frac{1}{2}$. Let's remember that. Now, if one fraction is less than or equal to $\frac{1}{2}$ and the other fraction is greater than or equal to $\frac{1}{2}$, then it's easy to compare. Is one fraction less than $\frac{1}{2}$ and the other greater than $\frac{1}{2}$?**	
Students	*OPTION 1:*	Yes. (Skip Option 2.)
	OPTION 2:	No.
Teacher	***OPTION 2:***	**If both fractions are less than $\frac{1}{2}$ or greater than $\frac{1}{2}$, then we have to look at the value of each fraction a little closer. Is one fraction greater in length or area than the other fraction?**
Students		Yes.
Teacher		**What do you notice about __** (first fraction) **compared to __** (second fraction)?
Students		__.
Teacher		**So, we can see that the value of the first fraction is different from the value of the second fraction.**
Teacher	**It's time to compare. What should we do?**	
Students	Compare.	
Teacher	**Is the __** (first fraction) **greater than, less than, or equal to that of the second fraction?**	
Students	__.	
Teacher	**If it's greater, that means __** (first number) **is greater than __** (second number). **If it's less, that means __** (first number) **is less than __** (second number). **If the numbers are the same, they are equal. What's the comparison?**	
Students	__ (greater than/less than/equal to).	
Teacher	**That's right! __** (first number) **is __** (greater than/less than/equal to) **__** (second number). **Let's say that together.**	
Students	__ is greater/less/equal to __.	
Teacher	**Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?**	
Students	__.	
Teacher	**Let's write the symbol between the two numbers.** (Write.)	
Teacher	**What does it mean to compare numbers?**	
Students	To determine if one number is greater than, less than, or equal to another number.	
Teacher	**How did we compare numbers in this example?**	

Students We compared each fraction and then determined whether one number was greater than, less than, or equal to the other number.

Example

$$\frac{7}{8} > \frac{3}{4}$$



Teacher **Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?**

Students To determine whether a number is greater than, less than, or equal to another number.

Teacher **Today, we'll compare numbers with this number line.**
(Show number line.)

Teacher **Before we place fractions on the number line, let's draw a number line. I'll mark this number line with 0, $\frac{1}{2}$, and 1. How will I mark the number line?**

Students With 0, $\frac{1}{2}$, and 1.

Teacher **Now, let's compare numbers. Let's compare $\frac{7}{8}$ and $\frac{3}{4}$. What numbers are we going to compare?**

Students $\frac{7}{8}$ and $\frac{3}{4}$.

Teacher **And for this comparison, we want to determine if $\frac{7}{8}$ is greater than, less than, or equal to $\frac{3}{4}$. What do we want to do?**

Students Determine if the first number is greater than, less than, or equal to the second number.

Teacher **Now, let's compare numbers. Let's draw the first number on a number line. How could I show $\frac{7}{8}$?**

Students You could make 8 equal parts and mark $\frac{7}{8}$ above the seventh one-eighth mark.

Teacher **I'll show $\frac{7}{8}$ by dividing the number line into 8 equal parts. Then, I'll write $\frac{7}{8}$ above the seventh equal part.**
(Draw and write.)

Teacher **Let's draw the second number on the same number line. How could I show $\frac{3}{4}$?**

Students	You could make 4 equal parts and mark $\frac{3}{4}$ above the third one-fourth mark.
Teacher	**I'll show $\frac{3}{4}$ by dividing the number line into 4 equal parts. Then, I'll write $\frac{3}{4}$ above the third equal part.** (Draw and write.)
Teacher	**Now, it's time to compare. What are we going to do?**
Students	Compare.
Teacher	**Let's think about the value of each fraction compared to the whole. Look at the first number, is $\frac{7}{8}$ less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?**
Students	Greater than.
Teacher	**The first number is greater than $\frac{1}{2}$. Let's remember that. Look at the second number, is $\frac{3}{4}$ less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?**
Students	Greater than.
Teacher	**The second number is greater than $\frac{1}{2}$. Let's remember that. Now, if one fraction is less than or equal to $\frac{1}{2}$ and the other fraction is greater than or equal to $\frac{1}{2}$, then it's easy to compare. Is one fraction less than $\frac{1}{2}$ and the other greater than $\frac{1}{2}$?**
Students	No.
Teacher	**If both fractions are less than $\frac{1}{2}$ or greater than $\frac{1}{2}$, then we have to look at the value of each fraction a little closer. Is one fraction greater in length or area than the other fraction?**
Students	Yes.
Teacher	**What do you notice about $\frac{7}{8}$ compared to $\frac{3}{4}$?**
Students	$\frac{7}{8}$ is greater in value or longer than $\frac{3}{4}$.
Teacher	**So, is $\frac{7}{8}$ greater, less, or equal to that of $\frac{3}{4}$?**
Students	Greater.
Teacher	**What's the comparison?**
Students	**$\frac{7}{8}$ is greater than $\frac{3}{4}$.**
Teacher	**That's right! $\frac{7}{8}$ is greater than $\frac{3}{4}$. Let's say that together.**
Students	$\frac{7}{8}$ is greater than $\frac{3}{4}$.
Teacher	**Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?**
Students	Greater than.
Teacher	**Let's write the symbol between the two numbers.** (Write.)
Teacher	**What does it mean to compare numbers?**
Students	To determine greater than, less than, or equal to.
Teacher	**How did we compare numbers in this example?**

Students	We compared each fraction using a number line and then determined whether one number was greater than, less than, or equal to the other number.

D. Problems for Use During Instruction

See Module 2 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 2 Vocabulary Cards.

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Module 2:
Comparison

Problem Sets

A. > , < , = for numbers less than 20 (30)
B. > , < , = for numbers from 20-1,500 (30)
C. > , < , = for fractions with like denominators (15)
D. > , < , = for fractions with unlike denominators (15)
E. > , < , = for decimals to thousandths (15)

A.

6 5

A.

3 15

A.

13 3

A.

9 14

A.

2 11

A.

16 9

A.

5 13

A.

20 12

A.

7 4

A.

18 5

A.

6 16

A.

2 12

A.

14 14

A.

12 7

A.

8 19

A.

8 8

A.

10 20

A.

5 6

A.

9 9

A.

1 18

A.

15 6

A.

5 15

A.

12 12

A.

18 11

A.

7 17

A.

19 9

A.

4 4

A.

12 10

A.

14 13

A.

7 15

B.

545 534

B.

344 423

B.

287 287

B.

674 676

B.

882 828

B.

582 633

B.

656 562

B.

633 633

B.

535 553

B.

644 624

B.

448 484

B.

599 595

B.

737 735

B.

23 123

B.

45 45

B.

346 364

B.

870 807

B.

1,305 1,543

B.

128 112

B.

894 904

B.

1,321 1,321

B.

332 32

B.

65 98

B.

45 24

B.

39 29

B.

140 410

B.

285 345

B.

167 16

B.

145 45

B.

770 770

c.

$$\frac{9}{4} \qquad \frac{8}{4}$$

c.

$$\frac{7}{2} \qquad \frac{6}{2}$$

c.

$$\frac{3}{2} \qquad \frac{4}{2}$$

c.

$$\frac{6}{8} \qquad \frac{2}{8}$$

c.

$$\frac{1}{2} \qquad \frac{1}{2}$$

c.

$$\frac{2}{6} \qquad \frac{5}{6}$$

c.

$$\frac{11}{5} \qquad \frac{2}{5}$$

c.

$$\frac{8}{2} \qquad \frac{2}{2}$$

c.

$$\frac{2}{3} \qquad \frac{10}{3}$$

c.

$$\frac{5}{4} \qquad \frac{7}{4}$$

c.

$$\frac{8}{6} \qquad \frac{4}{6}$$

c.

$$\frac{2}{5} \qquad \frac{2}{5}$$

c.

$$\frac{11}{3} \qquad \frac{7}{3}$$

c.

$$\frac{8}{5} \qquad \frac{11}{5}$$

c.

$$\frac{10}{4} \qquad \frac{3}{4}$$

D.

$$\frac{1}{10} \qquad \frac{1}{2}$$

D.

$$\frac{1}{7} \qquad \frac{1}{4}$$

D.

$$\frac{2}{6} \qquad \frac{2}{8}$$

D.

$$\frac{1}{9} \qquad \frac{1}{4}$$

D.

$$\frac{4}{6} \qquad \frac{2}{8}$$

D.

$$\frac{2}{5} \qquad \frac{4}{10}$$

D.

$$\frac{2}{3} \qquad \frac{1}{4}$$

D.

$$\frac{4}{6} \qquad \frac{9}{12}$$

D.

$$\frac{2}{8} \qquad \frac{1}{2}$$

D.

$$\frac{4}{10} \qquad \frac{1}{3}$$

D.

$$\frac{4}{5} \qquad \frac{7}{8}$$

D.

$$\frac{1}{8} \qquad \frac{2}{6}$$

D.

$$\frac{7}{12} \qquad \frac{1}{5}$$

D.

$$\frac{5}{10} \qquad \frac{1}{2}$$

D.

$$\frac{8}{12} \qquad \frac{9}{10}$$

E.

5.6   5.2

E.

0.13 0.132

E.

0.899 0.889

E.

2.40 2.04

E.

104.5 150.4

E.

3.67 3.59

E.

0.657 0.756

E.

0.82 0.81

E.

1.906 1.903

E.

76.5 79.8

E.

5.60 5.06

E.

14.9 13.8

E.

405.4 540.4

E.

0.145 0.141

E.

1.29 1.32

Module 2:

Comparison

Vocabulary Cards

compare
denominator
digit
equal
equal sign
equivalent
fraction
greater than
hundreds
less than
number line
numerator
ones
place value
rational number
tens
thousands

compare

To examine differences between numbers, quantities, or values to decide if one quantity is greater than, less than, or equal to another quantity.

61	>	8	37	<	80	3	=	3
	greater than			less than			equal to	

---

denominator

The term in a fraction that tells the number of equal parts in a whole.

2 / 3 $\frac{2}{3}$ **In these fractions, 3 is the denominator.**

digit

A symbol used to show numbers.

0 1 2 3 4 5 6 7 8 9

---

equal

When the number, quantity, or value on the left side of the equal sign is the same as the number, quantity, or value on the right side of the equal sign.

$$3 = 3$$

equal to

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$12 + 8 = 20$$

= is the equal sign

---

equivalent

Two numbers that have the same value.

$$\frac{1}{4} = \frac{2}{8} \qquad \frac{2}{3} = \frac{8}{12}$$

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \qquad \frac{10}{12} \qquad \frac{8}{3}$$

---

greater than

When the number, quantity, or value on one side of the equal sign is larger than the number, quantity, or value on the other side of the equal sign.

$$61 > 8$$

greater than

hundreds

The digit representing 100.

---

less than

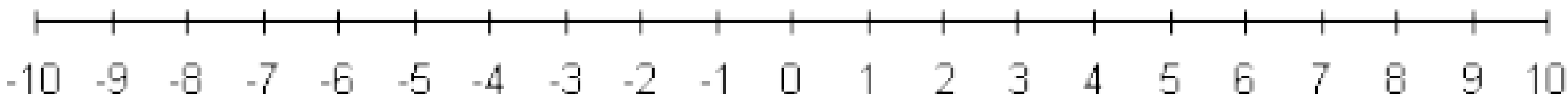
When the number, quantity, or value on one side of the equal sign is smaller than the number, quantity, or value on the other side of the equal sign.

$$37 < 80$$

less than

number line

A straight line with numbers placed at equal intervals along its length.



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numerator

The term in a fraction that tells how many parts of a fraction.

2 / 3 $\frac{2}{3}$ **In these fractions, 2 is the numerator.**

ones

The digit representing 1.

---

place value

The value of a digit depending on its place in a number.

thousands	hundreds	tens	ones	.	tenths	hundredths	thousandths
8	7	6	5	.	4	3	2

rational number

Any number that can be written as a fraction.

$$\frac{3}{6} \qquad \frac{10}{12} \qquad \frac{8}{3}$$

---

tens

The digit representing 10.

thousands

The digit representing 1,000.

Instructional Routines for Mathematics Intervention

MODULE 3

Representing Fractions

Module 3: Representing Fractions
Mathematics Routines

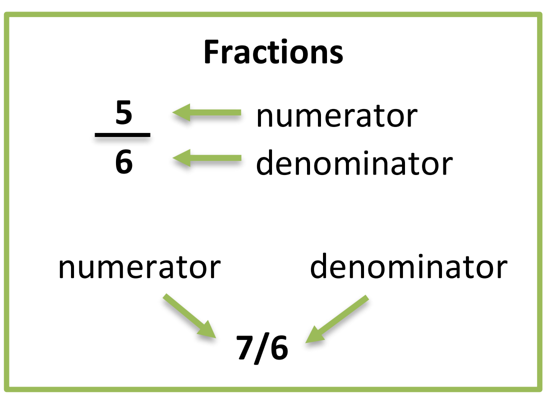
A. Important Vocabulary with Definitions

Term	**Definition**
denominator	The term in a fraction that tells the number of equal parts in a whole.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
improper fraction	Any fraction in which the numerator is greater than or equal to the denominator.
mixed number	A whole number and a fraction combined.
numerator	The term in a fraction that tells how many parts of a fraction.
proper fraction	A fraction where the numerator is less than the denominator.

B. Background Information

In this module, we focus on representing fractions. We use three models of fractions: (1) length model, (2) area model, and (3) set model.

When referring to fractions, be sure to use proper vocabulary. Also, present fractions in different ways.



C. Routines and Examples

(1) Length Model

Routine

Materials:

- Module 3 Problem Sets
- Module 3 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles, Cuisenaire rods, or number lines

ROUTINE WITH FRACTION TILES

Teacher	**Let's show different fractions. What's a fraction?**
Students	A fraction is a number with a numerator and denominator.
Teacher	**A fraction is a number – just like 2 is a number or 13 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?**
Students	A numerator and denominator.
Teacher	**The denominator tells us about the equal parts in the whole. What does the denominator tell us?**
Students	The equal parts in the whole.
Teacher	**And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**
Students	How many equal parts we show for a specific fraction.
Teacher	**So, let's show different fractions. We'll use these fraction tiles first.** (Show manipulatives.)
Teacher	**When we show fractions with the fraction tiles, let's first start by showing the whole. What should we show?**
Students	The whole.
Teacher	**I'll place the whole in the middle of my workspace.** (Show whole.)
Teacher	**Let's show this fraction.** (Show fraction.)
Teacher	**What fraction?**
Students	___.
Teacher	**What's the denominator of the fraction?**
Students	___.
Teacher	**That means we want to divide this whole into __** (denominator) **equal parts. How many equal parts?**

Students __.

Teacher **So, how could we divide this whole into __** (denominator) **equal parts?**

Students Divide the whole into __ equal parts with __ fraction tiles.
(Show whole divided into equal parts. Place equal parts over the whole or above the whole.)

Teacher **We showed the whole divided into __ equal parts. Now, what's the numerator of the fraction?**

Students __.

Teacher **That means we want to show __** (numerator) **of the equal parts in order to show the fraction __. How many equal parts do we want to show?**

Students __.

Teacher **Let's show __ of the __ equal parts.**
(Show the equal parts of the numerator. Leave equal parts over the whole or above the whole.)

Teacher **What fraction did we show?**

Students __.

Teacher **We showed the fraction __. First, we divided the whole into __ equal parts. Then, we showed the __ equal parts of the fraction. How did we show this fraction?**

Students We divided the whole into equal parts. Then, we showed the number of equal parts of the fraction.

Teacher **Now, let's think about this fraction. We should interpret the left side of the whole tile as zero – like zero on a number line. What number?**

Students Zero.

Teacher **And if we have only one whole, we should interpret the right side of the whole tile as one – like one on a number line. What number?**

Students One.

Teacher **Remember to think about the whole as a number line from 0 to 1** (then 1 to 2, then 2 to 3…). **That helps us learn the value of the fraction __. Where would the fraction $\frac{1}{2}$ be on this whole?**

Students In the middle between 0 and 1.

Teacher **Think about $\frac{1}{2}$. What do you notice about __** (fraction) **compared to $\frac{1}{2}$?**

Students __ is greater/less than $\frac{1}{2}$.

Teacher **Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about __** (fraction)**?**

Students __ is greater/less than __ (benchmark fraction).

Teacher **Great work! Using these fraction tiles helps you understand the value of different fractions. Let's review. What's a denominator?**

Students The equal parts in the whole.

Teacher **What's a numerator?**

Students How many equal parts we show for a specific fraction.

Teacher **How can you use the fraction tiles to show a fraction?**

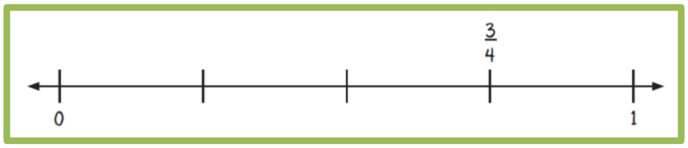
Students First, you show the whole. Then, you look at the denominator and divide the whole into equal parts. Then, you show the numerator of the fraction by showing the equal parts in the numerator.

ROUTINE WITH CUISENAIRE RODS

Teacher **Let's show different fractions. What's a fraction?**
Students A fraction is a number with a numerator and denominator.
Teacher **A fraction is a number – just like 4 is a number or 65 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?**
Students A numerator and denominator.
Teacher **The denominator tells us about the equal parts in the whole. What does the denominator tell us?**
Students The equal parts in the whole.
Teacher **And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**
Students How many equal parts we show for a specific fraction.
Teacher **So, let's show different fractions. We'll use Cuisenaire rods.**
(Show manipulatives.)
Teacher **When we show fractions with the Cuisenaire rods, let's first start by thinking about the whole. What should we focus on?**
Students The whole.
Teacher **Let's show this fraction.**
(Show fraction.)
Teacher **What fraction?**
Students __.
Teacher **What's the denominator of the fraction?**
Students __.
Teacher **That means we want to show a whole with __** (denominator) **equal parts. How many equal parts?**
Students __.
Teacher **So, how could we show a whole with __** (denominator) **equal parts?**
Students Show __ (denominator) equal parts. Then, find a Cuisenaire rod with a whole that's the same length as the __ equal parts.
Teacher **Let's first find __** (denominator) **equal parts. We'll line those up and find a whole that's the same length as the __** (denominator) **equal parts.**
(Show whole divided into equal parts. Place equal parts over the whole or above the whole.)
Teacher **So, this rod** (say color) **shows the denominator. What does this rod show?**

Students	The denominator.
Teacher	**Now, what's the numerator of the fraction?**
Students	__.
Teacher	**That means we want to show __** (numerator) **of the equal parts in order to show the fraction __. How many equal parts do we want to show?**
Students	__.
Teacher	**Let's show __ of the __ equal parts.** (Show the equal parts of the numerator. Leave equal parts over the whole or above the whole.)
Teacher	**What fraction did we show?**
Students	__.
Teacher	**We showed the fraction __. First, we divided the whole into __ equal parts. Then, we showed the __ equal parts of the fraction. How did we show this fraction?**
Students	We divided the whole into equal parts. Then, we showed the equal parts of the fraction.
Teacher	**Now, let's think about this fraction. We should interpret the left side of the whole as zero – like zero on a number line. What number?**
Students	Zero.
Teacher	**And if we have only one whole, we should interpret the right side of the whole as one – like one of a number line. What number?**
Students	One.
Teacher	**Remember to think about the whole as a number line from 0 to 1** (then 1 to 2, then 2 to 3…)**. That helps us learn the value of the fraction __. Where would the fraction $\frac{1}{2}$ be on this whole?**
Students	In the middle between 0 and 1.
Teacher	**Think about $\frac{1}{2}$. What do you notice about __** (fraction) **compared to $\frac{1}{2}$?**
Students	__ is greater/less than $\frac{1}{2}$.
Teacher	**Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about __** (fraction)**?**
Students	__ is greater/less than __ (benchmark fraction).
Teacher	**Great work! Using these Cuisenaire rods helps you understand the value of different fractions. Let's review. What's a denominator?**
Students	The equal parts in the whole.
Teacher	**What's a numerator?**
Students	How many equal parts we show for a specific fraction.
Teacher	**How can you use the Cuisenaire rods to show a fraction?**
Students	First, you determine the equal parts of the whole and find a whole of the same length. Then, you look at the denominator and divide the whole into equal parts. Then, you show the numerator of the fraction by showing the number of equal parts by the numerator.

ROUTINE WITH NUMBER LINE



Teacher **Let's show different fractions. What's a fraction?**

Students A fraction is a number with a numerator and denominator.

Teacher **A fraction is a number – just like 5 is a number or 17 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?**

Students A numerator and denominator.

Teacher **The denominator tells us about the equal parts in the whole. What does the denominator tell us?**

Students The equal parts in the whole.

Teacher **And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**

Students How many equal parts we show for a specific fraction.

Teacher **So, let's show different fractions. Today, let's draw fractions with a number line.**
(Draw a number line.)

Teacher **When we show fractions with a number line, let's draw a 0 and 1 on the number line.**
(Draw 0 and 1.)

Teacher **Now, let's start by thinking about the whole. What should we focus on?**

Students The whole.

Teacher **Let's show this fraction.**
(Show fraction.)

Teacher **What fraction?**

Students __.

Teacher **What's the denominator of the fraction?**

Students __.

Teacher **That means we want to show a whole with __** (denominator) **equal parts. How many equal parts?**

Students __.

Teacher **So, how could we show a whole with __** (denominator) **equal parts?**

Students Draw __ (denominator) equal parts.

Teacher **Let's divide this number line into __** (denominator) **equal parts.**
(Divide number line into equal parts by drawing tick marks.)

Teacher **So, our number line shows __ equal parts or the denominator. What does the number line show?**

Students The denominator.

Teacher **Now, what's the numerator of the fraction?**

Students __.

Teacher	**That means we want to show __** (numerator) **of the equal parts in order to show the fraction __. How many equal parts do we want to show?**
Students	__.
Teacher	**Let's draw where the fraction __ is on the number line. I count __ equal parts.** (Draw the fraction on the number line.)
Teacher	**What fraction did we show?**
Students	__.
Teacher	**We showed the fraction __. First, we drew a number line. Then, we divided the whole into __ equal parts. Then, we showed the fraction. How did we show this fraction?**
Students	We divided the number line into equal parts. Then, we determine the numerator – or the number of equal parts – and drew the fraction on the number line.
Teacher	**Let's think about this fraction on a number line from 0 to 1** (then 1 to 2, then 2 to 3…)**. That helps us learn the value of the fraction __. Where would the fraction $\frac{1}{2}$ be on this whole?**
Students	In the middle between 0 and 1.
Teacher	**Think about $\frac{1}{2}$. What do you notice about __** (fraction) **compared to $\frac{1}{2}$?**
Students	__ is greater/less than $\frac{1}{2}$.
Teacher	**Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about __** (fraction)**?**
Students	__ is greater/less than __ (benchmark fraction).
Teacher	**Great work! Using the number line helps you understand the value of different fractions. Let's review. What's a denominator?**
Students	The equal parts in the whole.
Teacher	**What's a numerator?**
Students	How many equal parts we show for a specific fraction.
Teacher	**How can you use the number line to show a fraction?**
Students	First, draw the equal parts of the whole. Then, draw the fraction on the number line.

Example

$\frac{3}{4}$

EXAMPLE WITH FRACTION TILES

Teacher **Let's show different fractions. What's a fraction?**

Students A fraction is a number with a numerator and denominator.

Teacher **A fraction is a number, except with a fraction, the number has a numerator and denominator. What does a fraction have?**

Students A numerator and denominator.

Teacher **The denominator tells us about the equal parts in the whole. What does the denominator tell us?**

Students The equal parts in the whole.

Teacher **And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**

Students How many equal parts we show for a specific fraction.

Teacher **So, let's show different fractions. We'll use these fraction tiles first.**
(Show manipulatives.)

Teacher **When we show fractions with the fraction tiles, let's first start by showing the whole. What should we show?**

Students The whole.

Teacher **I'll place the whole in the middle of my workspace.**
(Show whole.)

Teacher **Let's show this fraction.**
(Show fraction.)

Teacher **What fraction?**

Students $\frac{3}{4}$.

Teacher **What's the denominator of the fraction?**

Students 4.

Teacher **That means we want to divide this whole into 4 equal parts. How many equal parts?**

Students 4.

Teacher **So, how could we divide this whole into 4 equal parts?**

Students Divide the whole into 4 equal parts with 4 one-fourth fraction tiles.
(Show whole divided into equal parts. Place equal parts over the whole or above the whole.)

Teacher **We showed the whole divided into 4 equal parts. Each equal part is one-fourth of the whole. Now, what's the numerator of the fraction?**

Students 3.

Teacher	**That means we want to show 3 of the equal parts in order to show the fraction $\frac{3}{4}$. How many equal parts do we want to show?**
Students	3.
Teacher	**Let's show 3 of the 4 equal one-fourth parts.** (Show the equal parts of the numerator. Leave equal parts over the whole or above the whole.)
Teacher	**What fraction did we show?**
Students	$\frac{3}{4}$.
Teacher	**We showed the fraction $\frac{3}{4}$. First, we divided the whole into 4 equal parts. Then, we showed the 3 equal parts of the fraction. How did we show this fraction?**
Students	We divided the whole into 4 equal parts. Then, we showed the 3 equal parts of the fraction.
Teacher	**Now, let's think about this fraction. We should interpret the left side of the whole tile as zero – like zero on a number line. What number?**
Students	Zero.
Teacher	**And if we have only one whole, we should interpret the right side of the whole tile as one – like one on a number line. What number?**
Students	One.
Teacher	**Remember to think about the whole as a number line from 0 to 1. That helps us learn the value of the fraction $\frac{3}{4}$. Where would the fraction $\frac{1}{2}$ be on this whole?**
Students	In the middle between 0 and 1.
Teacher	**Think about $\frac{1}{2}$. What do you notice about $\frac{3}{4}$ compared to $\frac{1}{2}$?**
Students	$\frac{3}{4}$ is greater than $\frac{1}{2}$.
Teacher	**Yes, I see $\frac{3}{4}$ is greater than $\frac{1}{2}$. What else do you notice about $\frac{3}{4}$?**
Students	$\frac{3}{4}$ is less than 1.
Teacher	**Excellent! Using these fraction tiles helps you understand the value of different fractions. Let's review. What's a denominator?**
Students	The equal parts in the whole.
Teacher	**What's a numerator?**
Students	How many equal parts we show for a specific fraction.
Teacher	**How can you use the fraction tiles to show a fraction?**
Students	First, you show the whole. Then, you look at the denominator and divide the whole into equal parts. Then, you show the numerator of the fraction by showing the equal parts in the numerator.

(2) Area Model

Routine

Materials:

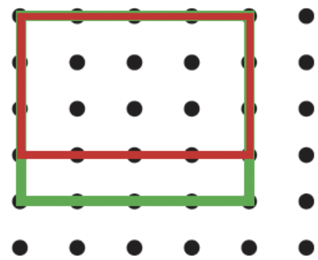
- [Module 3 Problem Sets]
- [Module 3 Vocabulary Cards]
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction circles, geoboards, or pattern blocks

ROUTINE WITH FRACTION CIRCLES

Teacher	**Let's show different fractions. What's a fraction?**
Students	A fraction is a number with a numerator and denominator.
Teacher	**A fraction is a number – just like 2 is a number or 13 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?**
Students	A numerator and denominator.
Teacher	**The denominator tells us about the equal parts in the whole. What does the denominator tell us?**
Students	The equal parts in the whole.
Teacher	**And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**
Students	How many equal parts we show for a specific fraction.
Teacher	**So, let's show different fractions. We'll use these fraction circles.** (Show manipulatives.)
Teacher	**These fraction circles can help us see fractions by area. Any area – a circle, triangle, rectangle, or any other area – can also be used to show fractions. What's an area that could be used to show fractions?**
Students	Rectangle or triangle.
Teacher	**When we show fractions with the fraction circles, let's first start by showing the whole. What should we show?**
Students	The whole.
Teacher	**I'll place the whole in the middle of my workspace.** (Show whole.)
Teacher	**Let's show this fraction.** (Show fraction.)
Teacher	**What fraction?**

Students	__.
Teacher	**What's the denominator of the fraction?**
Students	__.
Teacher	**That means we want to divide this whole into __** (denominator) **equal parts. How many equal parts?**
Students	__.
Teacher	**So, how could we divide this whole into __** (denominator) **equal parts?**
Students	Divide the whole into __ equal parts with __ fraction tiles. (Show whole divided into equal parts. Place equal parts over the whole.)
Teacher	**We showed the whole divided into __ equal parts. Now, what's the numerator of the fraction?**
Students	__.
Teacher	**That means we want to show __** (numerator) **of the equal parts in order to show the fraction __. How many equal parts do we want to show?**
Students	__.
Teacher	**Let's show __ of the __ equal parts.** (Show the equal parts of the numerator. Leave equal parts over the whole.)
Teacher	**What fraction did we show?**
Students	__.
Teacher	**We showed the fraction __. First, we divided the whole into __ equal parts. Then, we showed the __ equal parts of the fraction. How did we show this fraction?**
Students	We divided the whole into equal parts. Then, we showed the equal parts of the fraction.
Teacher	**Now, let's think about this fraction. We're looking at a fraction by area. What would be $\frac{1}{2}$ of this area**
Students	Half of the circle.
Teacher	**What do you notice about __** (fraction) **compared to $\frac{1}{2}$?**
Students	__ is greater/less than $\frac{1}{2}$.
Teacher	**Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about __?**
Students	__ is greater/less than 1.
Teacher	**Great work! Using these fraction circles helps you understand the area of different fractions. Let's review. What's a denominator?**
Students	The equal parts in the whole.
Teacher	**What's a numerator?**
Students	How many equal parts we show for a specific fraction.
Teacher	**How can you use the fraction tiles to show a fraction?**
Students	First, you show the whole area. Then, you look at the denominator and divide the whole into equal parts. Then, you show the numerator of the fraction by placing the equal parts of the numerator on the whole.

ROUTINE WITH GEOBOARDS



Teacher	**Let's show different fractions. What's a fraction?**
Students	A fraction is a number with a numerator and denominator.
Teacher	**A fraction is a number – just like 5 is a number or 25 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?**
Students	A numerator and denominator.
Teacher	**The denominator tells us about the equal parts in the whole. What does the denominator tell us?**
Students	The equal parts in the whole.
Teacher	**And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**
Students	How many equal parts we show for a specific fraction.
Teacher	**So, let's show different fractions. We'll use this geoboard.** (Show manipulatives.)
Teacher	**This geoboard can help us see fractions by area. Any area – a circle, triangle, rectangle, or any other area – can also be used to show fractions. What's an area that could be used to show fractions?**
Students	Rectangle or triangle or circle or trapezoid.
Teacher	**When we show fractions with the geoboard, let's first start by showing the whole. What should we show?**
Students	The whole.
Teacher	**Let's show this fraction.** (Show fraction.)
Teacher	**What fraction?**
Students	__.
Teacher	**What's the denominator of the fraction?**
Students	__.
Teacher	**That means we want to make an area that can be divided into __** (denominator) **equal parts. How many equal parts?**
Students	__.
Teacher	**So, how could we make an area divided into __** (denominator) **equal parts?**
Students	Ring the rubber band around __ equal parts. (Use rubber band to ring whole divided into equal parts.)
Teacher	**We showed an area or whole divided into __ equal parts. Now, what's the numerator of the fraction?**

Students __.

Teacher **That means we want to show __** (numerator) **of the equal parts in order to show the fraction __. How many equal parts do we want to show?**

Students __.

Teacher **Let's show __ of the __ equal parts using a different colored rubber band.** (Use rubber band to ring equal parts of the numerator.)

Teacher **What fraction did we show?**

Students __.

Teacher **We showed the fraction __. First, we showed an area divided into __ equal parts. Then, we showed the __ equal parts of the fraction. How did we show this fraction?**

Students We showed the area of the whole. Then, we showed the equal parts of the fraction.

Teacher **Now, let's think about this fraction. What's the area of this fraction? What would be $\frac{1}{2}$ of this area?**

Students Half of this shape.

Teacher **What do you notice about __** (fraction) **compared to $\frac{1}{2}$?**

Students __ is greater/less than $\frac{1}{2}$.

Teacher **Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about __?**

Students __ is greater/less than 1.

Teacher **Awesome! Using a geoboard helps you understand the area of different fractions. Let's review. What's a denominator?**

Students The equal parts in the whole.

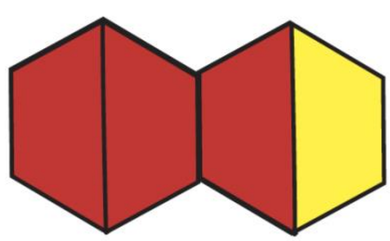
Teacher **What's a numerator?**

Students How many equal parts we show for a specific fraction.

Teacher **How can you use the geoboard to show a fraction?**

Students First, you show the whole. Then, you use a rubber band to show the denominator by dividing the whole into equal parts. Then, you use a second rubber band of a different color to show the fraction.

ROUTINE WITH PATTERN BLOCKS



Teacher **Let's show different fractions. What's a fraction?**

Students A fraction is a number with a numerator and denominator.

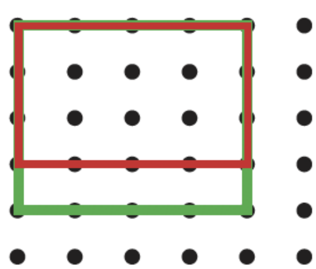
Teacher	**A fraction is a number – just like 14 is a number or 8 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?**
Students	A numerator and denominator.
Teacher	**The denominator tells us about the equal parts in the whole. What does the denominator tell us?**
Students	The equal parts in the whole.
Teacher	**And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**
Students	How many equal parts we show for a specific fraction.
Teacher	**So, let's show different fractions. We'll use these pattern blocks.** (Show manipulatives.)
Teacher	**These pattern blocks can help us see fractions by area. Any area – a circle, triangle, rectangle, or any other area – can also be used to show fractions. What's an area that could be used to show fractions?**
Students	Octagon or pentagon or triangle.
Teacher	**When we show fractions with the pattern blocks, let's first start by showing the whole. What should we show?**
Students	The whole.
Teacher	**Let's show this fraction.** (Show fraction.)
Teacher	**What fraction?**
Students	__.
Teacher	**What's the denominator of the fraction?**
Students	__.
Teacher	**That means we want to make an area that can be divided into __** (denominator) **equal parts. How many equal parts?**
Students	__.
Teacher	**So, how could we make an area divided into __** (denominator) **equal parts?**
Students	Use __ shape.
Teacher	**With pattern blocks, you could use one shape to make the whole. Or you could combine shapes to make a whole.** (Show whole.)
Teacher	**We showed an area or whole divided into __ equal parts. Now, what's the numerator of the fraction?**
Students	__.
Teacher	**That means we want to show __** (numerator) **of the equal parts in order to show the fraction __. How many equal parts do we want to show?**
Students	__.
Teacher	**Let's show __ of the __ equal parts by placing the numerator shapes over the whole.** (Place shapes over the whole.)
Teacher	**What fraction did we show?**
Students	__.

Teacher	**We showed the fraction __. First, we showed an area divided into __ equal parts. Then, we showed the __ equal parts of the fraction. How did we show this fraction?**
Students	We showed the area of the whole. Then, we showed the equal parts of the fraction.
Teacher	**Now, let's think about this fraction. What's the area of this fraction? What would be $\frac{1}{2}$ of this area?**
Students	Half of the shape.
Teacher	**What do you notice about __** (fraction) **compared to $\frac{1}{2}$?**
Students	__ is greater/less than $\frac{1}{2}$.
Teacher	**Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about __?**
Students	__ is greater/less than 1.
Teacher	**Great work! Using pattern blocks helps you understand the area of different fractions. Let's review. What's a denominator?**
Students	The equal parts in the whole.
Teacher	**What's a numerator?**
Students	How many equal parts we show for a specific fraction.
Teacher	**How can you use the pattern blocks to show a fraction?**
Students	First, you show the whole with a shape or shapes. Then, you show the numerator of the fraction by placing shapes on top of the whole area.

Example

$\frac{3}{4}$

EXAMPLE WITH GEOBOARDS



Teacher	**Let's show different fractions. What's a fraction?**
Students	A fraction is a number with a numerator and denominator.
Teacher	**A fraction is a number but the number has a numerator and denominator. What does a fraction have?**
Students	A numerator and denominator.

Teacher	**The denominator tells us about the equal parts in the whole. What does the denominator tell us?**
Students	The equal parts in the whole.
Teacher	**And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**
Students	How many equal parts we show for a specific fraction.
Teacher	**So, let's show different fractions. We'll use this geoboard.** (Show manipulatives.)
Teacher	**When we show fractions with the geoboard, let's first start by showing the whole. What should we show?**
Students	The whole.
Teacher	**Let's show this fraction.** (Show fraction.)
Teacher	**What fraction?**
Students	$\frac{3}{4}$.
Teacher	**What's the denominator of the fraction?**
Students	4.
Teacher	**That means we want to make an area that can be divided into 4 equal parts. How many equal parts?**
Students	4.
Teacher	**So, how could we make an area divided into 4 equal parts?**
Students	Ring the rubber band around 4 equal parts. (Use rubber band to ring whole divided into equal parts.)
Teacher	**We showed an area or whole divided into 4 equal parts. Now, what's the numerator of the fraction?**
Students	3.
Teacher	**That means we want to show 3 of the equal parts in order to show the fraction $\frac{3}{4}$. How many equal parts do we want to show?**
Students	3.
Teacher	**Let's show 3 of the 4 equal parts using a different colored rubber band.** (Use rubber band to ring equal parts of the numerator.)
Teacher	**What fraction did we show?**
Students	$\frac{3}{4}$.
Teacher	**We showed the fraction $\frac{3}{4}$. First, we showed an area divided into 4 equal parts. Then, we showed the 3 equal parts of the fraction. How did we show this fraction?**
Students	We showed the area of the whole. Then, we showed the equal parts of the fraction.
Teacher	**Now, let's think about this fraction. What would be $\frac{1}{2}$ of this area?**
Students	Half of the shape.
Teacher	**What do you notice about $\frac{3}{4}$ compared to $\frac{1}{2}$?**

Students	$\frac{3}{4}$ is greater than $\frac{1}{2}$.
Teacher	**Yes, I see $\frac{3}{4}$ is greater than $\frac{1}{2}$. What else do you notice about $\frac{3}{4}$?**
Students	$\frac{3}{4}$ is less than 1.
Teacher	**Awesome! Using a geoboard helps you understand the area of different fractions. Let's review. What's a denominator?**
Students	The equal parts in the whole.
Teacher	**What's a numerator?**
Students	How many equal parts we show for a specific fraction.
Teacher	**How can you use the geoboard to show a fraction?**
Students	First, you show the whole by ringing a rubber band around an area. Then, you show the numerator of the fraction by ringing the area of the numerator.

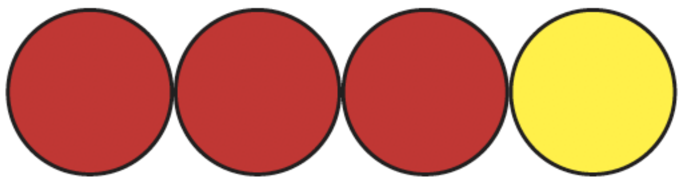
(3) Set Model

Routine

Materials:

- Module 3 Problem Sets
- Module 3 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like two-color counters or colored cubes

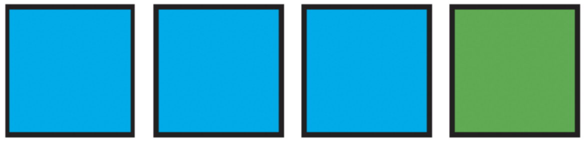
ROUTINE WITH TWO-COLOR COUNTERS



Teacher	**Let's show different fractions. What's a fraction?**
Students	A fraction is a number with a numerator and denominator.
Teacher	**A fraction is a number – just like 3 is a number or 300 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?**
Students	A numerator and denominator.
Teacher	**The denominator tells us about the equal parts in the whole. What does the denominator tell us?**
Students	The equal parts in the whole.
Teacher	**And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**
Students	How many equal parts we show for a specific fraction.
Teacher	**So, let's show different fractions. We'll use these two-color counters.** (Show manipulatives.)
Teacher	**When we show fractions with the two-color counters, let's first start by showing the whole. What should we show?**
Students	The whole.
Teacher	**Let's show this fraction.** (Show fraction.)
Teacher	**What fraction?**
Students	__.
Teacher	**What's the denominator of the fraction?**
Students	__.
Teacher	**That means we want to show __** (denominator) **equal parts. How many equal parts?**
Students	__.

Teacher	**So, how could we show a set with __** (denominator) **equal parts?**
Students	Use __ counters.
Teacher	**With two-color counters, we can create a set of __** (denominator) **counters. We'll use the yellow side to show the denominator.** (Show yellow counters in a set. Place the counters so they are touching one another.)
Teacher	**We showed a set divided into __ equal parts. Now, what's the numerator of the fraction?**
Students	__.
Teacher	**That means we want to show __** (numerator) **of the equal parts in order to show the fraction __. How many equal parts do we want to show?**
Students	__.
Teacher	**We can show the numerator by turning over the counter to the red side.** (Turn counters to red side.)
Teacher	**What fraction did we show?**
Students	__.
Teacher	**We showed the fraction __. First, we showed a set divided into __ equal parts. Then, we showed the __ equal parts of the fraction. How did we show this fraction?**
Students	We showed a set of yellow counters for the denominator, then we turned over counters to the red side to show the numerator.
Teacher	**Now, let's think about this fraction. What would be $\frac{1}{2}$ of this set?**
Students	Half of the counters.
Teacher	**What do you notice about __** (fraction) **compared to $\frac{1}{2}$?**
Students	__ is greater/less than $\frac{1}{2}$.
Teacher	**Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about __?**
Students	__ is greater/less than 1.
Teacher	**Great work! Using the two-color counters helps you understand how sets can be used to show different fractions. Let's review. What's a denominator?**
Students	The equal parts in the whole.
Teacher	**What's a numerator?**
Students	How many equal parts we show for a specific fraction.
Teacher	**How can you use the two-color counters to show a fraction?**
Students	We showed a set of yellow counters to show the denominator, then we turned over the counters to the red side to show the numerator or fraction.

ROUTINE WITH COLORED CUBES



Teacher	**Let's show different fractions. What's a fraction?**
Students	A fraction is a number with a numerator and denominator.
Teacher	**A fraction is a number – just like 150 is a number or 15 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?**
Students	A numerator and denominator.
Teacher	**The denominator tells us about the equal parts in the whole. What does the denominator tell us?**
Students	The equal parts in the whole.
Teacher	**And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**
Students	How many equal parts we show for a specific fraction.
Teacher	**So, let's show different fractions. We'll use these colored cubes.** (Show manipulatives.)
Teacher	**When we show fractions with the cubes, let's first start by showing the whole. What should we show?**
Students	The whole.
Teacher	**Let's show this fraction.** (Show fraction.)
Teacher	**What fraction?**
Students	__.
Teacher	**What's the denominator of the fraction?**
Students	__.
Teacher	**That means we want to show __** (denominator) **equal parts. How many equal parts?**
Students	__.
Teacher	**So, how could we show a set with __** (denominator) **equal parts?**
Students	Use __ cubes.
Teacher	**With the cubes, let's use one color to show the denominator. Let's place __** (denominator) __ (color) **cubes in a set.** (Show cubes in a set. Place the cubes so they are close to one another – to show a set of objects.)
Teacher	**We showed a set divided into __ equal parts. Now, what's the numerator of the fraction?**
Students	__.
Teacher	**That means we want to show __** (numerator) **of the equal parts in order to show the fraction __. How many equal parts do we want to show?**
Students	__.

Teacher **With the cubes, let's use another color to show the numerator. We'll replace __** (denominator) **cubes with __** (new color) **cubes.**
(Show numerator cubes in a different color by replacing the denominator cubes.)

Teacher **What fraction did we show?**

Students __.

Teacher **We showed the fraction __. First, we showed a set of cubes divided into __ equal parts. Then, we showed the __ equal parts of the fraction. How did we show this fraction?**

Students We showed a set of cubes for the denominator, then we used different colored cubes to show the numerator.

Teacher **Now, let's think about this fraction. What would be $\frac{1}{2}$ of this set?**

Students Half of the cubes.

Teacher **What do you notice about __** (fraction) **compared to $\frac{1}{2}$?**

Students __ is greater/less than $\frac{1}{2}$.

Teacher **Yes, I see __ is greater/less than $\frac{1}{2}$. What else do you notice about __?**

Students __ is greater/less than 1.

Teacher **Super! Using the colored cubes helps you understand how sets can be used to show different fractions. Let's review. What's a denominator?**

Students The equal parts in the whole.

Teacher **What's a numerator?**

Students How many equal parts we show for a specific fraction.

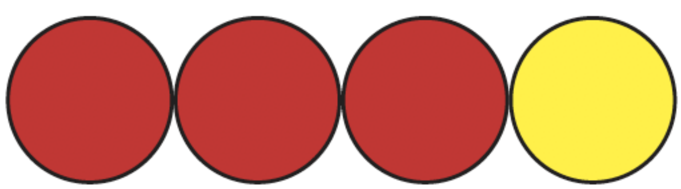
Teacher **How can you use the colored cubes to show a fraction?**

Students First, you show a set of the whole with cubes. Then, you use a different colored cube to show the numerator within that set.

Example

$\frac{3}{4}$

EXAMPLE WITH TWO-COLOR COUNTERS



Teacher **Let's show different fractions. What's a fraction?**

Students A fraction is a number with a numerator and denominator.

Teacher	**A fraction is a number with a numerator and denominator. What does a fraction have?**
Students	A numerator and denominator.
Teacher	**The denominator tells us about the equal parts in the whole. What does the denominator tell us?**
Students	The equal parts in the whole.
Teacher	**And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?**
Students	How many equal parts we show for a specific fraction.
Teacher	**So, let's show different fractions. We'll use these two-color counters.** (Show manipulatives.)
Teacher	**When we show fractions with the two-color counters, let's first start by showing the whole. What should we show?**
Students	The whole.
Teacher	**Let's show this fraction.** (Show fraction.)
Teacher	**What fraction?**
Students	$\frac{3}{4}$.
Teacher	**What's the denominator of the fraction?**
Students	4.
Teacher	**That means we want to show 4 equal parts. How many equal parts?**
Students	4.
Teacher	**So, how could we show a set with 4 equal parts?**
Students	Use 4 counters.
Teacher	**With two-color counters, we can create a set of 4 counters. We'll use the yellow side to show the denominator.** (Show 4 yellow counters in a set. Place the counters so they are touching one another.)
Teacher	**We showed a set divided into 4 equal parts. Now, what's the numerator of the fraction?**
Students	3.
Teacher	**That means we want to show 3 of the equal parts in order to show the fraction $\frac{3}{4}$. How many equal parts do we want to show?**
Students	3.
Teacher	**We can show the numerator by turning over 3 counters to the red side.** (Turn 3 counters to red side.)
Teacher	**What fraction did we show?**
Students	$\frac{3}{4}$.
Teacher	**We showed the fraction $\frac{3}{4}$. First, we showed a set divided into 4 equal parts. Then, we showed the 3 equal parts of the fraction. How did we show this fraction?**

Students	We showed a set of 4 yellow counters for the denominator, then we turned over 3 counters to the red side to show the numerator.
Teacher	**Now, let's think about this fraction. What would be $\frac{1}{2}$ of this set?**
Students	Half of the counters – or 2 counters.
Teacher	**What do you notice about $\frac{3}{4}$ compared to $\frac{1}{2}$?**
Students	$\frac{3}{4}$ is greater than $\frac{1}{2}$.
Teacher	**Yes, I see $\frac{3}{4}$ is greater than $\frac{1}{2}$. What else do you notice about $\frac{3}{4}$?**
Students	$\frac{3}{4}$ is less than 1.
Teacher	**Great work! Using the two-color counters helps you understand how sets can be used to show different fractions. Let's review. What's a denominator?**
Students	The equal parts in the whole.
Teacher	**What's a numerator?**
Students	How many equal parts we show for a specific fraction.
Teacher	**How can you use the two-color counters to show a fraction?**
Students	First, you show a set of the whole by showing a set with yellow counters. Then, you turn over counters to the red side for the numerator.

D. Problems for Use During Instruction

See Module 3 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 3 Vocabulary Cards.

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Module 3:
Representing Fractions

Problem Sets

A.

$$\frac{2}{5}$$

A.

$$\frac{6}{10}$$

A.

$$\frac{3}{6}$$

A.

$$\frac{1}{4}$$

A.

$$\frac{1}{12}$$

A.

$$\frac{2}{5}$$

A.

$$\frac{3}{8}$$

A.

$$\frac{4}{10}$$

A.

$$\frac{2}{12}$$

A.

$$\frac{7}{8}$$

A.

$$\frac{5}{6}$$

A.

$$\frac{3}{5}$$

A.

$$\frac{1}{6}$$

A.

$$\frac{4}{12}$$

A.

$$\frac{3}{10}$$

A.

$$\frac{2}{10}$$

A.

$$\frac{3}{9}$$

A.

$$\frac{1}{12}$$

A.

$$\frac{3}{8}$$

A.

$$\frac{9}{12}$$

B.

$$\frac{7}{5}$$

B.

$$\frac{9}{8}$$

B.

$$\frac{8}{6}$$

B.

$$\frac{5}{4}$$

B.

$$\frac{13}{10}$$

B.

$$\frac{8}{5}$$

B.

$$\frac{9}{6}$$

B.

$$\frac{6}{2}$$

B.

$$\frac{13}{12}$$

B.

$$\frac{11}{10}$$

B.

$$\frac{5}{3}$$

B.

$$\frac{9}{2}$$

B.

$$\frac{7}{3}$$

B.

$$\frac{6}{5}$$

B.

$$\frac{11}{4}$$

B.

$$\frac{9}{5}$$

B.

$$\frac{8}{6}$$

B.

$$\frac{12}{8}$$

B.

$$\frac{13}{5}$$

B.

$$\frac{15}{10}$$

c.

$$4\frac{3}{12}$$

c.

$$2\frac{3}{5}$$

c.

$$3\frac{5}{6}$$

c.

$$4\frac{1}{5}$$

c.

$$3\frac{5}{12}$$

c.

$$1\frac{4}{5}$$

c.

$$1\frac{3}{4}$$

c.

$$7\frac{5}{6}$$

c.

$$2\frac{2}{4}$$

c.

$$3\frac{5}{8}$$

c.

$$1\frac{7}{8}$$

c.

$$4\frac{1}{4}$$

c.

$$7\frac{1}{2}$$

c.

$$1\frac{5}{6}$$

c.

$$5\frac{1}{2}$$

c.

$$3\frac{4}{10}$$

c.

$$2\frac{5}{12}$$

c.

$$3\frac{1}{2}$$

c.

$$6\frac{5}{8}$$

c.

$$4\,\frac{5}{10}$$

Module 3:
Representing Fractions

Vocabulary Cards

denominator
equivalent
fraction
improper fraction
mixed number
numerator
proper fraction

denominator

The term in a fraction that tells the number of equal parts in a whole.

2 / 3 $\qquad \frac{2}{3}$ **In these fractions, 3 is the denominator.**

---

equivalent

Two numbers that have the same value.

$$\frac{1}{4} = \frac{2}{8} \qquad \frac{2}{3} = \frac{8}{12}$$

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \qquad \frac{10}{12} \qquad \frac{8}{3}$$

---

improper fraction

Any fraction in which the numerator is greater than or equal to the denominator.

$$\frac{9}{4} \qquad \frac{17}{12} \qquad \frac{10}{3}$$

mixed number

A whole number and a fraction combined.

$$1\frac{1}{6} \qquad 4\frac{5}{12} \qquad 12\frac{4}{3}$$

---

numerator

The term in a fraction that tells how many parts of a fraction.

2 / 3 $$\frac{2}{3}$$ **In these fractions, 2 is the numerator.**

proper fraction

A fraction where the numerator is less than the denominator.

$$\frac{3}{4} \qquad \frac{5}{6} \qquad \frac{8}{21}$$

Instructional Routines for Mathematics Intervention

MODULE 4

Concepts of Additions

Module 4: Concepts of Addition
Mathematics Routines

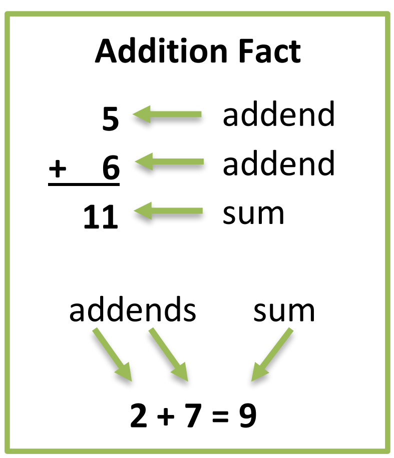
A. Important Vocabulary with Definitions

Term	Definition
add/addition	To put amounts together to find the sum or to increase a set.
addend	Any numbers that are added together.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
join	To add to an existing set.
plus sign	The symbol that tells you to add.
sum	The result of adding two or more numbers or the total number when you combine sets.
together	To combine sets or numbers.

B. Background Information

Students need to learn two concepts of addition: (1) addition as combining and (2) addition as joining to a set. Typically, students first learn about adding as combining parts together. Then, students learn about adding as joining to a set.

For learning the concepts of addition, we recommend using *mathematics facts*. We define an addition mathematics fact as single-digit addends added for a single- or double-digit sum. You may present addition facts vertically or horizontally.



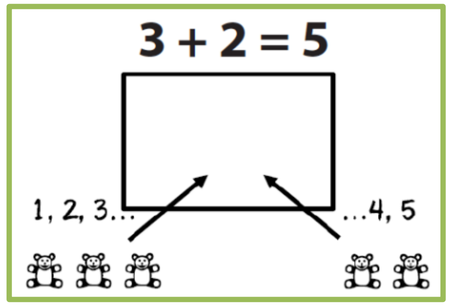
C. Routines and Examples

(1) Addition as Combining

Routine

Materials:

- Module 4 Addition Problems
- Module 4 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., clips, candies, cubes)



Teacher	**Let's work on addition. Today, let's think about addition as combining. What does it mean to combine?**
Students	Put together.
Teacher	**When we combine, we put things together. When you cook, you put ingredients together. For example, to make macaroni and cheese, you combine what?**
Students	Macaroni noodles and cheese!
Teacher	**That's right. You combine macaroni and cheese! Now, let's think about combining numbers. Look at this problem.** (Show problem.)
Teacher	**First, I notice a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**We'll add by combining. Let's show each addend with our clips. An addend is one of the numbers we add. Then we'll combine the clips for a sum. Let's do this together.** (Move clips to workspace.)
Teacher	**Our first addend is __. What's our first addend?**
Students	__.
Teacher	**Let's show this addend by showing __ clips.** (Show clips.)
Teacher	**How many clips?**
Students	__.
Teacher	**Our second addend is __. What's our second addend?**
Students	__.
Teacher	**Let's show the second addend by showing __ clips.** (Show clips.)
Teacher	**How many clips?**
Students	__.

Teacher	**So, we have __ plus __. Let's add by combining. What does combining mean?**
Students	To put together.
Teacher	**Yes. Let's combine, or put together, the __ clips and __ clips.** (Move two sets of clips together.)
Teacher	**To learn the sum, let's count the clips.** (Count clips.)
Teacher	**How many clips are there in total or altogether?**
Students	__.
Teacher	**Yes! There are __ clips. So, __ plus __ equals __. Let's say that together.**
Students	__ plus __ equals __.
Teacher	**Let's say it together again.**
Students	__ plus __ equals __.
Teacher	**So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or parts in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets.
Teacher	**What does it mean to combine?**
Students	To put together.
Teacher	**How could you explain combining to a friend?**
Students	We started with two different sets of clips. We combined the sets by putting all the clips together. The sum is the total number of clips.

Example

$$\begin{array}{r} 7 \\ +\ \ 4 \\ \hline 11 \end{array}$$

Teacher	**Let's work on addition. Today, let's think about addition as combining. What does it mean to combine?**
Students	Put together.
Teacher	**When we combine, we put things together. Let's think about combining numbers. Look at this problem.** (Show problem.)
Teacher	**First, I notice a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**We'll add by combining. Let's show each addend with our frogs. What's an addend?**
Students	An addend is one of the numbers we add.
Teacher	**Our first addend is 7. What's our first addend?**

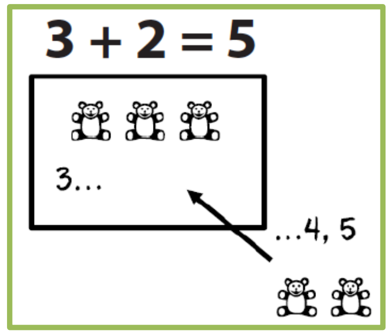
Students 7.
Teacher **Let's show this addend by showing 7 frogs.**
(Show 7 frogs.)
Teacher **How many frogs?**
Students 7.
Teacher **Our second addend is 4. What's our second addend?**
Students 4.
Teacher **Let's show the second addend by showing 4 frogs.**
(Show 4 frogs.)
Teacher **How many frogs?**
Students 4.
Teacher **So, we have 7 plus 4. Let's add by combining. What does combining mean?**
Students To put together.
Teacher **Yes. Let's combine, or put together, the 7 frogs and the 4 frogs.**
(Move two sets of frogs together.)
Teacher **To learn the sum, let's count the frogs. Count with me.**
(Count: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.)
Teacher **How many frogs are there in total or altogether?**
Students 11.
Teacher **Yes! There are 11 frogs. So, 7 plus 4 equals 11. Let's say that together.**
Students 7 plus 4 equals 11.
Teacher **Let's say it together again.**
Students 7 plus 4 equals 11.
Teacher **So, if you have a set of 7 and a set of 4, when you combine (or put together) the sets, the sum is 11. 7 plus 4 equals 11. Let's review. What's an addend?**
Students One of the sets or parts in an addition problem.
Teacher **What's a sum?**
Students The total number when you combine sets.
Teacher **What does it mean to combine?**
Students To put together.
Teacher **How could you explain combining to a friend?**
Students We started with two different sets of frogs. We combined the sets by putting all the frogs together. The sum is the total number of frogs.

(2) Addition as Joining

Routine

Materials:

- Module 4 Problems
- Module 4 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., clips, candies, cubes)



Teacher	**Let's work on addition. Today, let's think about addition as joining. What does it mean to join?**
Students	To add more to a set.
Teacher	**When we join, we add more to a group. When you're at recess and you want to join your friends, you walk to your friends and join their group. For example, if you want to join a sports team, what does that mean?**
Students	Become a member of the team and join other people to play a sport.
Teacher	**That's right. If you want to join a team, you become a member of the team. There are now more members on the team. Now, let's think about joining in addition. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Today we'll add by joining, but there are other ways to add. Let's start by showing the first addend with our candies and then joining more candies to that set for a sum. Let's do this together.** (Move candies to workspace.)
Teacher	**Our first addend is __. What's our first addend?**
Students	__.
Teacher	**Let's show this addend by showing __ candies.** (Show candies.)
Teacher	**How many candies?**
Students	__.
Teacher	**Our second addend is __. What's our second addend?**
Students	__.
Teacher	**Let's show the second addend by showing __ candies.** (Show candies.)
Teacher	**How many candies?**
Students	__.
Teacher	**Now, let's join the second addend to the first set of candies. We'll add by joining. What does joining mean?**

Students	To add more to a set.
Teacher	**Yes. Let’s join the second addend to the first set. We started with __ candies. How many candies?**
Students	__. (first addend)
Teacher	**To join, we count on from the first set. So, we started with __ candies and we join the second set of candies by counting on from __. Watch me: __** (first addend)**, __, __, __, …** (Add second set of candies to first set.)
Teacher	**The sum is the last number we said. We counted __. What’s the sum?**
Students	__.
Teacher	**How many candies are there in total or altogether?**
Students	__.
Teacher	**Yes! There are __ candies. So, __ plus __ equals __. Let’s say that together.**
Students	__ plus __ equals __.
Teacher	**Let’s say it together again.**
Students	__ plus __ equals __.
Teacher	**So, if you have a set of __ and join __ to the set, the sum is __. __ plus __ equals __. Let’s review. What’s an addend?**
Students	One of the sets or parts in an addition problem.
Teacher	**What’s a sum?**
Students	The total number when you join sets.
Teacher	**What does it mean to join?**
Students	To add more to a set.
Teacher	**How could you explain joining to a friend?**
Students	We started with one set of candies. We joined more candies to that set. The sum is the total number of candies.

Example

$$\begin{array}{r} 7 \\ +\ \ 4 \\ \hline 11 \end{array}$$

Teacher	**Let’s work on addition. Today, let’s think about addition as joining. What does it mean to join?**
Students	To add more to a set.
Teacher	**When we join, we add more to a group. Now, let’s think about joining in addition. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.

Teacher	**Today we'll add by joining, but there are other ways to add. Let's start by showing the first addend with our cubes and then joining more cubes to that set for a sum. Let's do this together.** (Move cubes to workspace.)
Teacher	**Our first addend is 7. What's our first addend?**
Students	7.
Teacher	**Let's show this addend by showing 7 cubes.** (Show 7 cubes.)
Teacher	**How many cubes?**
Students	7.
Teacher	**Our second addend is 4. What's our second addend?**
Students	4.
Teacher	**Let's show the second addend by showing 4 cubes.** (Show 4 cubes.)
Teacher	**How many cubes?**
Students	4.
Teacher	**Now, let's join the second addend to the first set of cubes. We'll add by joining. What does joining mean?**
Students	To add more to a set.
Teacher	**Yes. Let's join the second addend to the first set. We started with 7 cubes. How many cubes?**
Students	7.
Teacher	**To join, we count on from the first set. So, we started with 7 cubes and we join the second set of cubes by counting on from 7. Watch me: 7** (point to set of 7)**: 8** (add 1 cube)**, 9** (add 1 cube)**, 10** (add 1 cube)**, 11** (add 1 cube)**.**
Teacher	**The sum is the last number we said. We counted 11. What's the sum?**
Students	11.
Teacher	**How many cubes are there in total or altogether?**
Students	11.
Teacher	**Yes! There are 11 cubes. So, 7 plus 4 equals 11. Let's say that together.**
Students	7 plus 4 equals 11.
Teacher	**Let's say it together again.**
Students	7 plus 4 equals 11.
Teacher	**So, if you have a set of 7 and join 4 to the set, the sum is 11. 7 plus 4 equals 11. Let's review. What's an addend?**
Students	One of the sets or parts in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you join sets.
Teacher	**What does it mean to join?**
Students	To add more to a set.
Teacher	**How could you explain joining to a friend?**
Students	We started with one set of cubes. We joined more cubes to that set. The sum is the total number of cubes.

D. Problems for Use During Instruction

See Module 4 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 4 Vocabulary Cards.

F. Supplementary

COUNTING UP
Addition

1. Put the greater addend in your fist and say it.
2. Count up the other addend on your fingers.
3. The sum is the last number you say.

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Module 4:

Concepts of Addition

Problem Sets

$$\begin{array}{r} 1 \\ +\quad 0 \\ \hline \end{array}$$

$$
\begin{array}{r}
5 \\
+\quad 4 \\
\hline
\end{array}
$$

$$\begin{array}{r} 7 \\ +\quad 5 \\ \hline \end{array}$$

$$
\begin{array}{r}
2 \\
+\quad 1 \\
\hline
\end{array}
$$

$$\begin{array}{r} 7 \\ +\quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ + \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ +\quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +\quad 3 \\ \hline \end{array}$$

$$
\begin{array}{r} 7 \\ +\quad 9 \\ \hline \end{array}
$$

$$\begin{array}{r} 4 \\ +\quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ +\quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +\quad 6 \\ \hline \end{array}$$

$$
\begin{array}{r}
7 \\
+\quad 8 \\
\hline
\end{array}
$$

$$\begin{array}{r} 4 \\ +\quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ +\quad 8 \\ \hline \end{array}$$

$$
\begin{array}{r}
4 \\
+ \quad 7 \\
\hline
\end{array}
$$

$$\begin{array}{r} 3 \\ +\quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +\quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +\quad 3 \\ \hline \end{array}$$

$$
\begin{array}{r}
9 \\
+\quad 6 \\
\hline
\end{array}
$$

$$\begin{array}{r} 8 \\ +\quad 5 \\ \hline \end{array}$$

$$
\begin{array}{r}
3 \\
+ \quad 0 \\
\hline
\end{array}
$$

$$\begin{array}{r} 6 \\ + \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ +\quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ +\quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ + \quad 7 \\ \hline \end{array}$$

$$
\begin{array}{r} 0 \\ +\quad 5 \\ \hline \end{array}
$$

$$\begin{array}{r} 2 \\ +\quad 0 \\ \hline \end{array}$$

$$
\begin{array}{r} 7 \\ +\quad 4 \\ \hline \end{array}
$$

$$\begin{array}{r} 3 \\ +\quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ +\quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +\quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ +\quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +\quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ + \quad 1 \\ \hline \end{array}$$

$$
\begin{array}{r} 6 \\ +\quad 5 \\ \hline \end{array}
$$

$$\begin{array}{r} 8 \\ +\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ +\quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ + \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +\quad 3 \\ \hline \end{array}$$

$$
\begin{array}{r} 8 \\ +\quad 8 \\ \hline \end{array}
$$

$$\begin{array}{r} 5 \\ +\quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +\quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ +\quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +\quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +\quad 2 \\ \hline \end{array}$$

$$
\begin{array}{r} 0 \\ +\quad 9 \\ \hline \end{array}
$$

$$\begin{array}{r} 6 \\ +\quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +\quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +\quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ + \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ +\quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +\quad 5 \\ \hline \end{array}$$

$$
\begin{array}{r}
2 \\
+\quad 4 \\
\hline
\end{array}
$$

$$\begin{array}{r} 0 \\ +\quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +\quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +\quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +\quad 1 \\ \hline \end{array}$$

$$
\begin{array}{r}
5 \\
+\quad 6 \\
\hline
\end{array}
$$

Module 4:

Concepts of Addition

Vocabulary Cards

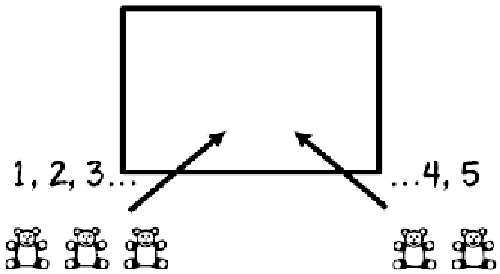
add/addition
addend
equal sign
join
plus sign
sum
together

add/addition

To put amounts together to find the sum or to increase a set.

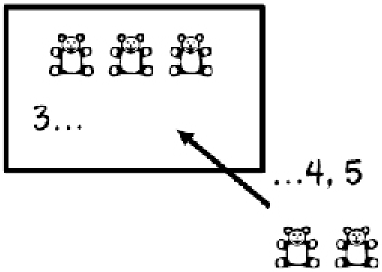
To put amounts together

3 + 2 = 5



To increase a set

3 + 2 = 5



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addend

Any numbers that are added together.

6 + 2 = 8

6 and 2 are addends

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

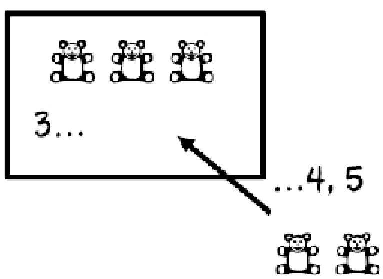
$$12 + 8 = 20$$

= is the equal sign

---

join

To add to an existing set.



plus sign

The symbol that tells you to add.

$$5 + 4 = 9$$

+ is the plus sign

---

sum

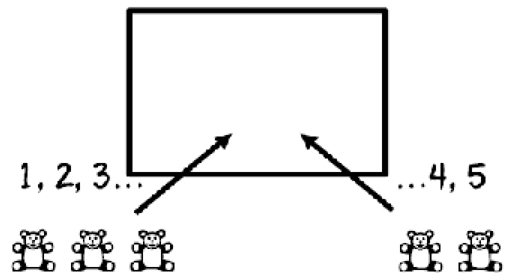
The result of adding two or more numbers or the total number when you combine sets.

$$7 + 2 + 1 = 10$$

10 is the sum

together

To combine sets or numbers.



Instructional Routines for Mathematics Intervention

MODULE 5

Addition of Whole Numbers

Module 5: Addition of Whole Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
add/addition	To put amounts together to find the sum or to increase a set.
addend	Any numbers that are added together.
algorithm	A procedure or description of steps that can be used to solve a problem.
computation	The action used to solve a problem.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
hundreds column	The column with digits in the hundreds place.
join	To add to an existing set.
ones column	The column with digits in the ones place.
plus sign	The symbol that tells you to add.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
sum	The result of adding two or more numbers or the total number when you combine sets.
tens column	The column with digits in the tens place.
together	To combine sets or numbers.

B. Background Information

If your focus is on the conceptual understanding of addition, see *Module 4: Concepts of Addition*. This module, *Module 5*, focuses on addition computation of whole numbers. As you focus on computation, continue to emphasize addition as combining and addition as joining to a set because students will see these concepts within word problems.

For learning computation with addition, we recommend presenting problems vertically. Some students may require explicit instruction on translating a horizontal problem (e.g., 17 + 59) to the vertical presentation (see below). Depending upon the algorithm, leave enough space above or below the problem for students to complete their written work.

Every student should develop efficiency with an addition computation strategy. In the following sections, we provide examples of (1) addition with a traditional algorithm – no regrouping, (2) addition with a traditional algorithm – regrouping, and (3) addition with a partial sums algorithm. Teachers should understand both the traditional and partial sums algorithms and help students develop competency with at least one algorithm.

Addition Computation

```
   1
   17  ←  addend
+  59  ←  addend
   76  ←  sum
```

C. Routines and Examples

(1) Addition with Traditional Algorithm – No Regrouping

Routine

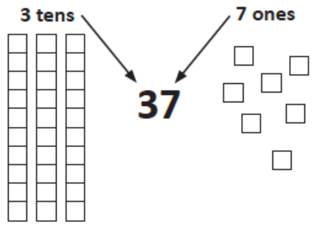
Materials:

- Module 5 Problem Sets
- Module 5 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
  - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT + 2-DIGIT: ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**With our Base-10 blocks, the rods represent tens. What do the rods represent?**
Students	Tens.
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**
Students	Ones.
Teacher	**Our first addend is __. What's our first addend?**
Students	__.

Teacher	**Let's show this addend by showing __ tens and __ ones.** (Show with Base-10 blocks.)



Teacher	**How many?**
Students	__.
Teacher	**Our second addend is __. What's our second addend?**
Students	__.
Teacher	**Let's show the second addend by showing __ tens and __ ones.** (Show with Base-10 blocks. Place Base-10 blocks under the first addend.)
Teacher	**How many?**
Students	__.
Teacher	**So, we have __ plus __. Let's add by combining. What does combining mean?**
Students	To put together.
Teacher	**Yes. Let's combine or put together. First, let's combine the ones. That means we put all the ones together.** (Move two sets of ones together.)
Teacher	**Let's count to learn the sum of the ones.** (Count ones.)
Teacher	**How many ones are there in total or altogether?**
Students	__.
Teacher	**Yes! There are __ ones. If we have more than 9 ones, we have to regroup. With addition, we regroup 10 ones for 1 ten. Do we have more than 9 ones?**
Students	No.
Teacher	**We don't have more than 9 ones, so we don't have to regroup. Now, let's combine the tens. That means we put all the tens together.** (Move two sets of tens together.)
Teacher	**How many tens are there in total or altogether?**
Students	__.
Teacher	**There are __ tens. If we have more than 9 tens, we have to regroup. Do we have more than 9 tens?**
Students	No.
Teacher	**We don't have more than 9 tens, so we don't have to regroup. So, let's count the tens and ones to learn the sum. Ready?** (Count the tens, then count the ones.)
Teacher	**That means __ plus __ equals __. Let's say that together.**
Students	__ plus __ equals __.
Teacher	**Let's say it together again.**
Students	__ plus __ equals __.

Teacher	**So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**How could you explain solving this problem to a friend?**
Students	We started by showing each addend. Then, we added the ones. We did not have to regroup. Then, we added the tens. We did not have to regroup. The sum was the total of tens and ones.

2-DIGIT + 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by adding the ones. What should we add first?**
Students	The ones.
Teacher	**Which ones do we add?**
Students	__ plus __.
Teacher	**What's __ plus __?** (If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up __ more. Ready? __: __, __, __.** See Counting Up poster at the end of Module 4 for more information.)
Teacher	**How many ones are there in total or altogether?**
Students	__.
Teacher	**Yes! There are __ ones. If we have more than 9 ones, we have to regroup. Do we have more than 9 ones?**
Students	No.
Teacher	**We don't have more than 9 ones, so we don't have to regroup. Let's write the ones below the equal line.** (Writes.)
Teacher	**Now, let's add the tens. Which tens do we add?**

Students	__ plus __.
Teacher	**What's __ plus __?**
	(If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up __ more. Ready? __: __, __, __.** See Counting Up poster at the end of Module 4 for more information.)
Teacher	**How many tens are there in total or altogether?**
Students	__.
Teacher	**There are __ tens. If we have more than 9 tens, we have to regroup. Do we have more than 9 tens?**
Students	No.
Teacher	**We don't have more than 9 tens, so we don't have to regroup. Let's write the tens below the equal line.**
	(Write.)
Teacher	**So, what's __ plus __?**
Students	__.
Teacher	**That's right. __ plus __ equals __. Let's say that together.**
Students	__ plus __ equals __.
Teacher	**So, if you have a set of __ and a set of __, when you combine (or join) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**How could you explain solving this problem to a friend?**
Students	First, we combined the ones. Then, we combined the tens. The sum is the total number of tens and ones.

Example

$$\begin{array}{r} 224 \\ +\ \ 63 \\ \hline 287 \end{array}$$

3-DIGIT + 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.**
	(Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.

Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens and the tens from the hundreds. Let's draw a vertical line between the ones column and the tens column. Then, draw a vertical line between the tens column and the hundreds column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by adding the ones. What should we add first?**
Students	The ones.
Teacher	**Which ones do we add?**
Students	4 plus 3.
Teacher	**What's 4 plus 3?** (If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up 3 more. Ready? 4: 5, 6, 7.** See Counting Up poster at the end of Module 4 for more information.)
Teacher	**How many ones are there in total or altogether?**
Students	7.
Teacher	**Yes! There are 7 ones. If we have more than 9 ones, we have to regroup. Do we have more than 9 ones?**
Students	No.
Teacher	**We don't have more than 9 ones, so we don't have to regroup. Let's write the ones below the equal line in the one place.** (Write.)
Teacher	**Now, let's add the tens. Which tens do we add?**
Students	2 plus 6.
Teacher	**What's 2 plus 6?** (If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up 2 more. Ready? 6: 7, 8.** See Counting Up poster at the end of Module 4 for more information.)
Teacher	**How many tens are there in total or altogether?**
Students	8.
Teacher	**There are 8 tens. If we have more than 9 tens, we have to regroup. Do we have more than 9 tens?**
Students	No.
Teacher	**We don't have more than 9 tens, so we don't have to regroup. Let's write the tens below the equal line in the tens place.** (Write.)
Teacher	**Now, let's add the hundreds. Which hundreds do we add?**
Students	2.
Teacher	**Yes. There's only 2 in the hundreds column. We can think of this as 2 plus 0 or 2. Let's write the hundreds below the equal line in the hundreds place.** (Write.)
Teacher	**So, let's look at our sum. What's 224 plus 63?**
Students	287.
Teacher	**That's right. 224 plus 63 equals 287. Let's say that together.**

Students	224 plus 63 equals 287.
Teacher	**So, if you have a set of 224 and a set of 63, when you combine (or join) the sets, the sum is 287. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**How could you explain solving this problem to a friend?**
Students	First, we combined the ones. Then, we combined the tens. Then, we added the hundreds. The sum is the total number of hundreds, tens, and ones.

(2) Addition with Traditional Algorithm – Regrouping

Routine

Materials:

- Module 5 Problem Sets
- Module 5 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
  - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT + 2-DIGIT: ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**With our Base-10 blocks, the rods represent tens. What do the rods represent?**
Students	Tens.
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**
Students	Ones.
Teacher	**Our first addend is __. What's our first addend?**
Students	__.
Teacher	**Let's show this addend by showing __ tens and __ ones.** (Show with Base-10 blocks.)
Teacher	**How many?**

Students __.
Teacher **Our second addend is __. What's our second addend?**
Students __.
Teacher **Let's show the second addend by showing __ tens and __ ones.**
(Show with Base-10 blocks. Place Base-10 blocks under the second addend.)
Teacher **How many?**
Students __.
Teacher **So, we have __ plus __. Let's add by combining. What does combining mean?**
Students To put together.
Teacher **Yes. Let's combine or put together. First, let's combine the ones. That means we put all the ones together.**
(Move two sets of ones together.)
Teacher **Let's count to learn the sum of the ones.**
(Count ones.)
Teacher **How many ones are there in total or altogether?**
Students __.
Teacher **Yes! There are __ ones. If we have more than 9 ones, we have to regroup. Do we have more than 9 ones?**
Students Yes.
Teacher **We have more than 9 ones. That means we have to regroup. To regroup, we count 10 ones and regroup/trade/exchange the 10 ones for 1 ten. Let's do that together. Let's count out 10 ones.**
(Count 10 ones.)
Teacher **Let's regroup/trade/exchange the 10 ones for 1 ten. See how 1 ten is the same as 10 ones?**
Students Yes.
Teacher **We leave the remaining ones here. But we can't put this 1 ten in the ones place. The ones place is only for ones. So, we place the 1 ten in the tens column. I like to place the 1 ten above the other tens.**
(Place 1 ten above tens column.)
Teacher **Now, let's combine the tens. That means we put all the tens together.**
(Move sets of tens together.)
Teacher **How many tens are there in total or altogether?**
Students __.
Teacher **There are __ tens. If we have more than 9 tens, we have to regroup. Do we have more than 9 tens?**
Students No.
Teacher **So, let's count the tens and ones to learn the sum. Ready?**
(Count the tens, then count the ones.)
Teacher **That means __ plus __ equals __. Let's say that together.**
Students __ plus __ equals __.
Teacher **Let's say it together again.**
Students __ plus __ equals __.

Teacher	**So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**What does it mean to regroup/trade/exchange?**
Students	You can regroup/trade/exchange 10 ones for 1 ten.
Teacher	**How could you explain solving this problem to a friend?**
Students	We started by showing each addend. Then, we combined the ones. We regrouped 10 ones for 1 ten. Then, we combined the tens. The sum was the total number of tens and ones.

2-DIGIT + 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by adding the ones. What should we add first?**
Students	The ones.
Teacher	**Which ones do we add?**
Students	__ plus __.
Teacher	**What's __ plus __?** (If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up __ more. Ready? __: __, __, __.** See Counting Up poster at the end of Module 4 for more information.)
Teacher	**How many ones are there in total or altogether?**
Students	__.
Teacher	**Yes! There are __ ones. If we have more than 9 ones, we have to regroup. Do we have more than 9 ones?**
Students	Yes.

Teacher	**We have more than 9 ones. That means we have to regroup. We think of our ones sum as 1 ten and __ ones. We write the ones in the ones column under the equal line.** (Write ones under equal line.)
Teacher	**We regroup the 1 ten to the tens column. We write the 1 ten in the tens column above the other tens.** (Write 1 above tens column.)
Teacher	**Now, let's add the tens. Which tens do we add?**
Students	__ plus __ plus __.
Teacher	**What's __ plus __ plus __?**
Students	__.
Teacher	**How many tens are there in total or altogether?**
Students	__.
Teacher	**There are __ tens. If we have more than 9 tens, we have to regroup. Do we have more than 9 tens?**
Students	No.
Teacher	**Let's write the tens below the equal line in the tens column.** (Write.)
Teacher	**So, let's look at the problem. What's __ plus __?**
Students	__.
Teacher	**That's right. __ plus __ equals __. Let's say that together.**
Students	__ plus __ equals __.
Teacher	**So, if you have a set of __ and a set of __, when you combine (or join) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**What does it mean to regroup/trade/exchange?**
Students	You can regroup/trade/exchange 10 ones for 1 ten.
Teacher	**How could you explain solving this problem to a friend?**
Students	First, we combined the ones. We regrouped 10 ones for 1 ten. Then, we combined the tens. The sum was the total number of tens and ones.

Example

$$\begin{array}{r} 153 \\ +\ \ 79 \\ \hline 232 \end{array}$$

3-DIGIT + 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.

Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens and the tens from the hundreds. Let's draw a vertical line between the ones column and the tens column. Then, let's draw a vertical line between the tens column and the hundreds column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by adding the ones. What should we add first?**
Students	The ones.
Teacher	**Which ones do we add?**
Students	3 plus 9.
Teacher	**What's 3 plus 9?** (If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up 3 more. Ready? 9: 10, 11, 12.** See Counting Up poster in Module 4 for more information.)
Teacher	**How many ones are there in total or altogether?**
Students	12.
Teacher	**Yes! There are 12 ones. If we have more than 9 ones, we have to regroup. Do we have more than 9 ones?**
Students	Yes.
Teacher	**We have more than 9 ones. That means we have to regroup. We think of our ones sum as 1 ten and 2 ones. We write the 2 in the ones column under the equal line.** (Write ones under equal line.)
Teacher	**We regroup the 1 ten to the tens column. We write the 1 ten in the tens column above the other tens.** (Write 1 above tens column.)
Teacher	**Now, let's add the tens. Which tens do we add?**
Students	1 plus 5 plus 7.
Teacher	**That's right. Don't forget to add the 1 ten you just regrouped. What's 1 plus 5 plus 7?**
Teacher	**How many tens are there in total or altogether?**
Students	13.
Teacher	**There are 13 tens. If we have more than 9 tens, we have to regroup. Do we have more than 9 tens?**
Students	Yes.
Teacher	**We have 13 tens. That means we have to regroup. We think of our tens sum as 1 hundred and 3 tens. 13 tens is the same as 1 hundred and 3 tens. We write the 3 in the tens column under the equal line.** (Write tens under equal line.)

Teacher	**We regroup the 1 hundred to the hundreds column. We write the 1 hundred in the hundreds column above the other hundreds.** (Write 1 above hundreds column.)
Teacher	**Let's add the hundreds. Which hundreds do we add?**
Students	1 plus 1.
Teacher	**That's right! Don't forget to add the 1 hundred you just regrouped. What's 1 plus 1?**
Students	2.
Teacher	**How many hundreds are there in total or altogether?**
Students	2.
Teacher	**If you have more than 9 hundreds, we have to regroup. Do we have more than 9 hundreds?**
Students	No.
Teacher	**We don't have to regroup. Let's just write 2 under the equal line.** (Write hundreds under equal line.)
Teacher	**So, let's look at the problem. What's 153 plus 79?**
Students	232.
Teacher	**That's right. 153 plus 79 equals 232. Let's say that together.**
Students	153 plus 79 equals 232.
Teacher	**Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**What does it mean to regroup/trade/exchange?**
Students	You can regroup/trade/exchange 10 ones for 1 ten.
Teacher	**How could you explain solving this problem to a friend?**
Students	First, we combined the ones. We regrouped 10 ones for 1 ten. Then, we combined the tens. We regrouped 10 tens for 1 hundred. Then, we added the hundreds. The sum was the total of hundreds, tens, and ones.

(3) Addition with Partial Sums Algorithm

Routine

Materials:

- Module 5 Problem Sets
- Module 5 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
  - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT + 2-DIGIT: ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**With our Base-10 blocks, the rods represent tens. What do the rods represent?**
Students	Tens.
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**
Students	Ones.
Teacher	**Our first addend is __. What's our first addend?**
Students	__.
Teacher	**Let's show this addend by showing __ tens and __ ones.** (Show with Base-10 blocks.)
Teacher	**How many?**
Students	__.
Teacher	**Our second addend is __. What's our second addend?**
Students	__.
Teacher	**Let's show the second addend by showing __ tens and __ ones.** (Show with Base-10 blocks. Place Base-10 blocks under the first addend.)
Teacher	**How many?**
Students	__.
Teacher	**So, we have __ plus __. Let's add by combining the partial sums. What does combining mean?**
Students	To put together.
Teacher	**Yes. Let's combine or put together. First, let's combine the tens. This will be our first partial sum. It's the sum for part of the problem. Adding the tens means we put all the tens together.** (Move two sets of tens together.)
Teacher	**Let's count to learn the sum of the tens.** (Count tens.)
Teacher	**How many tens are there in total or altogether?**
Students	__.
Teacher	**This __ is one of our partial sums. It's the sum of the tens. Now, let's combine the ones. That means we put all the ones together.** (Move ones together.)
Teacher	**How many ones are there in total or altogether?**
Students	__.

Teacher	**This __ is one of our partial sums. It's the sum of the ones. To determine the total sum, we add __ plus __.** (Start with tens and add ones.)
Teacher	**That means __ plus __ equals __. Let's say that together.**
Students	__ plus __ equals __.
Teacher	**Let's say it together again.**
Students	__ plus __ equals __.
Teacher	**So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**How could you explain solving this problem to a friend?**
Students	We started by showing each addend. Then, we combined the tens. Then, we combined the ones. We added the partial sums of the tens and ones. The sum was the total number of tens and ones.

2-DIGIT + 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw a vertical line to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.** (Draw vertical lines to separate place value columns.)
Teacher	**Today, let's use the partial sums strategy. With the partial sum strategy, we add the tens then we add the ones. Then, we add the partial sums from the tens and ones together. Now, we start by adding the greatest place value in the problem - the tens. What should we add first?**
Students	The tens.
Teacher	**Which tens do we add?**
Students	__ plus __.
Teacher	**What's __ plus __?** (If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up __ more. Ready? __: __, __, __.** See Counting Up poster at the end of Module 4 for more information.)

Teacher **How many tens are there in total or altogether?**
Students __.
Teacher **So, let's write __ under the equal line.**
(Write tens.)
Teacher **Now, let's add the ones. Which ones do we add?**
Students __ plus __.
Teacher **What's __ plus __?**
(If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up __ more. Ready? __: __, __, __.** See Counting Up poster at the end of Module 4 for more information.)
Teacher **How many ones are there in total or altogether?**
Students __.
Teacher **So, let's write __ under the equal line.**
(Write ones.)
Teacher **Now, let's add the partial sums. What's __ plus __?**
Students __.
Teacher **That's right. __ plus __ equals __. Let's write the total sum.**
Students (Writes sum.)
Teacher **So, if you have a set of __ and a set of __, when you combine (or join) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students One of the sets or numbers added together in an addition problem.
Teacher **What's a sum?**
Students The total number when you combine sets, or the result of adding two or more numbers together.
Teacher **How could you explain solving this problem to a friend?**
Students We combined the tens. Then, we combined the ones. We added the partial sums of the tens and ones. The sum was the total number of tens and ones.

Example

$$\begin{array}{r} 259 \\ +\ \ 75 \\ \hline 334 \end{array}$$

3-DIGIT + 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher **Let's work on addition. What does it mean to add?**
Students To put together or to join to a set.
Teacher **Addition means to put together or to join to a set. Look at this problem.**
(Show problem.)
Teacher **First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students To add.

Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens and the tens from the hundreds. Let's draw a vertical line between the ones column and the tens column. Then, let's draw a vertical line between the tens column and the hundreds column.** (Draw vertical lines to separate place value columns.)
Teacher	**Today, let's use the partial sums strategy. We'll add the hundreds to determine a partial sum. Then, we'll add the tens to determine a partial sum. Then, we'll add the ones to determine a partial sum. To calculate the total sum, we add all the partial sums. What's this strategy called?**
Students	Partial sums.
Teacher	**Now, we start the partial sums strategy by adding the greatest place value. What should we add first?**
Students	The hundreds
Teacher	**Which hundreds do we add?**
Students	200 plus 0 hundreds.
Teacher	**We have 200 added to 0 hundreds. What's 200 plus 0?**
Students	200.
Teacher	**So, let's write 200 under the equal line. Make sure to place the 2 in the hundreds column, the 0 in the tens column, and the other 0 in the ones column.** (Write 200.)
Teacher	**200 is the partial sum when you add the hundreds. Now, let's add the tens. Which tens do we add?**
Students	50 plus 70.
Teacher	**That's right. We had 50 plus 70. 5 tens is 50 and 7 tens is 70. What's 50 plus 70?**
Students	120.
Teacher	**120 is how many hundreds, tens, and ones?**
Students	1 hundred, 2 tens, and 0 ones.
Teacher	**So, write 1 hundred, 2 tens, and 0 ones below the 200.** (Write 120.)
Teacher	**120 is the partial sum when you add the tens. Now, let's add the ones. Which ones do we add?**
Students	9 plus 5.
Teacher	**What's 9 plus 5?**
Students	14.
Teacher	**14 is how many tens and ones?**
Students	1 ten and 4 ones.
Teacher	**Let's write 1 tens and 4 ones below the 120.** (Write 14.)
Teacher	**Now, let's add the partial sums. Let's add in steps. What's 200 plus 120?**
Students	320.
Teacher	**Then, what's 320 plus 14?**

Students 334.

Teacher That's right. 200 plus 120 plus 14 equals 334. That's the total sum!

Students (Write 334.)

Teacher So, if you have a set of 259 and a set of 75, when you combine (or join) the sets, the sum is 334. 259 plus 75 is 334. Let's review. What's an addend?

Students One of the sets or numbers added together in an addition problem.

Teacher What's a sum?

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher How could you explain solving this problem to a friend?

Students We added the hundreds. Then, we added the tens. Then, we added the ones. We added the partial sums of the hundreds, tens, and ones. The sum was the total of the partial sums.

D. Problems for Use During Instruction

See Module 5 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 5 Vocabulary Cards.

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Module 5:
Addition of Whole Numbers

Problem Sets

A.

$$\begin{array}{r} 52 \\ +\ 32 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 46 \\ +\ 51 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 42 \\ +\ 12 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 53 \\ +\ 31 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 82 \\ +\ 11 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 35 \\ +\ 22 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 25 \\ +\ 33 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 48 \\ +\ 20 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 30 \\ +\ 18 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 60 \\ +\ 19 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 87 \\ +\ 10 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 11 \\ +\ 56 \\ \hline \end{array}$$

A.

$$
\begin{array}{r} 10 \\ +\ 66 \\ \hline \end{array}
$$

A.

$$\begin{array}{r} 65 \\ +\ 12 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 70 \\ +\ 17 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 29 \\ +\ 10 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 11 \\ +\ 36 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 39 \\ +\ 50 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 46 \\ +\ 42 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 22 \\ +\ 33 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 15 \\ +\ 89 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 52 \\ +\ 78 \\ \hline \end{array}$$

B.

$$
\begin{array}{r}
74 \\
+\ 67 \\
\hline
\end{array}
$$

B.

$$\begin{array}{r} 97 \\ +\ 56 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 84 \\ +\ 36 \\ \hline \end{array}$$

B.

$$
\begin{array}{r}
54 \\
+\ 88 \\
\hline
\end{array}
$$

B.

$$\begin{array}{r} 98 \\ +\ 93 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 19 \\ +\ 92 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 43 \\ +\ 67 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 54 \\ +\ 57 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 44 \\ +\ 78 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 48 \\ +\ 92 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 39 \\ +\ 47 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 74 \\ +\ 96 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 44 \\ +\ 88 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 91 \\ +\ 39 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 62 \\ +\ 69 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 67 \\ +\ 77 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 56 \\ +\ 29 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 44 \\ +\ 66 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 854 \\ +\,130 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 220 \\ +\,542 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 226 \\ +\ 633 \\ \hline \end{array}$$

c.

$$
\begin{array}{r}
731 \\
+\ 241 \\
\hline
\end{array}
$$

c.

$$\begin{array}{r} 320 \\ +\ 139 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 395 \\ +\ 103 \\ \hline \end{array}$$

c.

$$
\begin{array}{r}
151 \\
+\,313 \\
\hline
\end{array}
$$

c.

$$\begin{array}{r} 703 \\ +\ 202 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 117 \\ +\ 120 \\ \hline \end{array}$$

c.

$$
\begin{array}{r} 100 \\ +\ 490 \\ \hline \end{array}
$$

D.

$$\begin{array}{r} 967 \\ +\ 244 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 134 \\ +\,519 \\ \hline \end{array}$$

D.

$$
\begin{array}{r}
806 \\
+\,586 \\
\hline
\end{array}
$$

D.

$$\begin{array}{r} 888 \\ +\,453 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 656 \\ +\,615 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 267 \\ +\ 155 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 338 \\ +\ 374 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 792 \\ +\,638 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 897 \\ +\ 565 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 907 \\ +\,444 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 965 \\ +\quad 30 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 610 \\ +\quad 43 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 700 \\ +\quad 97 \\ \hline \end{array}$$

E.

$$
\begin{array}{r} 418 \\ +\quad 60 \\ \hline \end{array}
$$

E.

$$\begin{array}{r} 506 \\ +\quad 43 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 409 \\ +\ \ 89 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 527 \\ +\quad 74 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 326 \\ +\quad 37 \\ \hline \end{array}$$

F.

$$
\begin{array}{r}
256 \\
+ \quad 44 \\
\hline
\end{array}
$$

F.

$$\begin{array}{r} 945 \\ +\quad 69 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 11 \\ +\quad 3 \\ \hline \end{array}$$

G.

$$
\begin{array}{r}
76 \\
+\quad 2 \\
\hline
\end{array}
$$

G.

$$\begin{array}{r} 83 \\ +\quad 5 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 37 \\ +\quad 2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 24 \\ +\quad 5 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 16 \\ +\quad 4 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 25 \\ +\quad 8 \\ \hline \end{array}$$

H.

$$
\begin{array}{r}
46 \\
+\quad 5 \\
\hline
\end{array}
$$

H.

$$\begin{array}{r} 58 \\ +\quad 7 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 83 \\ +\quad 9 \\ \hline \end{array}$$

Module 5:

Addition of Whole Numbers

Vocabulary Cards

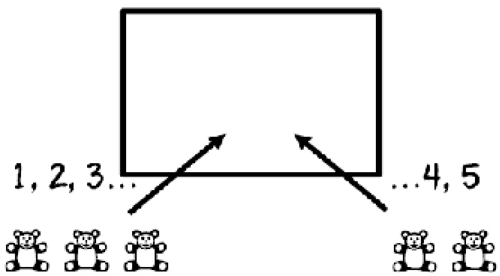
add/addition
addend
algorithm
computation
equal sign
hundreds column
join
ones column
plus sign
regroup/trade/exchange
sum
tens column
together

add/addition

To put amounts together to find the sum or to increase a set.

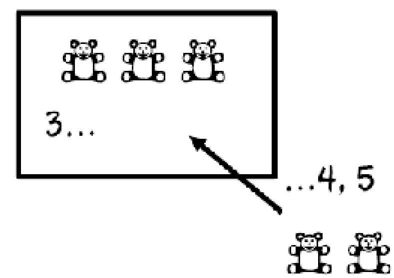
To put amounts together

3 + 2 = 5



To increase a set

3 + 2 = 5



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addend

Any numbers that are added together.

6 + 2 = 8

6 and 2 are addends

algorithm

A procedure or description of steps that can be used to solve a problem.

---

computation

The action used to solve a problem.

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$12 + 8 = 20$$

= is the equal sign

---

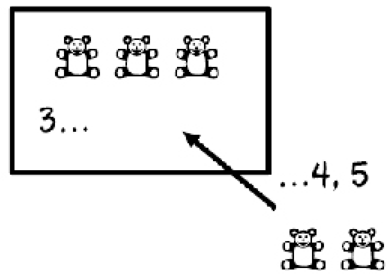
hundreds column

The column with digits in the hundreds place.

In the number 423, 4 is in the hundreds column.

join

To add to an existing set.



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ones column

The column with digits in the ones place.

In the number 423, 3 is in the ones place.

plus sign

The symbol that tells you to add.

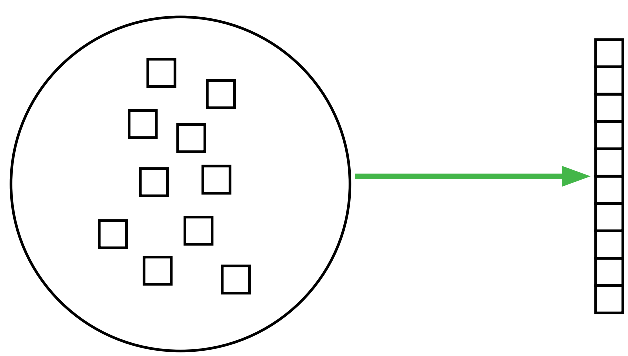
5 + 4 = 9

+ is the plus sign

---

regroup/trade/exchange

The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



sum

The result of adding two or more numbers or the total number when you combine sets.

$$7 + 2 + 1 = 10$$

10 is the sum

---

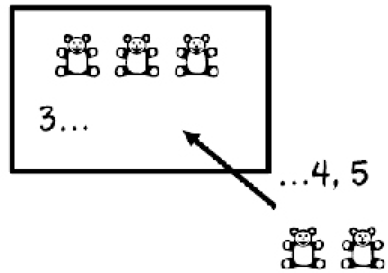
tens column

The column with digits in the tens place.

In the number 423, 2 is the in the tens column.

together

To combine sets or numbers.



Instructional Routines for Mathematics Intervention

MODULE 6

Addition of Rational Numbers

Module 6: Addition of Rational Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
add/addition	To put amounts together to find the sum or to increase a set.
addend	Any numbers that are added together.
algorithm	A procedure or description of steps that can be used to solve a problem.
computation	The action used to solve a problem.
decimal	A number based on powers of ten.
denominator	The term in a fraction that tells the number of equal parts in a whole.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
hundredths	The digit in representing $\frac{1}{100}$.
improper fraction	Any fraction in which the numerator is greater than or equal to the denominator.
join	To add to an existing set.
least common multiple	The common multiple with the least value.
mixed number	A whole number and a fraction combined.
multiple	The product of a number and any integer.
numerator	The term in a fraction that tells how many parts of a fraction.
ones	The digit representing 1.
plus sign	The symbol that tells you to add.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
sum	The result of adding two or more numbers.
tenths	The digit in representing $\frac{1}{10}$.
together	To combine sets or numbers.

B. Background Information

In this module, we focus on addition with fractions and decimals. As you focus on computation of rational numbers, continue to emphasize addition as combining and addition as joining to a set because students will see these concepts within word problems.

For addition of fractions, we recommend using several models of fractions to help students understand concepts related to addition of fractions. We also recommend demonstrating several algorithms for addition of decimals. Every student should develop efficiency with strategies for addition of fractions and decimals. In the following sections, we provide examples of (1) addition of fractions – like denominators, (2) addition of fractions – unlike denominators, (3) addition of decimals with traditional algorithm, and (4) addition of decimals with partial sums algorithm.

C. Routines and Examples

(1) Addition of Fractions – Like Denominators

Routine

Materials:

- Module 6 Problem Sets
- Module 6 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
    - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point)**. The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with fraction tiles.** (Move fraction tiles to workspace.)
Teacher	**Our first addend is __. What's our first addend?**
Students	__.
Teacher	**Let's show this addend by showing the fraction.** (Show fraction part compared to whole.)
Teacher	**What fraction?**

Students __.
Teacher **Our second addend is __. What's our second addend?**
Students __.
Teacher **Let's show the second addend by showing the fraction.**
(Show fraction part compared to whole.)
Teacher **What fraction?**
Students __.
Teacher **So, we have __ plus __. Let's add by combining. What does combining mean?**
Students To put together.
Teacher **Yes. Let's combine, or put together, the parts of the fraction. The parts of the fraction represent the numerator. When adding fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?**
Students Yes.
Teacher **The denominators are the same. Second, we want to add the parts or numerators of each fraction. That means we have to add __ one-__ parts and __ one-__ parts. What do we add?**
Students We add the parts or numerator of the fraction.
Teacher **Let's combine the parts together.**
Students (Combine parts, compare to whole.)
Teacher **So, we now have __, __, __, … one-__ parts. How many parts?**
Students __.
Teacher **When you have __ plus __, the sum is __. What's the sum?**
Students __.
Teacher **__ plus __ equals __. Let's say that together.**
Students __ plus __ equals __.
Teacher **So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students One of the sets or numbers added together in an addition problem.
Teacher **What's a sum?**
Students The total number when you combine sets, or the result of adding two or more numbers together.
Teacher **What do you add when you add fractions?**
Students The parts or numerator of each fraction.
Teacher **How could you explain solving this problem to a friend?**
Students We started by showing each addend. Then, we added the parts or numerator together to determine the sum.

ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on addition. What does it mean to add?**
Students To put together or to join to a set.

Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Our first addend is __. What's our first addend?**
Students	__.
Teacher	**Our second addend is __. What's our second addend?**
Students	__.
Teacher	**So, we have __ plus __. Let's add by combining. What does combining mean?**
Students	To put together.
Teacher	**Yes. Let's combine, or put together, the parts of the fraction. The parts of the fraction are the numerators. When adding fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?**
Students	Yes.
Teacher	**The denominators are the same. The denominator, __, will not change when we add the fractions. Let's go ahead and write the denominator for our sum.** (Write denominator.)
Teacher	**Now, we want to add the parts or numerator of each fraction. That means we have to add __ one-__ parts and __ one-__ parts. What do we add?**
Students	We add the parts or numerators of the fraction.
Teacher	**Let's combine the parts together. What's __ plus __?**
Students	__.
Teacher	**Let's write the parts we added together.** (Write parts.)
Teacher	**When you have __ plus __, the sum is __. What's the sum?**
Students	__.
Teacher	**__ plus __ equals __. Let's say that together.**
Students	__ plus __ equals __.
Teacher	**So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**What do you add when you add fractions?**
Students	The parts or numerator of each fraction.
Teacher	**How could you explain solving this problem to a friend?**
Students	We determined the denominators of the fraction were the same. We added the parts of the fraction to determine the sum.

Example

$$\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$

EXAMPLE WITH MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point)**. The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with fraction tiles.** (Move fraction tiles to workspace.)
Teacher	**Our first addend is $\frac{2}{8}$. What's our first addend?**
Students	$\frac{2}{8}$.
Teacher	**Let's show this addend by showing the fraction.** (Show 2 one-eighth parts compared to a whole.)
Teacher	**What fraction?**
Students	$\frac{2}{8}$.
Teacher	**Our second addend is $\frac{3}{8}$. What's our second addend?**
Students	$\frac{3}{8}$.
Teacher	**Let's show the second addend by showing the fraction.** (Show 3 one-eighth parts compared to a whole.)
Teacher	**What fraction?**
Students	$\frac{3}{8}$.
Teacher	**So, we have $\frac{2}{8}$ plus $\frac{3}{8}$. Let's add by combining. What does combining mean?**
Students	To put together.
Teacher	**Yes. Let's combine, or put together, the parts of the fraction. The parts of the fractions represent the numerators. When adding fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?**
Students	Yes.
Teacher	**Both denominators are 8. The denominators are the same or like denominators. Second, we want to add the numerators, or parts, of each fraction. That means we have to add 2 one-eighth parts and 3 one-eighth parts. What do we add?**
Students	We add the parts or numerators of the fraction.
Teacher	**Let's combine the parts together. That means we're combining the numerators.**

(Combine parts, compare to whole.)

Teacher **So, we now have 1, 2, 3, 4, 5 one-eighth parts. How many parts?**

Students 5 one-eighth parts.

Teacher **When you have $\frac{2}{8}$ plus $\frac{3}{8}$, the sum is $\frac{5}{8}$. What's the sum?**

Students $\frac{5}{8}$.

Teacher **$\frac{2}{8}$ plus $\frac{3}{8}$ equals $\frac{5}{8}$. Let's say that together.**

Students $\frac{2}{8}$ plus $\frac{3}{8}$ equals $\frac{5}{8}$.

Teacher **So, if you have a set of $\frac{2}{8}$ and a set of $\frac{3}{8}$, when you combine (or put together) the parts or numerators of each fraction, the sum is $\frac{5}{8}$. $\frac{2}{8}$ plus $\frac{3}{8}$ equals $\frac{5}{8}$. Let's review. What's an addend?**

Students One of the sets or numbers added together in an addition problem.

Teacher **What's a sum?**

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher **What do you add when you add fractions?**

Students The parts or numerators of each fraction.

Teacher **How could you explain solving this problem to a friend?**

Students We started by showing each addend. We checked whether there were like denominators, then added the parts or numerators together to determine the sum.

(2) Addition of Fractions – Unlike Denominators

Routine

Materials:

- Module 6 Problem Sets
- Module 6 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
  - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher **Let's work on addition. What does it mean to add?**

Students To put together or to join to a set.

Teacher **Addition means to put together or to join to a set. Look at this problem.** (Show problem.)

Teacher **First, I see a plus sign** (point)**. The plus sign tells us to add. What does the plus sign mean?**

Students To add.

Teacher **Let's do this problem with two-color counters.**

	(Move two-color counters to workspace.)
Teacher	**Our first addend is __. What's our first addend?**
Students	__.
Teacher	**Let's show this addend by showing the fraction.** (Show set compared to whole with white/yellow counters representing numerator and red counters representing denominator.)
Teacher	**What fraction?**
Students	__.
Teacher	**Our second addend is __. What's our second addend?**
Students	__.
Teacher	**Let's show the second addend by showing the fraction.** (Show set compared to whole with white/yellow counters representing numerator and red counters representing denominator.)
Teacher	**What fraction?**
Students	__.
Teacher	**So, we have __ plus __. Let's add by combining. What does combining mean?**
Students	To put together.
Teacher	**Yes. Let's combine, or put together, the parts of the fraction. Remember, the parts of the fractions represent the numerators. When adding fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?**
Students	No.
Teacher	**The denominators are not the same. To add, we should add parts or numerators with the same denominator. When the denominators are unlike, the parts or numerators do not have the same value. So, we will work to make the fractions have like denominators. Why do we want to add fractions with like denominators?**
Students	So, we can add the parts or numerators of the fraction.
Teacher	**To do this, let's write the first five multiples of each denominator. The first addend has a denominator of __, so let's write the first five multiples of __.** (Write multiples as __, __, __, __, __.)
Teacher	**What are the multiples of __? Say them with me.**
Students	__, __, __, __, __.
Teacher	**The second addend has a denominator of __, so let's write the first five multiples of __.** (Write multiples as __, __, __, __, __.)
Teacher	**What are the multiples of __? Say them with me.**
Students	__, __, __, __, __.
Teacher	**Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?**
Students	__.
Teacher	**So, __ is the least common multiple. Say that with me.**
Students	Least common multiple.

Teacher	**Sometimes we call the least common multiple the LCM. What do we call the least common multiple?**
Students	LCM.
Teacher	**The least common multiple, or LCM, helps us to determine the common denominator for the two fractions. What does the LCM help with?**
Students	Finding a common denominator for the two fractions.
Teacher	**The first addend has a denominator of __.**
	OPTION 1: **This is the original denominator. We don't have to do anything to this fraction.**
	OPTION 2: **This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.**
	What do we need to do?
Students	*OPTION 1:* We don't have to change the denominator.
	OPTION 2: We need to convert the fraction to a denominator of __.
Teacher	***OPTION 2:*** **To convert the fraction to a denominator of __, I determine how many groups of __** (original denominator) **I need to make __** (common denominator)**. I see I need to make __, __, __ groups of __** (original denominator)**. How many groups?**
Students	__.
Teacher	**So, I make __ groups of __ with the two-color counters. That means I iterate or copy the original fraction __ times. What does it mean to iterate?**
Students	To copy.
Teacher	**Our new fraction is __. Is __** (original fraction) **equivalent to __** (fraction with common denominator)**?**
Students	Yes.
Teacher	**How do you know the fractions are equivalent?**
Students	The fractions have the same value. They are equivalent.
Teacher	**So, we converted the first addend to a common denominator. Let's do the same with the second addend. What's the second addend?**
	__.
Teacher	**The second addend has a denominator of __.**
	OPTION 1: **This is the original denominator. We don't have to do anything to this fraction.**
	OPTION 2: **This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.**
	What do we need to do?
Students	*OPTION 1:* We don't have to change the denominator.
	OPTION 2: We need to convert the fraction to a denominator of __.
Teacher	***OPTION 2:*** **To convert the fraction to a denominator of __, I determine how many groups of __** (original denominator) **I need to make __** (common denominator)**. I see I need to make __, __, __ groups of __** (original denominator)**. How many groups?**
Students	__.

Teacher	**We make __ groups of __ with the two-color counters. That means I iterate or copy the original fraction __ times. How many times?**
Students	__.
Teacher	**Let's check our work. Is __** (original fraction) **equivalent to __** (fraction with common denominator)**?**
Students	Yes.
Teacher	**How do you know the fractions are equivalent?**
Students	The fractions have the same value. They are equivalent.

Teacher **Now that we have common denominators, we want to add the parts or numerators of each fraction. That means we have to add __ one-__ parts and __ one-__ parts. What do we add?**

Students We add the parts or numerators of the fraction.

Teacher **Let's combine the numerators together. With the two-color counters, we add the red one-__ parts. Because our common denominator is __, we make groups of __** (common denominator)**. We make groups of what?**

Students __.

Teacher **We add the one-__ parts. We now have __, __, __, … one-__ parts. How many parts?**

Students __.

Teacher **When you have __ plus __, the sum is __. What's the sum?**

Students __.

Teacher **__ plus __ equals __. Let's say that together.**

Students __ plus __ equals __.

Teacher **So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**

Students One of the sets or numbers added together in an addition problem.

Teacher **What's a sum?**

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher **What do you add when you add fractions?**

Students The parts or numerators of each fraction.

Teacher **How could you explain solving this problem to a friend?**

Students We started by showing each addend. We decided the denominators were not alike, so we determined a common denominator by using the least common multiples. Then, we added the parts together to determine the sum.

ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on addition. What does it mean to add?**

Students To put together or to join to a set.

Teacher **Addition means to put together or to join to a set. Look at this problem.**

	(Show problem.)
Teacher	**First, I see a plus sign** (point)**. The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Our first addend is __. What's our first addend?**
Students	__.
Teacher	**Our second addend is __. What's our second addend?**
Students	__.
Teacher	**So, we have __ plus __. Let's add by combining. What does combining mean?**
Students	To put together.
Teacher	**Yes. Let's combine, or put together, the parts of the fraction. Remember, the parts of a fraction represent the numerator. What do you add?**
Students	The parts or numerators of the fractions.
Teacher	**When adding fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?**
Students	No.
Teacher	**The denominators are not the same. To add, we should add parts or numerators with the same value. When the denominators are unlike, the parts or numerators do not represent the same value. So, we will work to make the fractions have like denominators. Why do we want to add fractions with like denominators?**
Students	So, we can add the parts or numerators of the fractions.
Teacher	**To do this, let's write the first five multiples of each denominator. The first addend has a denominator of __, so let's write the first five multiples of __.** (Write multiples as __, __, __, __, __.)
Teacher	**What are the multiples of __? Say them with me.**
Students	__, __, __, __, __.
Teacher	**The second addend has a denominator of __, so let's write the first five multiples of __.** (Write multiples as __, __, __, __, __.)
Teacher	**What are the multiples of __? Say them with me.**
Students	__, __, __, __, __.
Teacher	**Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?**
Students	__.
Teacher	**So, __ is the least common multiple. Say that with me.**
Students	Least common multiple.
Teacher	**Sometimes we call the least common multiple the LCM. What do we call the least common multiple?**
Students	LCM.
Teacher	**The least common multiple, or LCM, helps us determine the common denominator for the two fractions. What does the LCM help with?**
Students	Finding a common denominator for the two fractions.

Teacher **The first addend has a denominator of __.**
OPTION 1: **This is the original denominator. We don't have to do anything to this fraction.**
OPTION 2: **This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.**
What do we need to do?

Students *OPTION 1:* We don't have to change the denominator.
OPTION 2: We need to convert the fraction to a denominator of __.

Teacher ***OPTION 2:*** **To convert the fraction to a denominator of __, I determine how many groups of __** (original denominator) **I need to make __** (common denominator)**. I see I need to make __, __, __ groups of __** (original denominator)**. How many groups?**

Students __.

Teacher **So, I multiply the denominator times __ and the numerator times __. Let's multiply the denominator first. __** (original denominator) **times __ is what?**

Students __.

Teacher **That's right. __ times __ equals __. Our new denominator is __. What's our new denominator?**

Students __.

Teacher **Now, let's multiply the numerator times __. __** (original numerator) **times __ is what?**

Students __.

Teacher **Yes. __ times __ equals __. Our new numerator is __. What's the new numerator?**

Students __.

Teacher **Let's check our work. Is __** (original fraction) **equivalent to __** (fraction with common denominator)**? How do you know the fractions are equivalent?**

Students The fractions have the same value. They are equivalent.

Teacher **So, we converted the first addend to a common denominator. Let's do the same with the second addend. What's the second addend?**

__.

Teacher **The second addend has a denominator of __.**
OPTION 1: **This is the original denominator. We don't have to do anything to this fraction.**
OPTION 2: **This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.**
What do we need to do?

Students *OPTION 1:* We don't have to change the denominator.
OPTION 2: We need to convert the fraction to a denominator of __.

Teacher ***OPTION 2:*** **To convert the fraction to a denominator of __, I determine how many groups of __** (original denominator) **I need to make __**

(common denominator). **I see I need to make __, __, __ groups of __** (original denominator). **How many groups?**

Students __.

Teacher So, I multiply the denominator times __ and the numerator times __. Let's multiply the denominator first. __ (original denominator) **times __ is what?**

Students __.

Teacher That's right. __ times __ equals __. Our new denominator is __. What's our new denominator?

Students __.

Teacher Now, let's multiply the numerator times __. __ (original numerator) **times __ is what?**

Students __.

Teacher Yes. __ times __ equals __. Our new numerator is __. What's the new numerator?

Students __.

Teacher Let's check our work. Is __ (original fraction) **equivalent to __** (fraction with common denominator)**?**

Students Yes.

Teacher How do you know the fractions are equivalent?

Students The fractions have the same value. They are equivalent.

Teacher Now that we have common denominators, we want to add the parts or numerator of each fraction. That means we have to add __ one-__ parts and __ one-__ parts. What do we add?

Students We add the parts of the fraction.

Teacher Let's combine the parts or numerators together.

(Combine parts, compare to whole.)

Teacher So, we now have __, __, __, … one-__ parts. How many parts?

Students __.

Teacher When you have __ plus __, the sum is __. What's the sum?

Students __.

Teacher __ plus __ equals __. Let's say that together.

Students __ plus __ equals __.

Teacher So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?

Students One of the sets or numbers added together in an addition problem.

Teacher What's a sum?

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher What do you add when you add fractions?

Students The parts or numerator of each fraction.

Teacher How could you explain solving this problem to a friend?

Students	We started by showing each addend. We used least common multiples to help determine common denominators. Then, we added the parts together to determine the sum.

Example

$$\frac{3}{4} + \frac{1}{3} = \frac{13}{12}$$

EXAMPLE WITH MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point)**. The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with two-color counters.** (Move two-color counters to workspace.)
Teacher	**Our first addend is $\frac{3}{4}$. What's our first addend?**
Students	$\frac{3}{4}$.
Teacher	**Let's show this addend by showing the fraction. First, we have a denominator of 4, so let's show 4 yellow counters. How many?**
Students	4.
Teacher	**Then, we need to show 3 of the 4 parts as red to show $\frac{3}{4}$. How many should we make red?**
Students	3.
Teacher	**What fraction?**
Students	$\frac{3}{4}$.
Teacher	**Our second addend is $\frac{1}{3}$. What's our second addend?**
Students	$\frac{1}{3}$.
Teacher	**Let's show the second addend by showing the fraction. First, we have a denominator of 3, so let's show 3 yellow counters. How many?**
Students	3.
Teacher	**Then, we need to show 1 of the 3 parts as red to show $\frac{1}{3}$. How many should we make red?**
Students	1.
Teacher	**What fraction?**
Students	$\frac{1}{3}$.

Teacher	**So, we have $\frac{3}{4}$ plus $\frac{1}{3}$. Let's add by combining. What does combining mean?**
Students	To put together.
Teacher	**Yes. Let's combine, or put together, the parts of the fraction. When adding fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?**
Students	No.
Teacher	**How do you know the denominators are not alike?**
Students	We have a denominator of 4 and a denominator of 3. Those are not the same.
Teacher	**The denominators are not the same. To add, we should add parts or numerators with the same denominator. When the denominators are unlike, the parts or numerators do not represent the same value. So, we will work to make the fractions have like denominators. Why do we want to add fractions with like denominators?**
Students	So, we can add the parts or numerator of the fraction.
Teacher	**To do this, let's write the first five multiples of each denominator. The first addend has a denominator of 4, so let's write the first five multiples of 4.** (Write multiples as 4, 8, 12, 16, 20.)
Teacher	**What are the multiples of 4? Say them with me.**
Students	4, 8, 12, 16, 20.
Teacher	**The second addend has a denominator of 3, so let's write the first five multiples of 3.** (Write multiples as 3, 6, 9, 12, 15.)
Teacher	**What are the multiples of 3? Say them with me.**
Students	3, 6, 9, 12, 15.
Teacher	**Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?**
Students	12.
Teacher	**So, 12 is the least common multiple. What is 12?**
Students	The least common multiple.
Teacher	**Sometimes we call the least common multiple the LCM. What do we call the least common multiple?**
Students	LCM.
Teacher	**The least common multiple, or LCM, helps us determine the common denominator for the two fractions. What does the LCM help with?**
Students	Finding a common denominator for the two fractions.
Teacher	**The first addend has a denominator of 4, which is not the original denominator. We need to convert the fraction from a denominator of 4 to a denominator of 12. What do we need to do?**
Students	Convert the fraction from a denominator of 4 to a denominator of 12.
Teacher	**To convert the fraction to a denominator of 12, I determine how many groups of 4 I need to make 12. I see I need to make 1, 2, 3 groups of 4.** (Point to the multiples of 4, 8, and 12.) **How many groups?**

Students 3.

Teacher **Let's make 3 groups of the fraction $\frac{3}{4}$ with the two-color counters. We already have one group of $\frac{3}{4}$. Let's make a second group** (show 3 red counters and 1 yellow counter) **and a third group** (show 3 red counters and 1 yellow counter.) **Our new fraction is $\frac{9}{12}$. Is $\frac{9}{12}$ equivalent to $\frac{3}{4}$?**

Students Yes. The fractions are equivalent.

Teacher **So, we converted the first addend to a common denominator. Let's do the same with the second addend. What's the second addend?**

Students $\frac{1}{3}$.

Teacher **The second addend has a denominator of 3, which is not the original denominator. We need to convert the fraction from a denominator of 3 to a denominator of 12. What do we need to do?**

Students Convert the fraction from a denominator of 3 to a denominator of 12.

Teacher **To convert the fraction to a denominator of 12, I determine how many groups of 3 I need to make 12. I see I need to make 1, 2, 3, 4 groups of 3.** (Point to the multiples of 3, 6, 9, and 12.) **How many groups?**

Students 4.

Teacher **Let's make 4 groups of the fraction $\frac{1}{3}$ with the two-color counters. We already have one group of $\frac{1}{3}$. Let's make a second group** (show 1 red counter and 2 yellow counters), **a third group** (show 1 red counter and 2 yellow counters), **and a fourth group** (show 1 red counter and 2 yellow counters). **Our new fraction is $\frac{4}{12}$. Is $\frac{4}{12}$ equivalent to $\frac{1}{3}$?**

Students Yes. The fractions are equivalent.

Teacher **Now that we have common denominators, we want to add the parts or numerators of each fraction. That means we have to add 9 one-twelfth parts and 4 one- twelfth parts. What do we add?**

Students We add the parts or numerators of the fractions.

Teacher **Let's combine the parts or numerators together. With the two-color counters, we add the red one-twelfth parts. Because our common denominator is 12, we make groups of 12** (common denominator). **We make groups of what?**

Students 12.

Teacher **We add the one-twelfth parts. We now have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 one-twelfth parts. How many parts?**

Students 13.

Teacher **When you have $\frac{9}{12}$ plus $\frac{4}{12}$, the sum is $\frac{13}{12}$. What's the sum?**

Students $\frac{13}{12}$.

Teacher **$\frac{9}{12}$ plus $\frac{4}{12}$ equals $\frac{13}{12}$. Let's say that together.**

Students $\frac{9}{12}$ plus $\frac{4}{12}$ equals $\frac{13}{12}$.

$\frac{13}{12}$ **is also equivalent to** $1\frac{1}{12}$**.**

Teacher **If you have a set of** $\frac{3}{4}$ **and a set of** $\frac{1}{3}$**, when you combine (or put together) the sets, the sum is** $\frac{13}{12}$**.** $\frac{9}{12}$ **plus** $\frac{4}{12}$ **equals** $\frac{13}{12}$**. Let's review. What's an addend?**

Students One of the sets or numbers added together in an addition problem.

Teacher **What's a sum?**

Students The total number when you combine sets, or the result of adding two or more numbers together.

Teacher **What do you add when you add fractions?**

Students The parts or numerator of each fraction.

Teacher **How could you explain solving this problem to a friend?**

Students We started by showing each addend. We determined the denominators were not alike. So, we used least common multiples to find a common denominator. After converting both fractions to a common denominator, we added the parts or numerators together to determine the sum.

(3) Addition of Decimals with Traditional Algorithm

Routine

Materials:

- Module 6 Problem Sets
- Module 6 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or money
  - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher **Let's work on addition. What does it mean to add?**

Students To put together or to join to a set.

Teacher **Addition means to put together or to join to a set. Look at this problem.** (Show problem.)

Teacher **First, I see a plus sign** (point)**. The plus sign tells us to add. What does the plus sign mean?**

Students To add.

Teacher **Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)

Teacher **When we use the Base-10 blocks with decimals, we can shift the meaning of each type of block. Today, let's use the flats to represent ones. What do the flats represent?**

Students Ones.

Teacher **We'll use the rods to represent tenths. What do the rods represent?**

Students Tenths.

Teacher **How can we use the rods to represent tenths?**

Students 1 rod equals 1 tenth.

Teacher **What do you notice about the relationship between the rods and the flat?**

Students There are 10 tenths in 1 in the same way there are 10 rods in 1 flat.

Teacher **With our Base-10 blocks, the units represent hundredths. What do the units represent?**

Students Hundredths.

Teacher **What do you notice about the relationship between the units and the rods?**

Students There are 10 hundredths in 1 tenth in the same way there are 10 units in 1 rod.

Teacher **Our first addend is __. What's our first addend?**

Students __.

Teacher **Let's show this addend by showing __ ones, __ tenths, and __ hundredths.**
(Show with Base-10 blocks.)

Teacher **How many?**

Students __.

Teacher **Our second addend is __. What's our second addend?**

Students __.

Teacher **Let's show the second addend by showing __ ones, __ tenths, and __ hundredths.**
(Show with Base-10 blocks. Place Base-10 blocks under the first addend.)

Teacher **How many?**

Students __.

Teacher **So, we have __ plus __. Let's add by combining. What does combining mean?**

Students To put together.

Teacher **Yes. Let's combine or put together. First, let's combine the least place value. That means the place value with the least or smallest value. What's the least place value in this problem?**

Students Hundredths.

Teacher **Let's add the hundredths together.**
(Move two sets of hundredths together.)

Teacher **Let's count to learn the sum of the hundredths.**
(Count hundredths.)

Teacher **How many hundredths are there in total or altogether?**

Students __.

Teacher **Yes! There are __ hundredths. If we have more than 9 hundredths, we have to regroup. Do we have more than 9 hundredths?**

Students Yes.

Teacher **We have more than 9 hundredths. That means we have to regroup. To regroup, we count 10 hundredths and regroup/trade/exchange the 10 hundredths for 1 tenth. Let's do that together. Let's count out 10 hundredths.**
(Count 10 hundredths.)

Teacher **Let's regroup/trade/exchange the 10 hundredths for 1 tenth. See how 1 tenth is the same as 10 hundredths?**

Students	Yes.
Teacher	**We leave the remaining hundredths here. But we can't put this 1 tenth in the hundredths place. The hundredths place is only for hundredths. So, we place the 1 tenth in the tenths column. I like to place the 1 tenth above the other tenths.** (Place 1 tenth above tenths column.)
Teacher	**Now, let's combine the tenths. That means we put all the tenths together.** (Move sets of tenths together.)
Teacher	**How many tenths are there in total or altogether?**
Students	__.
Teacher	**There are __ tenths. If we have more than 9 tenths, we have to regroup. Do we have more than 9 tenths?**
Students	No.
Teacher	**Now, let's combine the ones. Let's put all the ones together.** (Move sets of ones together.)
Teacher	**How many ones are there in total or altogether?**
Students	__.
Teacher	**So, let's count the ones, tenths, and hundredths to learn the sum. Ready?** (Count the ones, then tenths, then hundredths.)
Teacher	**That means __ plus __ equals __. Let's say that together.**
Students	__ plus __ equals __.
Teacher	**Let's say it together again.**
Students	__ plus __ equals __.
Teacher	**So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**What does it mean to regroup/trade/exchange?**
Students	You can regroup/trade/exchange 10 hundredths for 1 tenth.
Teacher	**How could you explain solving this problem to a friend?**
Students	We started by showing each addend. Then, we combined the hundredths. We regrouped 10 hundredths for 1 tenth. Then, we combined the tenths. Then, we combined the ones. The sum was the total number of ones, tenths, and hundredths.

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)

Teacher	**First, I see a plus sign** (point)**. The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the different place value columns. Let's draw a vertical line between the ones column and the tenths column and another line between the tenths column and the hundredths column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by adding the hundredths. What should we add first?**
Students	The hundredths.
Teacher	**Which hundredths do we add?**
Students	__ plus __.
Teacher	**What's __ plus __?** (If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up __ more. Ready? __: __, __, __.** See Counting Up poster at the end of Module 4 for more information.)
Teacher	**How many hundredths are there in total or altogether?**
Students	__.
Teacher	**Yes! There are __ hundredths. If we have more than 9 hundredths, we have to regroup. Do we have more than 9 hundredths?**
Students	Yes.
Teacher	**We have more than 9 hundredths. That means we have to regroup. We think of our hundredths sum as 1 tenth and __ hundredths. We write the hundredths in the hundredths column under the equal line.** (Write hundredths under equal line.)
Teacher	**We regroup the 1 tenth to the tenths column. We write the 1 tenth in the tenths column above the other tenths.** (Write 1 above tenths column.)
Teacher	**Now, let's add the tenths. Which tens do we add?**
Students	__ plus __ plus __.
Teacher	**What's __ plus __ plus __?**
Students	__.
Teacher	**How many tenths are there in total or altogether?**
Students	__.
Teacher	**There are __ tenths. If we have more than 9 tenths, we have to regroup. Do we have more than 9 tenths?**
Students	No.
Teacher	**Now, let's add the ones. Which ones do we add?**
Students	__ plus __.
Teacher	**What's __ plus __?**
Students	__.
Teacher	**How many ones are there in total or altogether?**
Students	__.

Teacher	**So, let's look at the problem. What's __ plus __?**
Students	__.
Teacher	**That's right. __ plus __ equals __. Let's say that together.**
Students	__ plus __ equals __.
Teacher	**So, if you have a set of __ and a set of __, when you combine (or join) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**What does it mean to regroup/trade/exchange?**
Students	You can regroup/trade/exchange 10 hundredths for 1 tenth.
Teacher	**How could you explain solving this problem to a friend?**
Students	First, we combined the hundredths. We regrouped 10 hundredths for 1 tenth. Then, we combined the tenths. Then, we combined the ones. The sum was the total number of ones, tenths, and hundredths.

Example

$$\begin{array}{r} 2.16 \\ +\ \ 4.78 \\ \hline 6.94 \end{array}$$

EXAMPLE WITH MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point)**. The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**When we use the Base-10 blocks with decimals, we can shift the meaning of each type of block. Today, let's use the flats to represent ones. What do the flats represent?**
Students	Ones.
Teacher	**We'll use the rods to represent tenths. What do the rods represent?**
Students	Tenths.
Teacher	**How can we use the rods to represent tenths? What do you notice about the relationship between the rods and the flat?**
Students	There are 10 tenths in 1 in the same way there are 10 rods in 1 flat.

Teacher	**With our Base-10 blocks, the units represent hundredths. What do the units represent?**
Students	Hundredths.
Teacher	**What do you notice about the relationship between the units and the rods?**
Students	There are 10 hundredths in 1 tenth in the same way there are 10 units in 1 rod.
Teacher	**Our first addend is 2 and 16 hundredths. What's our first addend?**
Students	2 and 16 hundredths.
Teacher	**Let's show this addend by showing 2 ones, 1 tenth, and 6 hundredths.** (Show with Base-10 blocks.)
Teacher	**How many?**
Students	2 and 16 hundredths.
Teacher	**Our second addend is 4 and 78 hundredths. What's our second addend?**
Students	4 and 78 hundredths.
Teacher	**Let's show the second addend by showing 4 ones, 7 tenths, and 8 hundredths.** (Show with Base-10 blocks. Place Base-10 blocks under the first addend.)
Teacher	**How many?**
Students	4 and 78 hundredths.
Teacher	**So, we have 2 and 16 hundredths plus 4 and 78 hundredths. Let's add by combining. What does combining mean?**
Students	To put together.
Teacher	**Yes. Let's combine or put together. First, let's combine the least place value. What's the least place value in this problem?**
Students	Hundredths.
Teacher	**Let's add the hundredths together. 6 hundredths plus 8 hundredths.** (Move two sets of hundredths together.)
Teacher	**Let's count to learn the sum of the hundredths.** (Count hundredths.)
Teacher	**How many hundredths are there in total or altogether?**
Students	14.
Teacher	**Yes! There are 14 hundredths. If we have more than 9 hundredths, we have to regroup. Do we have more than 9 hundredths?**
Students	Yes.
Teacher	**We have more than 9 hundredths. That means we have to regroup. To regroup, we count 10 hundredths and regroup/trade/exchange the 10 hundredths for 1 tenth. Let's do that together. Let's count out 10 hundredths.** (Count 10 hundredths.)
Teacher	**Let's regroup/trade/exchange the 10 hundredths for 1 tenth. See how 1 tenth is the same as 10 hundredths?**
Students	Yes.
Teacher	**We leave the remaining hundredths here. But we can't put this 1 tenth in the hundredths place. The hundredths place is only for hundredths. So, we place the 1 tenth in the tenths column. I like to place the 1 tenth above the other tenths.** (Place 1 tenth above tenths column.)

Teacher	**Now, let's combine the tenths. That means we put all the tenths together.** (Move sets of tenths together.)
Teacher	**Let's add 1 tenth plus 1 tenth plus 7 tenths. How many tenths are there in total or altogether?**
Students	9.
Teacher	**There are 9 tenths. If we have more than 9 tenths, we have to regroup. Do we have more than 9 tenths?**
Students	No.
Teacher	**Now, let's combine the ones. Let's put all the ones together.** (Move sets of ones together.)
Teacher	**How many ones are there in total or altogether?**
Students	6.
Teacher	**So, let's count the ones, tenths, and hundredths to learn the sum. Ready?** (Count the ones, then tenths, then hundredths.)
Teacher	**That means 2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths. Let's say that together.**
Students	2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths.
Teacher	**Let's say it together again.**
Students	2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths.
Teacher	**Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**What does it mean to regroup/trade/exchange?**
Students	You can regroup/trade/exchange 10 hundredths for 1 tenth.
Teacher	**How could you explain solving this problem to a friend?**
Students	We started by showing each addend. Then, we combined the hundredths. We regrouped 10 hundredths for 1 tenth. Then, we combined the tenths. Then, we combined the ones. The sum was the total number of ones, tenths, and hundredths.

(4) Addition of Decimals with Partial Sums Algorithm

Routine

Materials:

- Module 6 Problem Sets
- Module 6 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or money
    - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point). **The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with money.** (Move money to workspace.)
Teacher	**When we use the money, the dollar bills represent ones. What do the dollar bills represent?**
Students	Ones.
Teacher	**We'll use the dimes to represent tenths. What do the dimes represent?**
Students	Tenths.
Teacher	**How can we use the dimes to represent tenths?**
Students	1 dime represents 1 tenth.
Teacher	**What do you notice about the relationship between the dimes and the dollar bill?**
Students	There are 10 dimes in 1 dollar.
Teacher	**With our money, the pennies represent hundredths. What do the pennies represent?**
Students	Hundredths.
Teacher	**What do you notice about the relationship between the pennies and the dimes?**
Students	There are 10 pennies in 1 dime.
Teacher	**Our first addend is __. What's our first addend?**
Students	__.
Teacher	**Let's show this addend by showing __ ones, __ tenths, and __ hundredths.** (Show with money.)
Teacher	**How many?**
Students	__.
Teacher	**Our second addend is __. What's our second addend?**
Students	__.
Teacher	**Let's show the second addend by showing __ ones, __ tenths, and __ hundredths.** (Show with money. Place under the first addend.)
Teacher	**How many?**
Students	__.
Teacher	**So, we have __ plus __. Let's add by combining. What does combining mean?**
Students	To put together.

Teacher	**Yes. Let's combine or put together. First, let's combine the ones. That means we combine the dollars. This will be our first partial sum. It's the sum for part of the problem. Adding the ones means we put all the ones together.** (Move two sets of ones together.)
Teacher	**Let's count to learn the sum of the ones.** (Count ones.)
Teacher	**How many ones are there in total or altogether?**
Students	__.
Teacher	**This __ is one of our partial sums. It's the sum of the ones. Now, let's combine the tenths. That means we put all the dimes together.** (Move dimes together.)
Teacher	**How many dimes are there in total or altogether?**
Students	__.
Teacher	**This __ is another of our partial sums. It's the sum of the tenths. What's a partial sum?**
Students	It's a sum of part of the problem.
Teacher	**Let's combine the hundredths or pennies. Let's put all the hundredths together to get the sum of the hundredths.** (Move pennies together.)
Teacher	**How many pennies are there in total or altogether?**
Students	__.
Teacher	**Now, we add the partial sums. Let's add the partial sums of the ones, tenths, and hundredths or the dollars, dimes, and pennies.** (Start with dollars, then add the dimes, then add the pennies.)
Teacher	**That means __ plus __ equals __. Let's say that together.**
Students	__ plus __ equals __.
Teacher	**Let's say it together again.**
Students	__ plus __ equals __.
Teacher	**So, if you have a set of __ and a set of __, when you combine (or put together) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**How could you explain solving this problem to a friend?**
Students	We started by showing each addend. Then, we combined the ones. Then, we combined the tenths. Then, we combined the hundredths. We added the partial sums of the ones, tenths, and hundredths by adding the dollars, dimes, and pennies. The sum was the total number of ones, tenths, and hundredths.

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point)**. The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the different place value columns. Let's draw a vertical line between the ones column and the tenths column and another line between the tenths column and the hundredths column.** (Draw vertical lines to separate place value columns.)
Teacher	**With the partial sums algorithm, we start by adding the greatest place value. What should we add first?**
Students	The ones.
Teacher	**Which ones do we add?**
Students	__ plus __.
Teacher	**What's __ plus __?** (If a student has difficulty with addition, say: **Start with the greater addend. Place that number in your fist, and let's count up __ more. Ready? __: __, __, __.** See Counting Up poster at the end of Module 4 for more information.)
Teacher	**How many ones are there in total or altogether?**
Students	__.
Teacher	**So, let's write __ under the equal line.** (Write ones.)
Teacher	**Now, let's add the tenths. Which tens do we add?**
Students	__ plus __.
Teacher	**What's __ plus __?**
Students	__.
Teacher	**Let's write __ under the equal line.** (Write tenths.)
Teacher	**Now, let's add the hundredths. Which hundredths do we add?**
Students	__ plus __.
Teacher	**What's __ plus __?**
Students	__.
Teacher	**Let's write __ under the equal line.** (Write hundredths.)
Teacher	**Now, let's add the partial sums. What's __ plus __ plus __?**
Students	__.
Teacher	**That's right. To review, __ plus __ equals __. Let's say that together.**
Students	__ plus __ equals __.

Teacher	**So, if you have a set of __ and a set of __, when you combine (or join) the sets, the sum is __. __ plus __ equals __. Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**What's a partial sum?**
Students	The sum of just the ones or the tenths or the hundredths.
Teacher	**How could you explain solving this problem to a friend?**
Students	First, we combined the ones. Then, we combined the tenths. Then, we combined the hundredths. The sum was the total number of ones, tenths, and hundredths.

$$\begin{array}{r} 2.16 \\ +\ \ 4.78 \\ \hline 6.94 \end{array}$$

Example

EXAMPLE WITH MANIPULATIVES

Teacher	**Let's work on addition. What does it mean to add?**
Students	To put together or to join to a set.
Teacher	**Addition means to put together or to join to a set. Look at this problem.** (Show problem.)
Teacher	**First, I see a plus sign** (point)**. The plus sign tells us to add. What does the plus sign mean?**
Students	To add.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**When we use the Base-10 blocks with decimals, we can shift the meaning of each type of block. Today, let's use the flats to represent ones. What do the flats represent?**
Students	Ones.
Teacher	**We'll use the rods to represent tenths. What do the rods represent?**
Students	Tenths.
Teacher	**How can we use the rods to represent tenths?**
Students	1 rod equals 1 tenth.
Teacher	**What do you notice about the relationship between the rods and the flat?**
Students	There are 10 tenths in 1 in the same way there are 10 rods in 1 flat.
Teacher	**With our Base-10 blocks, the units represent hundredths. What do the units represent?**
Students	Hundredths.
Teacher	**What do you notice about the relationship between the units and the rods?**
Students	There are 10 hundredths in 1 tenth in the same way there are 10 units in 1 rod.
Teacher	**Our first addend is 2 and 16 hundredths. What's our first addend?**
Students	2 and 16 hundredths.

Teacher **Let's show this addend by showing 2 ones, 1 tenth, and 6 hundredths.**
(Show with Base-10 blocks.)
Teacher **How many?**
Students 2 and 16 hundredths.
Teacher **Our second addend is 4 and 78 hundredths. What's our second addend?**
Students 4 and 78 hundredths.
Teacher **Let's show the second addend by showing 4 ones, 7 tenths, and 8 hundredths.**
(Show with Base-10 blocks. Place Base-10 blocks under the first addend.)
Teacher **How many?**
Students 4 and 78 hundredths.
Teacher **So, we have 2 and 16 hundredths plus 4 and 78 hundredths. Let's add by combining. What does combining mean?**
Students To put together.
Teacher **Yes. Let's combine or put together. We'll use the partial sums strategy. What strategy?**
Students Partial sums.
Teacher **With the partial sums strategy, we add the greatest place value first. What's the greatest place value in this problem?**
Students Ones.
Teacher **Let's add the ones together: 2 plus 4.**
(Move 2 flats and 4 flats together.)
Teacher **Let's count to learn the sum of the ones.**
(Count ones.)
Teacher **How many ones are there in total or altogether?**
Students 6.
Teacher **Yes! There are 6 ones. Now, let's combine the tenths. That means we put all the tenths together: 1 tenth and 7 tenths.**
(Move 1 rod and 7 rods together.)
Teacher **How many tenths are there in total or altogether?**
Students 8.
Teacher **There are 8 tenths. Now, let's combine the hundredths. Let's put all the hundredths together: 6 hundredths and 8 hundredths.**
(Move 6 units and 8 units together.)
Teacher **How many hundredths are there in total or altogether?**
Students 14.
Teacher **Notice that 14 hundredths is the same as what?**
Students 1 tenth and 4 hundredths.
Teacher **So, let's count the ones, tenths, and hundredths to learn the sum. Ready?**
6 and 10, 20, 30, 40, 50, 60, 70, 80, 90, 91, 92, 93, 94 hundredths.
Teacher **That means 2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths. Let's say that together.**
Students 2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths.
Teacher **Let's say it together again.**
Students 2 and 16 hundredths plus 4 and 78 hundredths equals 6 and 94 hundredths.

Teacher	**Let's review. What's an addend?**
Students	One of the sets or numbers added together in an addition problem.
Teacher	**What's a sum?**
Students	The total number when you combine sets, or the result of adding two or more numbers together.
Teacher	**What's a partial sum?**
Students	The sum of just the ones or the tenths or the hundredths.
Teacher	**How could you explain solving this problem to a friend?**
Students	We started by showing each addend. Then, we added the ones, then the tenths, and then the hundredths. The sum was the total number of ones, tenths, and hundredths.

D. Problems for Use During Instruction

See Module 6 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 6 Vocabulary Cards.

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Module 6:
Addition of Rational Numbers

Problem Sets

A. Proper fractions with like denominators and sums <1 (20)
B. Improper fractions with like denominators and sums >1 (10)
C. Mixed numbers with like denominators and sums >1 (10)
D. Proper fractions with unlike denominators and sums <1 (20)
E. Improper fractions with unlike denominators and sums >1 (10)
F. Mixed numbers with unlike denominators and sums >1 (10)

G. Decimals with tenths; no regrouping (20)
H. Decimals with tenths; regrouping (20)
I. Decimals with hundredths; no regrouping (20)
J. Decimals with hundredths; regrouping (20)
K. Decimals with tenths and hundredths; mix of regrouping (20)

A.

$$\frac{2}{5} + \frac{2}{5} =$$

A.

$$\frac{4}{10} + \frac{3}{10} =$$

A. $\frac{3}{6} + \frac{1}{6} =$

A.

$$\frac{2}{4} + \frac{1}{4} =$$

A.

$$\frac{1}{3} + \frac{1}{3} =$$

A.

$$\frac{2}{6} + \frac{3}{6} =$$

A.

$$\frac{3}{8} + \frac{4}{8} =$$

A.

$$\frac{4}{10} + \frac{1}{10} =$$

A.

$$\frac{2}{12} + \frac{4}{12} =$$

A.

$$\frac{7}{12} + \frac{3}{12} =$$

A.

$$\frac{5}{9} + \frac{2}{9} =$$

A.

$$\frac{3}{5} + \frac{1}{5} =$$

A.

$$\frac{2}{6} + \frac{1}{6} =$$

A.

$$\frac{4}{7} + \frac{1}{7} =$$

A.

$$\frac{3}{9} + \frac{4}{9} =$$

A.

$$\frac{5}{10} + \frac{2}{10} =$$

A.

$$\frac{1}{4} + \frac{1}{4} =$$

A.

$$\frac{1}{6} + \frac{1}{6} =$$

A.

$$\frac{2}{7} + \frac{3}{7} =$$

A.

$$\frac{1}{8} + \frac{2}{8} =$$

B.

$$\frac{6}{5} + \frac{7}{5} =$$

B. $$\frac{12}{8} + \frac{3}{8} =$$

B.

$$\frac{7}{6} + \frac{3}{6} =$$

B.

$$\frac{5}{4} + \frac{1}{4} =$$

B.

$$\frac{2}{3} + \frac{4}{3} =$$

B.

$$\frac{8}{6} + \frac{3}{6} =$$

B. $\frac{5}{8} + \frac{9}{8} =$

B. $$\frac{11}{10} + \frac{13}{10} =$$

B.

$$\frac{13}{12} + \frac{4}{12} =$$

B. $$\frac{10}{10} + \frac{5}{10} =$$

c.

$$7\frac{7}{12} + 4\frac{3}{12} =$$

c.

$$\frac{3}{5} + 2\frac{3}{5} =$$

c.

$$1\frac{2}{6} + 3\frac{5}{6} =$$

c.

$$\frac{8}{5} + 4\frac{1}{5} =$$

c.

$$1\frac{4}{9} + 2\frac{6}{9} =$$

c.

$$1\frac{10}{12} + 3\frac{5}{12} =$$

c.

$$1\frac{3}{4} + 1\frac{3}{4} =$$

c.

$$7\frac{5}{6} + 2\frac{7}{6} =$$

c.

$$\frac{3}{4} + 2\frac{3}{4} =$$

c.

$$1\frac{6}{8} + 3\frac{5}{8} =$$

D.

$$\frac{2}{4} + \frac{1}{3} =$$

D.

$$\frac{1}{2} + \frac{2}{6} =$$

D.

$$\frac{2}{12} + \frac{1}{4} =$$

D.

$$\frac{3}{10} + \frac{1}{5} =$$

D.

$$\frac{1}{6} + \frac{1}{3} =$$

D.

$$\frac{2}{10} + \frac{2}{5} =$$

D.

$$\frac{3}{8} + \frac{2}{4} =$$

D.

$$\frac{3}{6} + \frac{1}{3} =$$

D.

$$\frac{1}{3} + \frac{1}{2} =$$

D.

$$\frac{2}{4} + \frac{3}{8} =$$

D.

$$\frac{3}{10} + \frac{2}{5} =$$

D.

$$\frac{2}{12} + \frac{5}{6} =$$

D. $$\frac{1}{3} + \frac{2}{5} =$$

D.

$$\frac{1}{3} + \frac{1}{4} =$$

D.

$$\frac{2}{5} + \frac{2}{4} =$$

D.

$$\frac{1}{5} + \frac{1}{2} =$$

D.

$$\frac{2}{12} + \frac{2}{4} =$$

D.

$$\frac{5}{9} + \frac{1}{3} =$$

D.

$$\frac{1}{4} + \frac{5}{8} =$$

D.

$$\frac{2}{12} + \frac{2}{3} =$$

E. $\frac{4}{2} + \frac{5}{3} =$

E.

$$\frac{4}{5} + \frac{5}{4} =$$

E. $\frac{5}{2} + \frac{7}{4} =$

E.

$$\frac{6}{5} + \frac{5}{3} =$$

E. $$\frac{11}{4} + \frac{10}{8} =$$

E.

$$\frac{8}{7} + \frac{9}{5} =$$

E.

$$\frac{7}{4} + \frac{5}{8} =$$

E. $\frac{11}{8} + \frac{3}{2} =$

E. $$\frac{13}{5} + \frac{2}{4} =$$

E. $\frac{12}{10} + \frac{10}{4} =$

F.

$$1\frac{1}{2} + 1\frac{7}{8} =$$

F.

$$\frac{2}{5} + 4\frac{1}{4} =$$

F.

$$7\frac{1}{2} + 3\frac{1}{5} =$$

F.

$$1\frac{5}{6} + 1\frac{2}{4} =$$

F.

$$\frac{7}{8} + 2\frac{1}{2} =$$

F.

$$1\frac{4}{10} + 1\frac{2}{5} =$$

F.

$$7\frac{3}{8} + 2\frac{5}{12} =$$

F.

$$\frac{2}{3} + 3\frac{1}{9} =$$

F.

$$\frac{1}{2} + 2\frac{5}{6} =$$

F.

$$1\frac{2}{6} + 4\frac{5}{12} =$$

G.

$$\begin{array}{r} 0.3 \\ +\ 0.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 1.5 \\ +\ 2.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 3.2 \\ +\ 0.3 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 2.5 \\ +\ 4.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 0.1 \\ +\ 4.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 3.3 \\ +\ 4.6 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 0.8 \\ +\ 2.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 1.6 \\ +\ 4.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 6.3 \\ +\ 2.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 3.1 \\ +\ 1.8 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 5.8 \\ +\ 4.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 9.2 \\ +\ 0.4 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 1.7 \\ +\ 6.2 \\ \hline \end{array}$$

G.

$$
\begin{array}{r} 0.7 \\ +\ 0.2 \\ \hline \end{array}
$$

G.

$$\begin{array}{r} 5.3 \\ +\ 4.4 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 6.1 \\ +\ 3.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 5.4 \\ +\ 0.4 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 0.3 \\ +\ 0.6 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 2.2 \\ +\ 7.0 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 6.4 \\ +\ 3.3 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 4.2 \\ +\ 2.8 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 2.3 \\ +\ 6.7 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 1.5 \\ +\ 5.6 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 2.1 \\ +\ 3.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 4.8 \\ +\ 3.6 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 4.5 \\ +\ 3.9 \\ \hline \end{array}$$

H.

$$
\begin{array}{r} 2.9 \\ +\ 5.4 \\ \hline \end{array}
$$

H.

$$
\begin{array}{r} 6.2 \\ +\ 2.9 \\ \hline \end{array}
$$

H.

$$\begin{array}{r} 1.1 \\ +\ 6.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 4.2 \\ +\ 3.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 1.8 \\ +\ 7.4 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 5.1 \\ +\ 2.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 3.5 \\ +\ 5.7 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 4.1 \\ +\ 4.9 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 5.6 \\ +\ 4.7 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 6.9 \\ +\ 3.2 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 8.8 \\ +\ 1.6 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 2.2 \\ +\ 7.8 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 3.5 \\ +\ 6.6 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 1.2 \\ +\ 6.8 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 0.73 \\ +\ 0.21 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 1.46 \\ +\ 3.32 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 2.58 \\ +\ 6.11 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 9.82 \\ +\quad 0.01 \\ \hline \end{array}$$

1.

$$
\begin{array}{r}
0.31 \\
+\quad 8.22 \\
\hline
\end{array}
$$

1.

$$\begin{array}{r} 1.50 \\ +\quad 2.46 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 2.31 \\ +\quad 1.60 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 7.31 \\ +\ 2.47 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 9.13 \\ + \quad 0.60 \\ \hline \end{array}$$

1.

$$
\begin{array}{r} 12.46 \\ +\ \ 1.10 \\ \hline \end{array}
$$

1.

$$
\begin{array}{r}
23.20 \\
+\quad 6.04 \\
\hline
\end{array}
$$

1.

$$\begin{array}{r} 1.71 \\ +\quad 4.10 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 2.35 \\ +\quad 4.22 \\ \hline \end{array}$$

1.

$$
\begin{array}{r} 0.88 \\ +\ 1.01 \\ \hline \end{array}
$$

1.

$$
\begin{array}{r}
3.63 \\
+\quad 1.21 \\
\hline
\end{array}
$$

1.

$$\begin{array}{r} 10.13 \\ +\ 10.26 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 9.34 \\ +\quad 2.44 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 5.60 \\ +\ \ 1.22 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 6.31 \\ +\ 3.08 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 10.33 \\ +\quad 0.55 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 2.56 \\ +\quad 3.45 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 5.24 \\ +\ 1.37 \\ \hline \end{array}$$

J.

$$
\begin{array}{r} 5.45 \\ +\quad 3.78 \\ \hline \end{array}
$$

J.

$$
\begin{array}{r}
3.67 \\
+\quad 5.25 \\
\hline
\end{array}
$$

J.

$$\begin{array}{r} 6.14 \\ +\ 1.47 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 4.25 \\ +\quad 2.25 \\ \hline \end{array}$$

J.

$$
\begin{array}{r} 4.71 \\ +\quad 3.89 \\ \hline \end{array}
$$

J.

$$\begin{array}{r} 1.52 \\ +\ 3.77 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 2.84 \\ +\ 6.16 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 14.80 \\ +\quad 6.96 \\ \hline \end{array}$$

J.

$$
\begin{array}{r}
7.83 \\
+\quad 6.99 \\
\hline
\end{array}
$$

J.

$$\begin{array}{r} 8.95 \\ +\quad 9.80 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 12.80 \\ +\ 46.93 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 3.14 \\ +\quad 1.99 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 7.21 \\ +\quad 4.66 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 5.44 \\ +\quad 2.08 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 9.66 \\ +\quad 1.67 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 8.33 \\ +\quad 1.92 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 42.12 \\ +\ 10.09 \\ \hline \end{array}$$

J.

$$
\begin{array}{r}
6.87 \\
+\quad 2.33 \\
\hline
\end{array}
$$

K.

$$\begin{array}{r} 30.15 \\ +\quad 2.6 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 1.5 \\ +\ 2.49 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 14.58 \\ +\ \ 1.4 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 10.2 \\ +\ \ 5.73 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 5.4 \\ +\ \ .54 \\ \hline \end{array}$$

K.

$$
\begin{array}{r} 8.3 \\ +\ \ .91 \\ \hline \end{array}
$$

K.

$$\begin{array}{r} 4.6 \\ +\quad .64 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 9.38 \\ +\quad .19 \\ \hline \end{array}$$

K.

$$
\begin{array}{r}
10.21 \\
+\quad 5.6 \\
\hline
\end{array}
$$

K.

$$
\begin{array}{r}
1.9 \\
+\quad 2.01 \\
\hline
\end{array}
$$

K.

$$\begin{array}{r} 17.72 \\ +\ 12.58 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 42.1 \\ +\ 17.96 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 8.3 \\ +\quad 9.31 \\ \hline \end{array}$$

K.

$$
\begin{array}{r} 9.0 \\ +\ 8.12 \\ \hline \end{array}
$$

K.

$$
\begin{array}{r}
9.17 \\
+\ 2.7 \\
\hline
\end{array}
$$

K.

$$\begin{array}{r} 3.46 \\ +\quad 1.6 \\ \hline \end{array}$$

K.

$$
\begin{array}{r}
4.9 \\
+\quad 9.23 \\
\hline
\end{array}
$$

K.

$$\begin{array}{r} 15.5 \\ +\ 12.22 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 17.5 \\ + \quad 8.83 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 9.2 \\ +\quad 6.75 \\ \hline \end{array}$$

Module 6:
Addition of Rational Numbers

Vocabulary Cards

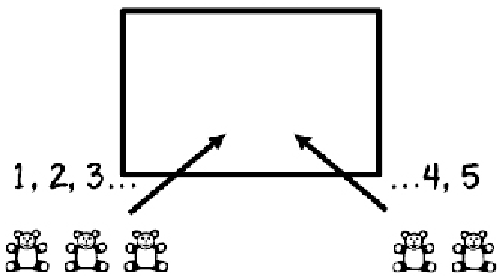
add/addition
addend
algorithm
computation
decimal
denominator
equal sign
equivalent
fraction
hundredths
improper fraction
join
least common multiple
mixed number
multiple
numerator
ones
plus sign
regroup/trade/exchange
sum
tenths
together

add/addition

To put amounts together to find the sum or to increase a set.

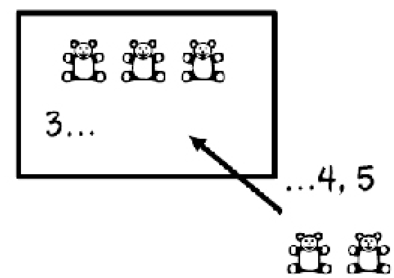
To put amounts together

3 + 2 = 5



To increase a set

3 + 2 = 5



---

addend

Any numbers that are added together.

6 + 2 = 8

6 and 2 are addends

algorithm

A procedure or description of steps that can be used to solve a problem.

---

computation

The action used to solve a problem.

decimal

A number based on powers of ten.

34.107

tens ones tenths hundredths thousandths

---

denominator

The term in a fraction that tells the number of equal parts in a whole.

2 / 3 $\frac{2}{3}$ **In these fractions, 3 is the denominator.**

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$12 + 8 = 20$$

= is the equal sign

---

equivalent

Two numbers that have the same value.

$$\frac{1}{4} = \frac{2}{8} \qquad \frac{2}{3} = \frac{8}{12}$$

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \qquad \frac{10}{12} \qquad \frac{8}{3}$$

---

hundredths

The digit in representing $\frac{1}{100}$.

In the number 4.23, 3 is in the hundredths place.

improper fraction

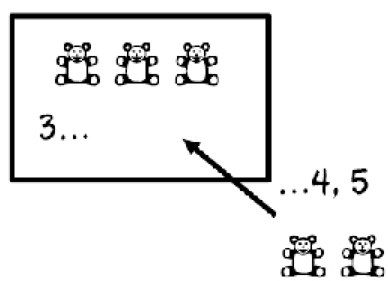
Any fraction in which the numerator is greater than or equal to the denominator.

$$\frac{9}{4} \qquad \frac{17}{12} \qquad \frac{10}{3}$$

---

join

To add to an existing set.



least common multiple

The common multiple with the least value.

6: 6, 12, 18, 24, 30
8: 8, 16, 24, 32, 40

With multiples of 6 and 8, the least common multiple is 24.

---

mixed number

A whole number and a fraction combined.

$$1\frac{1}{6} \qquad 4\frac{5}{12} \qquad 12\frac{4}{3}$$

multiple

The product of a number and any integer.

4: 4, 8, 12, 16, 20

---

numerator

The term in a fraction that tells how many parts in a fraction.

2 / 3 $\frac{2}{3}$ **In these fractions, 2 is the numerator.**

ones

The digit representing 1.

In the number 4.23, 4 is in the ones place.

---

plus sign

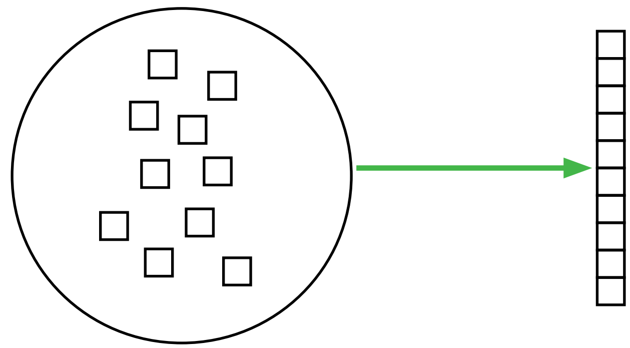
The symbol that tells you to add.

$$5 + 4 = 9$$

+ is the plus sign

regroup/trade/exchange

The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



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sum

The result of adding two or more numbers.

$$7 + 2 + 1 = 10$$

10 is the sum

tenths

The digit in representing $\frac{1}{10}$.

In the number 4.23, 2 is in the tenths place.

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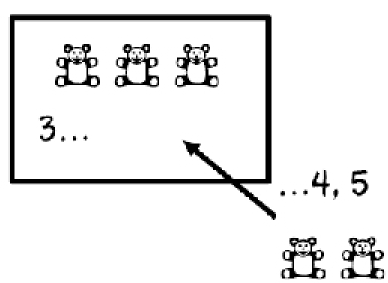
together

To combine sets or numbers.



Instructional Routines for Mathematics Intervention

MODULE 7

Concepts of Subtraction

Module 7: Concepts of Subtraction
Mathematics Routines

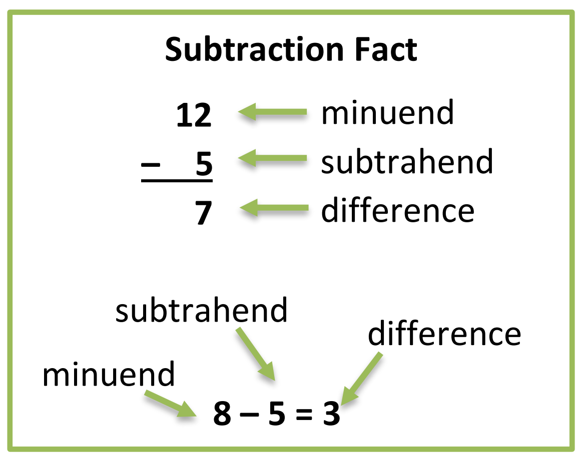
A. Important Vocabulary with Definitions

Term	Definition
compare	To find the difference between two sets.
difference	The result of subtracting one number from another number.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
minuend	The number from which another number is subtracted.
minus sign	The symbol that tells you to subtract.
separate	To start with a set and take away from that set.
subtract/subtraction	To compare two sets or to separate from a set.
subtrahend	The number to be subtracted.

B. Background Information

Students need to learn two concepts of subtraction: (1) subtraction as separating from a set and (2) subtraction as comparison for a difference. Typically, students first learn about subtraction as separating from a set. Then, students learn about comparing two sets for a difference.

For learning the concepts of subtraction, we recommend using *mathematics facts*. We define a subtraction mathematics fact as a single- or double-digit minuend less than 19 and a single-digit subtrahend. The subtrahend is subtracted from the minuend for a difference. You may present subtraction facts vertically or horizontally.



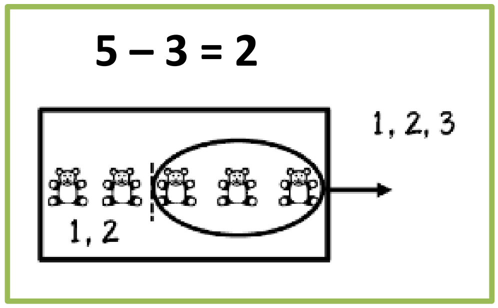
C. Routines and Examples

(1) Subtraction as Separating

Routine

Materials:

- Module 7 Subtraction Problems
- Module 7 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., clips, cubes, dinosaurs)



Teacher	**Let's work on subtraction. Today, let's think about subtraction as separating. What does it mean to separate?**
Students	To take some away.
Teacher	**When we separate, we take some away from a set. For example, you may separate your carrots from your celery. What are some things you separate?**
Students	I separate the blue candies from all the other candies.
Teacher	**When you separate, you take some away from a set. Now, let's think about separating numbers. Look at this problem.** (Show problem.)
Teacher	**First, I notice a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**We'll subtract by separating. Let's show the first number with our clips. The first number in a subtraction problem is called the minuend. Say that with me.**
Students	Minuend.
Teacher	**In a subtraction problem, we start with the minuend and separate some from the minuend.** (Move clips to workspace.)
Teacher	**Our minuend is __. What's our minuend?**
Students	__.
Teacher	**Let's show this minuend by showing __ clips.** (Show clips.)
Teacher	**How many clips?**
Students	__.
Teacher	**From the minuend we separate the subtrahend. Say that with me.**
Students	Subtrahend.
Teacher	**The subtrahend is the number after the minus sign. I remember it by thinking subtract the subtrahend. How could you remember it?**

Students	Subtract the subtrahend.
Teacher	**What's our subtrahend in this problem?**
Students	__.
Teacher	**Let's show the subtrahend by separating __ clips from our minuend. How many clips should we separate or take away?**
Students	__.
Teacher	**So, we need to separate __ clips from __ clips. What does separate mean?**
Students	To take away from a set.
Teacher	**Yes. Let's separate, or take away, __ clips from __ clips.** (Separate clips from original set.)
Teacher	**To learn the difference, let's count the remaining clips.** (Count clips.)
Teacher	**How many clips remain?**
Students	__.
Teacher	**Yes! There are __ clips. So, __ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**Let's say it together again.**
Students	__ minus __ equals __.
Teacher	**So, if you have a set of __ and separate __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The amount between the minuend and subtrahend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We started with a set of clips. Then, we separated some clips from that set. The difference is the number of clips remaining after we separated them from the original set.

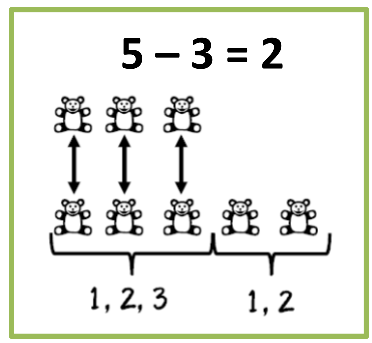
Example

$$\begin{array}{r} 10 \\ -\ \ 6 \\ \hline 4 \end{array}$$

Teacher	**Let's work on subtraction. Today, let's think about subtraction as separating. What does it mean to separate?**
Students	To take away from a set.
Teacher	**When we separate, we take some away from a set. Let's think about separating numbers. Look at this problem.** (Show problem.)
Teacher	**First, I notice a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**We'll subtract by separating. Let's show the minuend with our dinosaurs. What's the minuend?**
Students	The number you start with in a subtraction problem.
Teacher	**Our minuend is 10. What's our minuend?**
Students	10.
Teacher	**Let's show the minuend by showing 10 dinosaurs.** (Show 10 dinosaurs.)
Teacher	**How many dinosaurs?**
Students	10.
Teacher	**Now, let's focus on the subtrahend. What's the subtrahend?**
Students	The number you separate from the minuend.
Teacher	**And the subtrahend comes after which symbol?**
Students	The minus sign.
Teacher	**That's right. The subtrahend comes after the minus sign. We subtract the subtrahend. What's our subtrahend?**
Students	6.
Teacher	**Let's separate or take away 6 dinosaurs from the 10.** (Take away 6 dinosaurs. Move to side.)
Teacher	**How many dinosaurs do we have now? Let's count!**
Students	1, 2, 3, 4.
Teacher	**So, we subtracted 10 minus 6. We subtracted by separating the 6 dinosaurs from the 10 dinosaurs. What's the difference between 10 and 6?**
Students	4.
Teacher	**Yes! There are 4 dinosaurs remaining. So, 10 minus 6 equals 4. Let's say that together.**
Students	10 minus 6 equals 4.
Teacher	**Let's say it together again.**
Students	10 minus 6 equals 4.

Teacher	**So, if you have a set of 10 and separate, or take away, 6 from the set, the difference in 4. 10 minus 6 equals 4. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The amount or space between the minuend and subtrahend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We started with a set of dinosaurs. Then, we separated some dinosaurs from that set. The difference was the number of dinosaurs remaining after we separated them from the original set.

(2) Subtraction as Comparing



Routine

Materials:

- Module 7 Problems
- Module 7 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., clips, candies, cubes)

Teacher	**Let's work on subtraction. Today, let's think about subtraction as comparing. What does it mean to compare?**
Students	To find the difference between two sets.
Teacher	**When we compare, we find the differences between two sets. For example, you and your friend might compare your heights to see who is taller or shorter. What's another way you might compare?**
Students	I might compare who has more Legos; I could compare how much longer my jump rope is than my sister's jump rope.
Teacher	**When you compare, you find the difference between two sets. Now, let's think about comparing in subtraction. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Today we'll subtracting by comparing, but there are other ways to subtract. Let's start by showing the minuend with our candies and then comparing those candies to another set to find the difference. Let's do this together.**

(Move candies to workspace.)

Teacher **Our minuend is __. What's our minuend?**

Students __.

Teacher **Let's show this minuend by showing __ candies.**

(Show candies in a line.)

Teacher **How many candies?**

Students __.

Teacher **Our subtrahend is __. What's our subtrahend?**

Students __.

Teacher **Let's show the subtrahend by showing __ candies. I'm going to use different colored candies for the difference.**

Teacher **How many candies?**

Students __.

Teacher **Now, let's compare the first set of candies – the minuend – to the second set of candies – the subtrahend. What does comparing mean?**

Students To find the difference between two sets.

Teacher **Yes. Let's compare the sets of candies. I can count the difference as: __, __, __, … What's the difference between the two sets of candies?**

Students __.

Teacher **The difference is __ candies. So, __ minus __ equals __. Let's say that together.**

Students __ minus __ equals __.

Teacher **Let's say it together again.**

Students __ minus __ equals __.

Teacher **So, if you have a set of __ and compare __ to the set, the difference between the two sets is __. __ minus __ equals __. Let's review. What's a minuend?**

Students The number from which another is subtracted.

Teacher **What's a subtrahend?**

Students The number to be subtracted.

Teacher **What's a difference?**

Students The amount or space between the minuend and subtrahend.

Teacher **What does it mean to separate?**

Students To take away.

Teacher **How could you explain separating to a friend?**

Students We started with a set of candies. Then, we compared that set of candies to another set of candies. We counted the difference between the two sets.

Example

$$\begin{array}{r} 10 \\ -\ \ 6 \\ \hline 4 \end{array}$$

Teacher	**Let's work on subtraction. Today, let's think about subtracting as comparing. What does it mean to compare?**
Students	To find the difference between two sets.
Teacher	**When we compare, we look at two sets to determine the difference. Now, let's think about comparing in subtraction. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Today we'll subtract by comparing, but there are other ways to subtract. Let's start by showing the minuend with our cubes and then comparing the subtrahend with cubes to find the difference. Let's do this together.** (Move cubes to workspace.)
Teacher	**Our minuend is 10. What's our minuend?**
Students	10.
Teacher	**Let's show this minuend by showing 10 red cubes.** (Show 10 red cubes.)
Teacher	**How many red cubes?**
Students	10.
Teacher	**Our subtrahend is 6. What's our subtrahend?**
Students	6.
Teacher	**Let's show the subtrahend by showing 6 yellow cubes.** (Show 6 yellow cubes. Line up under the 10 red cubes.)
Teacher	**How many yellow cubes?**
Students	6.
Teacher	**Now, let's compare the two sets of cubes. What does comparing mean?**
Students	To find the difference between two sets.
Teacher	**Yes. Let's compare the 10 red cubes to the 6 yellow cubes. We have 1, 2, 3, 4 more red cubes. How many more red cubes?**
Students	4.
Teacher	**To compare, we count the difference between the two sets. The difference between 10 and 6 is 4. What's the difference?**
Students	4.
Teacher	**Yes! The difference is 4. So, 10 minus 6 equals 4. Let's say that together.**
Students	10 minus 6 equals 4.
Teacher	**Let's say it together again.**
Students	10 minus 6 equals 4.

Teacher	**So, if you compare 10 to 6, the difference is 4. 10 minus 6 equals 4. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The amount or space between the minuend and subtrahend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We showed 10 red cubes and 6 yellow cubes. We compared the difference between 10 and 6. The difference was 4.

D. Problems for Use During Instruction

See Module 7 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 7 Vocabulary Cards.

F. Supplementary

COUNTING UP Subtraction

1. Put the subtrahend in your fist and say it.

2. Count up your fingers to the minuend.

3. The difference is the number of fingers you have up.

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Module 7:
Concepts of Subtraction

Problem Sets

$$\begin{array}{r} 8 \\ -\quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ -\quad 1 \\ \hline \end{array}$$

$$
\begin{array}{r} 7 \\ -\quad 3 \\ \hline \end{array}
$$

$$\begin{array}{r} 9 \\ -\quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ -\quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ -\quad 1 \\ \hline \end{array}$$

$$
\begin{array}{r} 8 \\ -\quad 5 \\ \hline \end{array}
$$

$$\begin{array}{r} 5 \\ -\quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ -\quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ -\quad 2 \\ \hline \end{array}$$

$$
\begin{array}{r}
3 \\
-\quad 0 \\
\hline
\end{array}
$$

$$\begin{array}{r} 8 \\ - \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ -\quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ -\quad 3 \\ \hline \end{array}$$

$$
\begin{array}{r}
8 \\
-\quad 1 \\
\hline
\end{array}
$$

$$\begin{array}{r} 5 \\ -\quad 0 \\ \hline \end{array}$$

$$
\begin{array}{r}
4 \\
-\quad 1 \\
\hline
\end{array}
$$

$$\begin{array}{r} 6 \\ -\quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ -\quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ -\quad 6 \\ \hline \end{array}$$

$$
\begin{array}{r}
9 \\
-\quad 8 \\
\hline
\end{array}
$$

$$\begin{array}{r} 4 \\ -\quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ -\quad 2 \\ \hline \end{array}$$

$$
\begin{array}{r} 10 \\ -\quad 3 \\ \hline \end{array}
$$

$$\begin{array}{r} 12 \\ -\quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ -\quad 9 \\ \hline \end{array}$$

$$
\begin{array}{r}
12 \\
-\quad 6 \\
\hline
\end{array}
$$

$$\begin{array}{r} 15 \\ -\quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ -\quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 14 \\ -\quad 8 \\ \hline \end{array}$$

$$
\begin{array}{r}
16 \\
-\quad 8 \\
\hline
\end{array}
$$

$$\begin{array}{r} 11 \\ - \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ -\quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ -\quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ -\quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 18 \\ -\quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 16 \\ -\quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 15 \\ - \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 14 \\ -\quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ -\quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ -\quad 7 \\ \hline \end{array}$$

$$
\begin{array}{r}
11 \\
-\quad 7 \\
\hline
\end{array}
$$

$$\begin{array}{r} 11 \\ -\quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ -\quad 8 \\ \hline \end{array}$$

$$
\begin{array}{r} 13 \\ -\quad 3 \\ \hline \end{array}
$$

$$\begin{array}{r} 11 \\ -\quad 1 \\ \hline \end{array}$$

$$
\begin{array}{r}
15 \\
-\quad 2 \\
\hline
\end{array}
$$

$$\begin{array}{r} 16 \\ -\quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ -\quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ -\quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ -\quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ -\quad 3 \\ \hline \end{array}$$

$$
\begin{array}{r}
4 \\
-\quad 4 \\
\hline
\end{array}
$$

$$\begin{array}{r} 5 \\ -\quad 5 \\ \hline \end{array}$$

$$
\begin{array}{r} 6 \\ -\quad 6 \\ \hline \end{array}
$$

$$
\begin{array}{r} 7 \\ -\quad 7 \\ \hline \end{array}
$$

$$\begin{array}{r} 8 \\ -\quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - \quad 9 \\ \hline \end{array}$$

Module 7:
Concepts of Subtraction

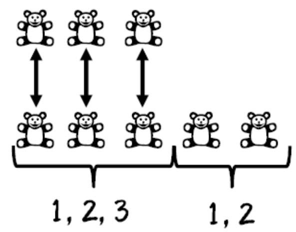
Vocabulary Cards

compare
difference
equal sign
minuend
minus sign
separate
subtract/subtraction
subtrahend

compare

To find the difference between two sets.

5 – 3 = 2



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difference

The result of subtracting one number from another number.

6 – 4 = 2

2 is the difference

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$12 - 8 = 4$$

= is the equal sign

---

minuend

The number from which another number is subtracted.

$$9 - 4 = 5$$

9 is the minuend

minus sign

The symbol that tells you to subtract.

9 – 4 = 5

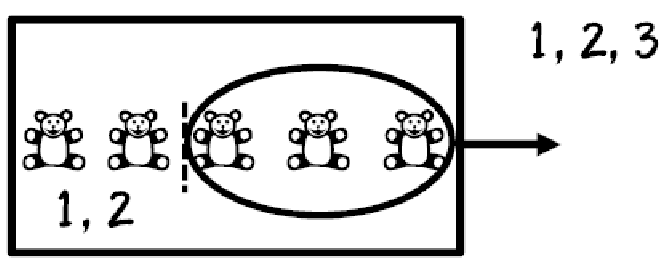
– is the minus sign

---

separate

To start with a set and take away from that set.

5 – 3 = 2

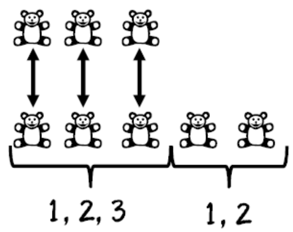


subtract/subtraction

To compare two sets or to take away from a set.

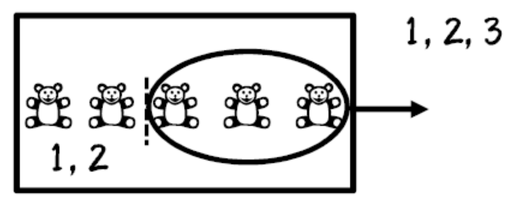
To compare two sets

5 – 3 = 2



To take away from a set

5 – 3 = 2



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subtrahend

The number to be subtracted.

9 – 4 = 5

4 is the subtrahend

Instructional Routines for Mathematics Intervention

MODULE 8

Subtraction of Whole Numbers

Module 8: Subtraction of Whole Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A procedure or description of steps that can be used to solve a problem.
compare	To find the difference between two sets.
computation	The action used to solve a problem.
difference	The result of subtracting one number from another number.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
hundreds column	The column with digits in the hundreds place.
minuend	The number from which another number is subtracted.
minus sign	The symbol that tells you to subtract.
ones column	The column with digits in the ones place.
regroup/trade/exchange	The process of exchanging 1 ten for 10 ones, 1 hundred for 10 tens, 1 thousand for 10 hundreds, etc.
separate	To start with a set and take away from that set.
subtract/subtraction	To compare two sets or to separate from a set.
subtrahend	The number to be subtracted.
tens column	The column with digits in the tens place.

B. Background Information

Background Information:

If your focus is on the conceptual understanding of subtraction, see *Module 7: Concepts of Subtraction*. This module, *Module 8*, focuses on subtraction computation of whole numbers. As you focus on computation, continue to emphasize subtraction as separating and subtraction as comparing because students will see these concepts within word problems.

For learning computation with subtraction, we recommend presenting problems vertically. Some students may require explicit instruction on translating a horizontal problem (e.g., 124 – 83) to the vertical presentation (see below). Depending upon the algorithm, leave enough space above or below the problem for students to complete their written work.

Every student should develop efficiency with a subtraction computation strategy. In the following sections, we provide examples of (1) subtraction with a traditional algorithm – no regrouping, (2) subtraction with a traditional algorithm – regrouping, (3) subtraction with partial differences algorithm, and (4) subtraction with an adding up algorithm. Teachers should understand different algorithms and help students to develop competency with at least one algorithm.

Subtraction Computation

1 11		
~~2~~~~1~~6	←	Minu end
– 73	←	subtrahend
143	←	difference

C. Routines and Examples

(1) Subtraction with Traditional Algorithm – No Regrouping

Routine

Materials:

- Module 8 Problem Sets
- Module 8 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
  - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT – 2-DIGIT: ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**With our Base-10 blocks, the rods represent tens. What do the rods represent?**
Students	Tens.
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**

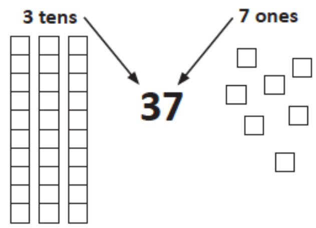
Students Ones.

Teacher Our minuend is __. What's our minuend?

Students __.

Teacher Let's show this minuend by showing __ tens and __ ones.

(Show with Base-10 blocks.)



Teacher How many?

Students __.

Teacher Our subtrahend is __. What's our subtrahend?

Students __.

Teacher Let's subtract the subtrahend. In this example, we'll think about subtraction as separating, but we could also think about subtraction as comparing. What do we subtract?

Students Subtrahend.

Teacher What's the subtrahend in this problem?

Students __.

Teacher Let's first subtract the ones of the subtrahend. We separate __ ones from the minuend. Do we have enough ones in the minuend to subtract __ ones?

Students Yes.

Teacher We have enough ones. Let's separate or take away __ ones.

(Remove ones.)

Teacher Now, let's subtract the tens of the subtrahend. We separate __ tens from the minuend. Do we have enough tens in the minuend to subtract __ tens?

Students Yes.

Teacher We have enough tens. Let's separate or take away __ tens.

(Remove tens.)

Teacher Let's count to learn the difference.

(Count the tens, then count the ones.)

Teacher That means __ minus __ equals __. Let's say that together.

Students __ minus __ equals __.

Teacher Let's say it together again.

Students __ minus __ equals __.

Teacher So, if you have a set of __ and separate __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?

Students The number from which another is subtracted.

Teacher What's a subtrahend?

Students The number to be subtracted.

Teacher What's a difference?

Students The result of subtracting a subtrahend from a minuend.

Teacher **What does it mean to separate?**
Students To take away.
Teacher **How could you explain separating to a friend?**
Students We started with a set of Base-10 blocks. We separated the ones and tens of the subtrahend. We counted to learn the difference.
Teacher **What's another way we could have solved this problem?**
Students We could have compared two sets.

2-DIGIT – 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on subtraction. What does it mean to subtract?**
Students To separate or compare.
Teacher **Subtraction means to separate from a set or to compare two sets. Look at this problem.**
(Show problem.)
Teacher **First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students To subtract.
Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.**
(Draw vertical lines to separate place value columns.)
Teacher **Now, we start by subtracting the ones. What should we subtract first?**
Students The ones.
Teacher **Which ones do we subtract?**
Students __ minus __.
Teacher **Do you have enough ones to subtract __ ones?**
Students Yes.
Teacher **You have enough ones to subtract or take away __ ones. We don't have to regroup. What's __ minus __?**
(If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Teacher **How many ones are remaining?**
Students __.
Teacher **Yes! There are __ ones. Let's write __ below the equal line.**
(Write.)
Teacher **Now, let's subtract the tens. Which tens do we subtract?**
Students __ minus __.
Teacher **Do you have enough tens to subtract __ tens?**
Students Yes.

Teacher	**You have enough tens to subtract or take away __ tens. We don't have to regroup. What's __ minus __?** (If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Teacher	**How many tens are remaining?**
Students	__.
Teacher	**There are __ tens. Let's write __ below the equal line.** (Write.)
Teacher	**So, what's __ minus __?**
Students	__.
Teacher	**That's right. __ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**So, if you have a set of __ and subtract __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We subtracted the ones and then we subtracted the tens to learn the difference between two numbers.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

$$\begin{array}{r} 458 \\ -\ \ 26 \\ \hline 432 \end{array}$$

Example

3-DIGIT – 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.

Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens and the tens from the hundreds. Let's draw a vertical line between the ones column and the tens column. Then, draw a vertical line between the tens column and the hundreds column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by subtracting the ones. What should we subtract first?**
Students	The ones.
Teacher	**Which ones do we subtract?**
Students	8 minus 6.
Teacher	**If you have 8 ones, can you subtract 6 ones?**
Students	Yes.
Teacher	**You have enough ones to subtract 6 ones. Let's subtract 8 minus 6.** (If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Teacher	**How many ones are remaining?**
Students	2.
Teacher	**Yes! There are 2 ones remaining. Let's write 2 under the equal line in the ones place.** (Write 2.)
Teacher	**Now, let's subtract the tens. Which tens do we subtract?**
Students	5 minus 2.
Teacher	**If you have 5 tens, can you subtract 2 tens?**
Students	Yes.
Teacher	**Great. You have enough tens to subtract 2 tens. What's 5 minus 2?** (If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Teacher	**How many tens are remaining?**
Students	3.
Teacher	**There are 3 tens. Let's write 3 under the equal line in the tens place.** (Write 3.)
Teacher	**Now, let's subtract the hundreds. Which hundreds do we subtract?**
Students	4 minus nothing or 0.
Teacher	**If you have 4 hundreds, can you subtract 0?**
Students	Yes.
Teacher	**You can subtract 4 minus 0. What's 4 minus 0?**
Students	4.
Teacher	(If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __:**

__, __, __. See Counting Up poster at the end of Module 7 for more information.)

Teacher	**How many hundreds are remaining?**
Students	4.
Teacher	**There are 4 hundreds. Let's write 4 under the equal line in the hundreds place.** (Write 4.)
Teacher	**What's 458 minus 26?**
Students	432.
Teacher	**That's right. 458 minus 26 equals 432. Let's say that together.**
Students	458 minus 26 equals 432.
Teacher	**So, if you have a set of 458 and separate 26, the difference is 432. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We subtracted the ones. Then, we subtracted the tens. Then, we subtracted the hundreds to learn the difference between 458 and 26.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

(2) Subtraction with Traditional Algorithm – Regrouping

Routine

Materials:

- Module 8 Problem Sets
- Module 8 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
    - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT – 2-DIGIT: ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.**

	(Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with Base-10 blocks.**
	(Move Base-10 blocks to workspace.)
Teacher	**With our Base-10 blocks, the rods represent tens. What do the rods represent?**
Students	Tens.
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**
Students	Ones.
Teacher	**Our minuend is __. What's our minuend?**
Students	__.
Teacher	**Let's show the minuend by showing __ tens and __ ones.**
	(Show with Base-10 blocks.)
Teacher	**How many?**
Students	__.
Teacher	**Now, we separate the subtrahend from the minuend. What's our subtrahend?**
Students	__.
Teacher	**Let's first subtract the ones of the subtrahend. We separate __ ones from the minuend. How many ones?**
Students	__.
Teacher	**Look at the minuend. Do we have enough ones in the minuend to subtract __ ones?**
Students	No!
Teacher	**We do not have enough ones. That means we have to regroup. To regroup, we take 1 ten and regroup/trade/exchange the 1 ten for 10 ones. Let's do that together.**
	(Show 1 ten is equivalent to 10 ones.)
Teacher	**Let's regroup/trade/exchange the 1 ten for 10 ones. See how 1 ten is the same as 10 ones?**
Students	Yes.
Teacher	**Now we have all these ones. But we can't leave the ones in the tens place. The tens place is only for tens. So, we place the 10 ones in the ones column. Where do we place the ones?**
Students	In the ones column.
Teacher	**Can we subtract __ ones now?**
Students	Yes.
Teacher	**Let's subtract __ ones.**
	(Separate ones.)
Teacher	**Now, let's subtract the tens of the subtrahend. How many tens do we need to subtract?**

Students __.

Teacher **Look at the tens of the minuend. Do we have enough tens in the minuend to subtract __ tens?**

Students Yes.

Teacher **We have enough tens. We do not have to regroup. Let's separate or subtract __ tens.**
(Separate tens.)

Teacher **So, let's count the remaining tens and ones to learn the difference. Ready?**
(Count the tens, then count the ones.)

Teacher **That means __ minus __ equals __. Let's say that together.**

Students __ minus __ equals __.

Teacher **Let's say it together again.**

Students __ minus __ equals __.

Teacher **So, if you have a set of __ and separate __ from the set, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**

Students The number from which another is subtracted.

Teacher **What's a subtrahend?**

Students The number to be subtracted.

Teacher **What's a difference?**

Students The result of subtracting a subtrahend from a minuend.

Teacher **What does it mean to separate?**

Students To take away.

Teacher **How could you explain separating to a friend?**

Students We subtracted the ones but we didn't have enough ones so we regrouped 1 ten for 10 ones. Then, we subtracted the tens. We figured out the difference between __ and __.

Teacher **What's another way we could have solved this problem?**

Students We could have compared two sets.

2-DIGIT – 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on subtraction. What does it mean to subtract?**

Students To separate or compare.

Teacher **Subtraction means to separate from a set or compare two sets. Look at this problem.**
(Show problem.)

Teacher **First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**

Students To subtract.

Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.**

	(Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by subtracting. What should we subtract first?**
Students	The ones.
Teacher	**Which ones do we subtract?**
Students	__ minus __.
Teacher	**Do you have enough ones to subtract __ ones?**
Students	No.
Teacher	**We do not have enough ones. That means we have to regroup. To regroup, we take 1 ten and regroup/trade/exchange the 1 ten for 10 ones. To take 1 ten, I subtract 1 ten from the tens column. __ minus 1 equals __. I like to cross out the __ and write a __ in the tens column.** (Show subtraction of 1 ten.)
Teacher	**Now, I imagine regrouping this 1 ten into 10 ones. If I have 10 ones and add these ones to the __ ones, how many ones do I have now?**
Students	__.
Teacher	**I like to show the __ ones by crossing out the __ and writing __ in the ones column.** (Show addition of 10 ones.)
Teacher	**Now, let's subtract the ones. What's __ minus __?** (If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Students	__.
Teacher	**Yes! There are __ ones. Let's write __ below the equal line.** (Write.)
Teacher	**Now, let's subtract the tens. Which tens do we subtract?**
Students	__ minus __.
Teacher	**Do you have enough tens to subtract __ tens?**
Students	Yes.
Teacher	**You have enough tens to subtract or take away __ tens. We don't have to regroup. What's __ minus __?**
Students	__.
Teacher	**There are __ tens. Let's write __ below the equal line.** (Write.)
Teacher	**That means __ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**Let's say it together again.**
Students	__ minus __ equals __.
Teacher	**So, if you have a set of __ and separate __ from the set, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.

Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We subtracted the ones but we didn't have enough ones so we regrouped 1 ten for 10 ones. Then, we subtracted the tens. We figured out the difference between __ and __.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

Example

$$\begin{array}{r} 236 \\ -\ \ 89 \\ \hline 147 \end{array}$$

3-DIGIT – 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw one vertical line between the ones column and the tens column and one vertical line between the tens column and the hundreds column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by subtracting. What should we subtract first?**
Students	The ones.
Teacher	**Which ones do we subtract?**
Students	__ minus __.
Teacher	**Do you have enough ones to subtract __ ones?**
Students	No.
Teacher	**We do not have enough ones. That means we have to regroup. To regroup, we take 1 ten and regroup/trade/exchange the 1 ten for 10 ones. To take 1 ten, I subtract 1 ten from the tens column. 3 minus 1 equals 2. I like to cross out the 3 and write a 2 in the tens column.**

	(Write 2 above tens column.)
Teacher	**Now, I imagine regrouping this 1 ten into 10 ones. If I have 10 ones and add these ones to the 6 ones, how many ones do I have now?**
Students	16.
Teacher	**I like to show the 16 ones by crossing out the 6 and writing 16 in the ones column.** (Write 16 above ones column.)
Teacher	**Now, let's subtract the ones. What's 16 minus 9?** (If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Students	7.
Teacher	**Yes! 16 minus 9 equals 7. Let's write 7 below the equal line.** (Write 7.)
Teacher	**Now, let's subtract the tens. Which tens do we subtract?**
Students	2 minus 8.
Teacher	**Do you have enough tens to subtract 8 tens?**
Students	No.
Teacher	**We do not have enough tens. That means we have to regroup. To regroup, we take 1 hundred and regroup/trade/exchange the 1 hundred for 10 tens. To take 1 hundred, I subtract 1 hundred from the hundreds column. 2 minus 1 equals 1. I like to cross out the 2 and write a 1 in the hundreds column.** (Write 1 above hundreds column.)
Teacher	**Now, I imagine regrouping this 1 hundred into 10 tens. If I have 10 tens and add these tens to the 2 tens, how many tens would you have?**
Students	12.
Teacher	**It's helpful to show the 12 tens by crossing out the 2 and writing 12 in the tens column.** (Write 12 above tens column.)
Teacher	**Now, let's subtract the tens. What's 12 minus 8?** (If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Students	4.
Teacher	**There are 4 tens. Let's write 4 below the equal line.** (Write 4.)
Teacher	**Are we finished subtracting?**
Students	No.
Teacher	**What do we subtract next?**
Students	Hundreds.
Teacher	**What do we subtract in the hundreds?**
Students	1 minus 0.

Teacher	**What's 1 minus 0?**
Students	1.
Teacher	**Let's write 1 below the equal line.** (Write 1.)
Teacher	**That means 236 minus 89 equals 147. Let's say that together.**
Students	236 minus 89 equals 147.
Teacher	**Let's say it together again.**
Students	236 minus 89 equals 147.
Teacher	**So, if you have a set of 236 and separate 89 from the set, the difference is 147. 236 minus 89 equals 147. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The amount between the minuend and subtrahend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We subtracted the ones but we didn't have enough ones so we regrouped 1 ten for 10 ones. Then, we subtracted the tens but we didn't have enough tens so we regrouped 1 hundred for 10 ones. Then, we subtracted the hundreds. The difference between 236 and 89 is 147.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

(3) Subtraction with Partial Differences* Algorithm

Routine

Materials:

- Module 8 Problem Sets
- Module 8 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like a number line
  - Note that drawings can be used alongside or instead of manipulatives

*This algorithm requires an understanding of positive and negative numbers. If students have difficulty interpreting numbers less than 0, do not use this algorithm.

2-DIGIT – 2-DIGIT: ROUTINE

Teacher **Let's work on subtraction. What does it mean to subtract?**
Students To separate or compare.
Teacher **Subtraction means to separate from a set or to compare two sets. Look at this problem.**
(Show problem.)
Teacher **First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students To subtract.
Teacher **Let's do this problem with our number line.**
(Show number line.)
Teacher **Our minuend is __. What's our minuend?**
Students __.
Teacher **We'll subtract the subtrahend from the minuend. What's our subtrahend?**
Students __.
Teacher **Let's subtract the subtrahend. In this example, we'll use the partial differences strategy. With partial differences, we subtract each place value and then combine the partial differences to find the difference.**
Teacher **Let's first subtract the tens of the subtrahend. That means we have __ tens** (from the minuend) **minus __ tens** (from the subtrahend). **Think about this on the number line. What's __ minus __?**
Students __.
Teacher **__ is one of our partial differences. It's the difference of the tens. Let's write __ below the equal line. I like to write a positive/negative symbol because this number is positive/negative.**
(Write.)
Teacher **Now, let's subtract the ones of the subtrahend. How many ones do we subtract?**
Students __.
Teacher **Yes, let's subtract __ ones** (from the minuend) **minus __ tens** (from the subtrahend). **Think about this on the number line. What's __ minus __?**
Students __.
Teacher **__ is one of our partial differences. It's the difference of the ones. Let's write __ below the equal line. I like to write a positive/negative symbol because this number is positive/negative.**
(Write.)
Teacher **Now, below the equal line we have __ plus/minus __. What's __ plus/minus __?**
Students __.
Teacher **That means __ minus __ equals __. Let's say that together.**
Students __ minus __ equals __.
Teacher **Let's say it together again.**
Students __ minus __ equals __.

Teacher	**So, if you have a set of __ and separate __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How can you use the partial differences algorithm?**
Students	You subtract the tens for a partial difference. You subtract the ones for a partial difference. You then combine the partial differences to find the difference.

Example

$$\begin{array}{r} 236 \\ -\ \ 89 \\ \hline +200 \\ -50 \\ -3 \\ \hline 147 \end{array}$$

3-DIGIT – 2-DIGIT: EXAMPLE

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's use the partial differences algorithm. What's the partial differences strategy?**
Students	We find each partial difference in each place value column. Then, we combine the partial differences to find the difference.
Teacher	**What's our minuend?**
Students	236.
Teacher	**So, in this problem, we'll subtract the hundreds then tens then ones. How will we work on this problem?**
Students	Subtract the hundreds then tens then ones.
Teacher	**Let's start with the hundreds. How many hundreds do we subtract from 200?**

Students 0.

Teacher Yes! We have 0 hundreds to subtract. Let's write 200 under the equal line because we subtracted 0 from 200.
(Write 200.)

Teacher 200 is one of our partial differences. What's 200?

Students The partial difference for the hundreds.

Teacher Let's subtract the tens of the subtrahend. How many tens do we need to subtract?

Students 8 tens.

Teacher 8 tens is the same as what?

Students 80.

Teacher We subtract 80 from 30. What's 30 minus 80?

Students -50.

Teacher 30 minus 80 is -50. Let's write -50 below the equal line.
(Write -50 below 200.)

Teacher -50 is one of our partial differences. It's the difference of the tens. What's -50?

Students The partial difference for the tens.

Teacher Now, let's subtract the ones of the subtrahend. How many ones do we need to subtract?

Students 9 ones.

Teacher We subtract 9 ones from 6 ones. What's 6 minus 9?

Students -3.

Teacher 6 minus 9 is -3. Let's write -3 below the equal line.
(Write -3 below -50.)

Teacher -3 is one of our partial differences. What's -3?

Students The partial difference for the ones.

Teacher Now, below the equal line we have 200 minus 50 minus 3. Let's do this in steps. What's 200 minus 50?

Students 150.

Teacher What's 150 minus 3?

Students 147.

Teacher Let's draw another equal line and write 147 below.
(Write 147.)

Teacher That means 236 minus 89 equals 147. Let's say that together.

Students 236 minus 89 equals 147.

Teacher Let's say it together again.

Students 236 minus 89 equals 147.

Teacher So, if you have a set of 236 and separate 89, the difference is 147. Let's review. What's a minuend?

Students The number from which another is subtracted.

Teacher What's a subtrahend?

Students The number to be subtracted.

Teacher What's a difference?

Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How can you use the partial differences algorithm?**
Students	You subtract the hundreds for a partial difference. Then, you subtract the tens for a partial difference. Then, you subtract the ones for a partial difference. You then combine to find the difference.

(4) Subtraction with Adding Up Algorithm

Routine

Materials:

- Module 8 Problem Sets
- Module 8 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative (e.g., money, Base-10 blocks)
  - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT – 2-DIGIT: ROUTINE

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Today, let's think about subtraction as the difference between two numbers. How can we interpret subtraction?**
Students	The difference between two numbers.
Teacher	**So, in this problem, subtraction is the difference between what two numbers?**
Students	__ and __.
Teacher	**Let's figure out the difference between __ and __. Let's do this with our Base-10 blocks.** (Show Base-10 blocks.)
Teacher	**When we think about subtraction as the difference between two numbers, let's start with our subtrahend. What's the subtrahend in this problem?**
Students	__.
Teacher	**Let's show the subtrahend with our Base-10 blocks. How many tens?**
Students	__.
Teacher	**How many ones?**

Students	__.
	(Show subtrahend with Base-10 blocks.)
Teacher	**Now, let's think about what we could add to the subtrahend to reach the minuend, __. I see that I could add __ ones to get to the nearest ten. I'll add the ones over here so I don't confuse this with the subtrahend ones.**
	(Add ones in separate pile.)
Teacher	**Now, what else could we add to reach the minuend, __? I see that I could add __ tens to get very close to the minuend of __. I'll add the tens over here so I don't confuse these tens with the subtrahend tens.**
	(Add tens.)
Teacher	**Have we reached the minuend yet?**
Students	No.
Teacher	**What could we add to reach the minuend?**
Students	__.
Teacher	**I could add __ ones to reach the minuend. Let's add the ones over here so I don't confuse these ones with the subtrahend ones.**
	(Add ones.)
Teacher	**So, the difference between __ and __ is: __, __, __, … What's the difference?**
Students	__.
Teacher	**That means __ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**Let's say it together again.**
Students	__ minus __ equals __.
Teacher	**With this strategy, called adding up, you figure out the difference between __ and __ by adding up. You add up to find the difference between __ and __. How do you find the difference?**
Students	Adding up from __ to __.
Teacher	**Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**How could you explain adding up to a friend?**
Students	You start with the subtrahend. You keep adding until you reach the minuend. You do this to find the difference between the minuend and subtrahend.

Example

236	89	
− 89	90	+1
	100	+10
	200	+100
	236	+36
		147

3-DIGIT – 2-DIGIT: EXAMPLE

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Today, let's think about subtraction as the difference between two numbers. How can we interpret subtraction?**
Students	The difference between two numbers.
Teacher	**So, in this problem, subtraction is the difference between what two numbers?**
Students	236 and 89.
Teacher	**Let's figure out the difference between 236 and 89.**
Teacher	**When we think about subtraction as the difference between two numbers, let's start with our subtrahend. What's the subtrahend in this problem?**
Students	89.
Teacher	**Let's write the subtrahend next to the problem. What should we write?**
Students	89.
Teacher	**Now, let's think about what we could add to 89 to reach the minuend, 236. I see that I could add 1 one to get to the nearest ten. I'll write +1 over here to show I wanted to add 1.** (Write +1.)
Teacher	**If I added 1 to 89, what's the sum?**
Students	90.
Teacher	**Let's write 90 below 89 to remember we're now at 90.** (Write 90 below 89.)
Teacher	**Let's figure out what we could add to 90 to reach the minuend, 236. Could we add 10 more to get to the nearest hundred?**
Students	Yes.
Teacher	**Let's write +10 to show we wanted to add 10.** (Write +10 below +1.)

Teacher	**If we added 10 to 90, what's the sum?**
Students	100.
Teacher	**Let's write 100 below 90 to remember we're now at 110.** (Write 100 below 90.)
Teacher	**Let's keep going. What could we add to 100 to reach the minuend?**
Students	100.
Teacher	**Great idea. Let's write +100 to show we wanted to add 100.** (Write +100.)
Teacher	**If I added 100 to 100, what's the sum?**
Students	200.
Teacher	**Let's write 200 below 100 to remember we're now at 200.** (Write 200 below 100.)
Teacher	**Are we getting closer to 236?**
Students	Yes.
Teacher	**What could we add to 200 to reach the minuend, 236?**
Students	36.
Teacher	**Let's write +36 to show we wanted to add 36.** (Write +36.)
Teacher	**If I added 36 to 200, what's the sum?**
Students	236.
Teacher	**Let's write 236 below 200 to remember we're now at 236.** (Write 236 below 200.)
Teacher	**Did we reach the minuend?**
Students	Yes!
Teacher	**Now, we add +1 and +10 and +100 and +36 to determine the difference. How could we add these numbers?**
Students	100 + 36 + 10 + 1 (or other responses).
Teacher	**So, the difference is 147. What's the difference?**
Students	147.
Teacher	**That means 236 minus 89 equals 147. Let's say that together.**
Students	236 minus 89 equals 147.
Teacher	**Let's say it together again.**
Students	236 minus 89 equals 147.
Teacher	**With this strategy, called adding up, you figure out the difference between 236 and 89 by adding up. How do you find the difference?**
Students	Adding up from 89 to 236.
Teacher	**Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**How could you explain adding up to a friend?**

Students	You start with the subtrahend. You keep adding until you reach the minuend. You do this to find the difference between the minuend and subtrahend.

D. Problems for Use During Instruction

See Module 8 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 8 Vocabulary Cards.

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Module 8:
Subtraction of Whole Numbers

Problem Sets

A.

$$\begin{array}{r} 68 \\ -\ \ 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 43 \\ -\quad 2 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 89 \\ -\quad 1 \\ \hline \end{array}$$

A.

$$
\begin{array}{r} 96 \\ -\quad 5 \\ \hline \end{array}
$$

A.

$$\begin{array}{r} 38 \\ -\quad 7 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 61 \\ - \quad 5 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 93 \\ -\quad 6 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 45 \\ -\quad 8 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 58 \\ -\quad 9 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 63 \\ -\quad 4 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 74 \\ -\ 31 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 84 \\ -\ 11 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 85 \\ -\ 70 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 97 \\ -\ 65 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 30 \\ -\ 20 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 91 \\ -\ 30 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 99 \\ -\ 38 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 55 \\ -\ 30 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 57 \\ -\ 10 \\ \hline \end{array}$$

c.

$$
\begin{array}{r} 98 \\ -\ 74 \\ \hline \end{array}
$$

c.

$$\begin{array}{r} 73 \\ -\ 32 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 85 \\ -\ 35 \\ \hline \end{array}$$

c.

$$
\begin{array}{r} 75 \\ -\ 62 \\ \hline \end{array}
$$

c.

$$\begin{array}{r} 77 \\ -\ 15 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 56 \\ -\ 26 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 65 \\ -\ 24 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 60 \\ -\ 30 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 97 \\ -\ 24 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 98 \\ -\ 40 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 69 \\ -\ 31 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 80 \\ -\ 24 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 72 \\ -\ 15 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 60 \\ -\ 58 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 75 \\ -\ 46 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 98 \\ -\ 79 \\ \hline \end{array}$$

D.

$$
\begin{array}{r} 96 \\ -\ 77 \\ \hline \end{array}
$$

D.

$$\begin{array}{r} 54 \\ -\ 46 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 80 \\ -\ 61 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 31 \\ -\ 18 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 71 \\ -\ 49 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 66 \\ -\ 59 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 26 \\ -\ 19 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 20 \\ -\ 16 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 96 \\ -\ 19 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 77 \\ -\ 18 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 75 \\ -\ 27 \\ \hline \end{array}$$

D.

$$
\begin{array}{r}
56 \\
-\ 49 \\
\hline
\end{array}
$$

D.

$$\begin{array}{r} 78 \\ -\ 49 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 91 \\ -\ 47 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 65 \\ -\ 57 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 195 \\ -\quad 63 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 694 \\ - \quad 20 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 384 \\ -\quad 21 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 499 \\ -\quad 18 \\ \hline \end{array}$$

E.

$$
\begin{array}{r}
750 \\
-\quad 20 \\
\hline
\end{array}
$$

F.

$$\begin{array}{r} 172 \\ -\quad 63 \\ \hline \end{array}$$

F.

$$
\begin{array}{r} 621 \\ -\quad 12 \\ \hline \end{array}
$$

F.

$$\begin{array}{r} 735 \\ -\quad 69 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 943 \\ -\quad 51 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 238 \\ -\quad 54 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 747 \\ -\ 115 \\ \hline \end{array}$$

G.

$$
\begin{array}{r}
509 \\
-\ 301 \\
\hline
\end{array}
$$

G.

$$\begin{array}{r} 773 \\ -\ 142 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 578 \\ -\ 427 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 685 \\ -\ 502 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 961 \\ -\ 151 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 323 \\ -\ 111 \\ \hline \end{array}$$

G.

$$
\begin{array}{r}
897 \\
-\ 530 \\
\hline
\end{array}
$$

G.

$$\begin{array}{r} 888 \\ -\ 184 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 350 \\ -\ 240 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 675 \\ -\ 328 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 582 \\ -\ 153 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 580 \\ -\ 321 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 777 \\ -\ 168 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 612 \\ -\ 223 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 202 \\ -\ 247 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 583 \\ -\ 108 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 490 \\ -\ 177 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 464 \\ -\ 215 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 609 \\ -\ 134 \\ \hline \end{array}$$

Module 8:

Subtraction of Whole Numbers

Vocabulary Cards

algorithm
compare
computation
difference
equal sign
hundred column
minuend
minus sign
ones column
regroup/trade/exchange
separate
subtract/subtraction
subtrahend
tens column

algorithm

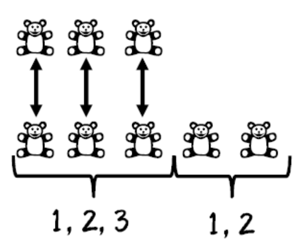
A procedure or description of steps that can be used to solve a problem.

---

compare

To find the difference between two sets.

5 – 3 = 2



computation

The action used to solve a problem.

---

difference

The result of subtracting one number from another number.

$$6 - 4 = 2$$

2 is the difference

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$12 - 8 = 4$$

= is the equal sign

---

hundreds column

The column with digits in the hundreds place.

In the number 423, 4 is in the hundreds place.

minuend

The number from which another number is subtracted.

9 – 4 = 5

9 is the minuend

---

minus sign

The symbol that tells you to subtract.

9 – 4 = 5

– is the minus sign

ones column

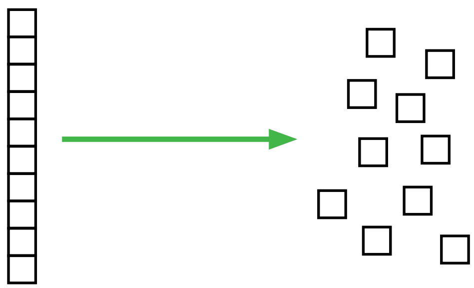
The column with digits in the ones place.

In the number 423, 3 is in the ones place.

---

regroup/trade/exchange

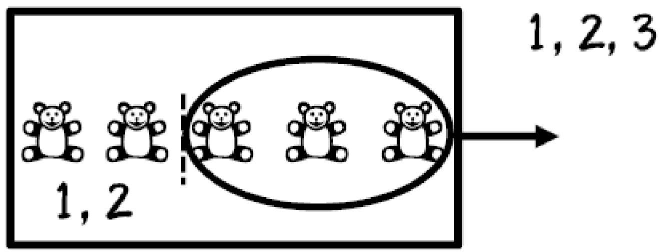
The process of exchanging 1 ten for 10 ones, 1 hundred for 10 tens, 1 thousand for 10 hundreds, etc.



separate

To start with a set and take away from that set.

5 – 3 = 2



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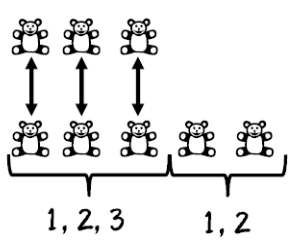
subtract/subtraction

To compare two sets or to separate from a set.

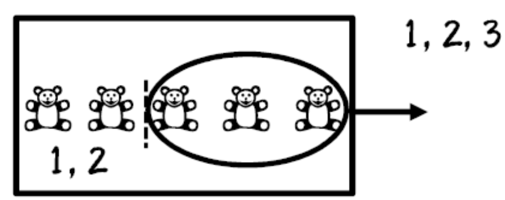
To compare two sets

5 – 3 = 2



To separate from a set

5 – 3 = 2



subtrahend

The number to be subtracted.

9 – 4 = 5

4 is the subtrahend

---

tens column

The column with digits in the tens place.

In the number 423, 2 is the in the tens column.

Instructional Routines for Mathematics Intervention

MODULE 9

Subtraction of Rational Numbers

Module 9: Subtraction of Rational Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A procedure or description of steps that can be used to solve a problem.
compare	To find the difference between two sets.
computation	The action used to solve a problem.
decimal	A number based on powers of ten.
denominator	The term in a fraction that tells the number of equal parts in a whole.
difference	The result of subtracting one number from another number.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
hundredths	The digit in representing $\frac{1}{100}$.
improper fraction	Any fraction in which the numerator is greater than the denominator.
least common multiple	The common multiple with the least value.
minuend	The number from which another number is subtracted.
minus sign	The symbol that tells you to subtract.
mixed number	A whole number and a fraction combined.
multiple	The product of a number and any integer.
numerator	The term in a fraction that tells how many parts of a fraction.
ones	The digit representing 1.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
separate	To start with a set and take away from that set.
subtract/subtraction	To compare two sets or to separate from a set.
subtrahend	The number to be subtracted.
tenths	The digit in representing $\frac{1}{10}$.

B. Background Information

Background Information:

In this module, we focus on subtraction with fractions and decimals. As you focus on computation of rational numbers, continue to emphasize subtraction as separating and subtraction as comparing because students will see these concepts within word problems.

For subtraction of fractions, we recommend using several models of fractions to help students understand concepts related to subtraction of fractions. We also recommend demonstrating several algorithms for subtraction of decimals. Every student should develop efficiency with strategies for subtraction of fractions and decimals. In the following sections, we provide examples of (1) subtraction of fractions – like denominators, (2) subtraction of fractions – unlike denominators, (3) subtraction of decimals with the traditional algorithm, and (4) subtraction of decimals with the adding up algorithm.

C. Routines and Examples

(1) Subtraction of Fractions – Like Denominators

Routine

Materials:

- Module 9 Problem Sets
- Module 9 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
    - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate from a set or to compare.
Teacher	**Subtraction means to separate from a set or to compare two numbers. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point)**. The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with fraction tiles.** (Move fraction tiles to workspace.)
Teacher	**First, our minuend is __. What's the minuend?**
Students	__.
Teacher	**Let's show this minuend using the fraction tiles.** (Show fraction part compared to whole.)

Teacher	**What fraction?**
Students	__.
Teacher	**Now, our subtrahend is __. What's our subtrahend?**
Students	__.
Teacher	**We'll subtract the subtrahend. When working with fractions, I like to show the subtrahend to know the quantity we will separate from the minuend. Let's show the subtrahend over here.** (Show fraction part compared to whole.)
Teacher	**What fraction?**
Students	__.
Teacher	**Let's subtract. When subtracting fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?**
Students	Yes.
Teacher	**The denominators are the same. When the denominators are the same, we can go ahead and subtract. So, let's look at our subtrahend. We want to subtract the subtrahend from the minuend. How many __ one-__ parts do we subtract?**
Students	__ one-__ parts.
Teacher	**Let's subtract __ one-__ parts.**
Students	(Subtract from the minuend set.)
Teacher	**So, we now have __, __, __, … one-__ parts remaining. How many parts?**
Students	__.
Teacher	**When you have __ minus __, the difference is __. What's the difference?**
Students	__.
Teacher	**__ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**So, if you have a set of __ and a set of __, when you subtract (or separate) the sets, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We started with a fraction showing fraction tiles. We checked whether the denominators were the same. Then, we separated __ one-__ parts from the minuend to learn the difference.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with our pencil. Our minuend is __. What's our minuend?**
Students	__.
Teacher	**Our subtrahend is __. What's our subtrahend?**
Students	__.
Teacher	**So, we have __ minus __. Let's subtract by separating. What does separating mean?**
Students	To remove some from a set.
Teacher	**Yes. Let's subtract, or separate, the subtrahend from the minuend. What do we subtract?**
Students	The subtrahend from the minuend.
Teacher	**Now, the parts of the fractions are the numerators. When we subtract fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?**
Students	Yes.
Teacher	**The denominators are the same. When the denominators are the same, we can go ahead and subtract. The denominator, __, will not change when we subtract the fractions. Let's go ahead and write the denominator for our difference.** (Write denominator.)
Teacher	**Now, we want to subtract the parts or numerator of the subtrahend from the minuend. That means we have to subtract __ one-__ parts from __ one-__ parts. What do we subtract?**
Students	We subtract the parts or numerators of the fractions.
Teacher	**Let's subtract the parts. What's __ minus __?**
Students	__.
Teacher	**Let's write the parts we subtracted.** (Write parts.)
Teacher	**When you have __ minus __, the difference is __. What's the difference?**
Students	__.
Teacher	**__ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**So, if you have a set of __ and a set of __, when you subtract (or separate) the subtrahend from the minuend, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**

Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We checked whether the denominators were the same. Then, we subtracted the parts of the subtrahend from the parts of the minuend to learn the difference between two numbers.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

Example

$$\frac{5}{6} - \frac{3}{6} = \frac{2}{6}$$

EXAMPLE WITH MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point)**. The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with fraction tiles.** (Move fraction tiles to workspace.)
Teacher	**Our minuend is $\frac{5}{6}$. What's our minuend?**
Students	$\frac{5}{6}$.
Teacher	**Let's show this minuend by showing the fraction.** (Show 5 one-sixth parts compared to a whole.)
Teacher	**What fraction?**
Students	$\frac{5}{6}$.
Teacher	**Our subtrahend is $\frac{3}{6}$. What's our subtrahend?**
Students	$\frac{3}{6}$.
Teacher	**Let's show the subtrahend by showing the fraction.** (Show 3 one-sixth parts compared to a whole.)

Teacher	**What fraction?**
Students	$\frac{3}{6}$.
Teacher	**So, we have $\frac{5}{6}$ minus $\frac{3}{6}$. Let's subtract the subtrahend from the minuend. What does subtracting mean?**
Students	To separate or compare.
Teacher	**Let's subtract, or separate, the parts of the fractions. The parts of the fractions represent the numerators. When subtracting fractions, first we want to determine whether the denominators are like or unlike. Are the denominators like or the same?**
Students	Yes.
Teacher	**Both denominators are 6. The denominators are the same or like denominators. When the denominators are the same, we can go ahead and subtract. Second, we want to subtract the numerators, or parts, or the subtrahend from the minuend. That means we have to subtract 3 one-sixth parts from 5 one-sixth parts. What do we subtract?**
Students	We subtract the parts or numerators of the fraction.
Teacher	**Let's subtract the 3 one-sixth parts from the 5 one-sixth parts. I'm not going to touch the subtrahend. Instead, I separate, or take away, 3 one-sixth parts from the minuend.** (Subtract parts, compare to whole.)
Teacher	**So, we now have 1, 2 one-sixth parts. How many parts?**
Students	2 one-sixth parts.
Teacher	**When you have $\frac{5}{6}$ minus $\frac{3}{6}$, the difference is $\frac{2}{6}$. What's the difference?**
Students	$\frac{2}{6}$.
Teacher	**$\frac{5}{6}$ minus $\frac{3}{6}$ equals $\frac{2}{6}$. Let's say that together.**
Students	$\frac{5}{6}$ minus $\frac{3}{6}$ equals $\frac{2}{6}$.
Teacher	**So, if you have a set of $\frac{5}{6}$ and you separate $\frac{3}{6}$, when you subtract the parts or numerators of the subtrahend from the minuend, the difference is $\frac{2}{6}$. $\frac{5}{6}$ minus $\frac{3}{6}$ equals $\frac{2}{6}$. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**

Students	We showed the minuend with fraction tiles and showed the subtrahend with fraction tiles. Then, we subtracted 3 one-sixth parts from 5 one-sixth parts. The difference was two-sixths.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

(2) Subtraction of Fractions – Unlike Denominators

Routine

Materials:

- Module 9 Problem Sets
- Module 9 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
  - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point)**. The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with two-color counters.** (Move two-color counters to workspace.)
Teacher	**Our minuend is __. What's our minuend?**
Students	__.
Teacher	**Let's show this minuend by showing the fraction.** (Show set compared to whole with white/yellow counters representing numerator and red counters representing denominator.)
Teacher	**What fraction?**
Students	__.
Teacher	**Our subtrahend is __. What's our subtrahend?**
Students	__.
Teacher	**Let's show the subtrahend by showing the fraction.** (Show set compared to whole with white/yellow counters representing numerator and red counters representing denominator.)
Teacher	**What fraction?**
Students	__.

Teacher	**So, we have __ minus __. Let's subtract by separating. What does separating mean?**
Students	To take away from a set.
Teacher	**Yes. Let's separate, or take away, the subtrahend from the minuend. Remember, the parts of the fractions represent the numerators. When subtracting fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?**
Students	No.
Teacher	**The denominators are not the same. To subtract, we need to subtract parts or numerators with the same denominator. When the denominators are unlike, the parts or numerators do not have the same value. So, we will work to make the fractions have like denominators. Why do we want to subtract fractions with like denominators?**
Students	So, we can subtract the parts or numerators of the fractions.
Teacher	**To do this, let's write the first five multiples of each denominator. The minuend has a denominator of __, so let's write the first five multiples of __.** (Write multiples as __, __, __, __, __.)
Teacher	**What are the multiples of __? Say them with me.**
Students	__, __, __, __, __.
Teacher	**The subtrahend has a denominator of __, so let's write the first five multiples of __.** (Write multiples as __, __, __, __, __.)
Teacher	**What are the multiples of __? Say them with me.**
Students	__, __, __, __, __.
Teacher	**Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?**
Students	__.
Teacher	**So, __ is the least common multiple. Say that with me.**
Students	Least common multiple.
Teacher	**Sometimes we call the least common multiple the LCM. What do we call the least common multiple?**
Students	LCM.
Teacher	**The least common multiple, or LCM, helps us to determine the common denominator for the two fractions. What does the LCM help with?**
Students	Finding a common denominator for the two fractions.
Teacher	**The minuend has a denominator of __.** ***OPTION 1:*** **This is the original denominator. We don't have to do anything to this fraction.** ***OPTION 2:*** **This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.** **What do we need to do?**
Students	*OPTION 1:* We don't have to change the denominator. *OPTION 2:* We need to convert the fraction to a denominator of __.

Teacher *OPTION 2:* **To convert the fraction to a denominator of __, I determine how many groups of __** (original denominator) **I need to make __** (common denominator)**. I see I need to make __, __, __ groups of __** (original denominator)**. How many groups?**

Students __.

Teacher **So, I make __ groups of __ with the two-color counters. That means I iterate or copy the original fraction __ times. What does it mean to iterate?**

Students To copy.

Teacher **Our new fraction is __. Is __** (original fraction) **equivalent to __** (fraction with common denominator)**?**

Students Yes.

Teacher **How do you know the fractions are equivalent?**

Students The fractions have the same value. They are equivalent.

Teacher **So, we converted the minuend to a common denominator. Let's do the same with the subtrahend. What's the subtrahend?**

__.

Teacher **The subtrahend has a denominator of __.**

OPTION 1: **This is the original denominator. We don't have to do anything to this fraction.**

OPTION 2: **This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.**

What do we need to do?

Students *OPTION 1:* We don't have to change the denominator.

OPTION 2: We need to convert the fraction to a denominator of __.

Teacher *OPTION 2:* **To convert the fraction to a denominator of __, I determine how many groups of __** (original denominator) **I need to make __** (common denominator)**. I see I need to make __, __, __ groups of __** (original denominator)**. How many groups?**

Students __.

Teacher **We make __ groups of __ with the two-color counters. That means I iterate or copy the original fraction __ times. How many times?**

Students __.

Teacher **Let's check our work. Is __** (original fraction) **equivalent to __** (fraction with common denominator)**?**

Students Yes.

Teacher **How do you know the fractions are equivalent?**

Students The fractions have the same value. They are equivalent.

Teacher **Now that we have common denominators, we want to subtract the parts or numerators of subtrahend from the minuend. That means we have to subtract __ one-__ parts from __ one-__ parts. What do we subtract?**

Students We subtract the parts or numerators of the fractions.

Teacher	**Let's subtract the numerators. I like to keep my subtrahend set where it is and only subtract from the minuend set. We need to subtract the red one-__ parts. How many parts do we have to subtract?**
Students	__.
Teacher	**We subtract __ one-__ parts. I subtract by turning over (to yellow) __ one-__ parts. How many parts?**
Students	__.
Teacher	**When you have __ minus __, the difference is __. What's the difference?**
Students	__.
Teacher	**__ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**So, if you have a set of __ and subtract a set of __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We showed the minuend and the subtrahend. Then, we determined the common denominator using the LCM. After converting the fractions to common denominators, we subtracted the subtrahend parts from the minuend parts to learn of the difference.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point)**. The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Our minuend is __. What's our minuend?**
Students	__.
Teacher	**Our subtrahend is __. What's our subtrahend?**
Students	__.

Teacher	**So, we have __ minus __. Let's subtract by separating. What does separating mean?**
Students	To take away.
Teacher	**Yes. Let's separate, or take away, the subtrahend from the minuend. Remember, the parts of fractions represent the numerator. What do you subtract?**
Students	The parts or numerators of the fractions.
Teacher	**When subtracting fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?**
Students	No.
Teacher	**The denominators are not the same. To subtract, we need to subtract the parts or numerators with the same value. When the denominators are unlike, the parts or numerators do not represent the same value. So, we will work to make the fractions have like denominators. Why do we want to subtract fractions with like denominators?**
Students	So we can subtract the parts or numerators of the fractions.
Teacher	**To do this, let's write the first five multiples of each denominator. The minuend has a denominator of __, so let's write the first five multiples of __.** (Write multiples as __, __, __, __, __.)
Teacher	**What are the multiples of __? Say them with me.**
Students	__, __, __, __, __.
Teacher	**The subtrahend has a denominator of __, so let's write the first five multiples of __.** (Write multiples as __, __, __, __, __.)
Teacher	**What are the multiples of __? Say them with me.**
Students	__, __, __, __, __.
Teacher	**Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?**
Students	__.
Teacher	**So, __ is the least common multiple. Say that with me.**
Students	Least common multiple.
Teacher	**Sometimes we call the least common multiple the LCM. What do we call the least common multiple?**
Students	LCM.
Teacher	**The least common multiple, or LCM, helps us determine the common denominator for the two fractions. What does the LCM help with?**
Students	Finding a common denominator for the two fractions.
Teacher	**The minuend has a denominator of __.** ***OPTION 1:*** **This is the original denominator. We don't have to do anything to this fraction.** ***OPTION 2:*** **This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.** **What do we need to do?**

Students	*OPTION 1:*	We don't have to change the denominator.
	OPTION 2:	We need to convert the fraction to a denominator of __.
Teacher	***OPTION 2:***	**To convert the fraction to a denominator of __, I determine how many groups of __** (original denominator) **I need to make __** (common denominator)**. I see I need to make __, __, __ groups of __** (original denominator)**. How many groups?**
Students		__.
Teacher		**So, I multiply the denominator times __ and the numerator times __. Let's multiply the denominator first. __** (original denominator) **times __ is what?**
Students		__.
Teacher		**That's right. __ times __ equals __. Our new denominator is __. What's our new denominator?**
Students		__.
Teacher		**Now, let's multiply the numerator times __. __** (original numerator) **times __ is what?**
Students		__.
Teacher		**Yes. __ times __ equals __. Our new numerator is __. What's the new numerator?**
Students		__.
Teacher		**Let's check our work. Is __** (original fraction) **equivalent to __** (fraction with common denominator)**? How do you know the fractions are equivalent?**
Students		The fractions have the same value. They are equivalent.
Teacher	**So, we converted the minuend to a common denominator. Let's do the same with the subtrahend. What's the subtrahend?**	
	__.	
Teacher	**The subtrahend has a denominator of __.**	
	OPTION 1:	**This is the original denominator. We don't have to do anything to this fraction.**
	OPTION 2:	**This is not the original denominator. We need to convert the fraction from a denominator of __ to a denominator of __.**
	What do we need to do?	
Students	*OPTION 1:*	We don't have to change the denominator.
	OPTION 2:	We need to convert the fraction to a denominator of __.
Teacher	***OPTION 2:***	**To convert the fraction to a denominator of __, I determine how many groups of __** (original denominator) **I need to make __** (common denominator)**. I see I need to make __, __, __ groups of __** (original denominator)**. How many groups?**
Students		__.
Teacher		**So, I multiply the denominator times __ and the numerator times __. Let's multiply the denominator first. __** (original denominator) **times __ is what?**
Students		__.

Teacher	**That's right. __ times __ equals __. Our new denominator is __. What's our new denominator?**
Students	__.
Teacher	**Now, let's multiply the numerator times __. __** (original numerator) **times __ is what?**
Students	__.
Teacher	**Yes. __ times __ equals __. Our new numerator is __. What's the new numerator?**
Students	__.
Teacher	**Let's check our work. Is __** (original fraction) **equivalent to __** (fraction with common denominator)**? How do you know the fractions are equivalent?**
Students	Yes.
Teacher	**How do you know the fractions are equivalent?**
Students	The fractions have the same value. They are equivalent.
Teacher	**Now that we have common denominators, we want to subtract the parts or numerator of the subtrahend from the minuend. That means we have to subtract __ one-__ parts from __ one-__ parts. What do we subtract?**
Students	We subtract the parts of the fractions.
Teacher	**Let's subtract the parts or numerators.** (Subtract parts, compare to whole.)
Teacher	**So, we now have __, __, __, … one-__ parts. How many parts?**
Students	__.
Teacher	**When you have __ minus __, the difference is __. What's the difference?**
Students	__.
Teacher	**__ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**So, if you have a set of __ and subtract a set of __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	After determining a common denominator, we subtracted the subtrahend from the minuend to learn the difference.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

Example

$$\frac{7}{8} - \frac{1}{4} = \frac{5}{8}$$

EXAMPLE WITH MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point)**. The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with two-color counters.** (Move two-color counters to workspace.)
Teacher	**Our minuend is $\frac{7}{8}$. What's our minuend?**
Students	$\frac{7}{8}$.
Teacher	**Let's show this minuend by showing the fraction. First, we have a denominator of 8, so let's show 8 yellow counters. How many?**
Students	8.
Teacher	**Then, we need to show 7 of the 8 parts as red to show $\frac{7}{8}$. How many should we make red?**
Students	7.
Teacher	**What fraction?**
Students	$\frac{7}{8}$.
Teacher	**Our subtrahend is $\frac{1}{4}$. What's our subtrahend?**
Students	$\frac{1}{4}$.
Teacher	**Let's show the subtrahend by showing the fraction. First, we have a denominator of 4, so let's show 4 yellow counters. How many?**
Students	4.
Teacher	**Then, we need to show 1 of the 4 parts as red to show $\frac{1}{4}$. How many should we make red?**
Students	1.
Teacher	**What fraction?**
Students	$\frac{1}{4}$.
Teacher	**So, we have $\frac{7}{8}$ minus $\frac{1}{4}$. Let's subtract by separating. What does separating mean?**
Students	To take away.

Teacher **Yes. Let's separate, or take away, the parts of the fractions. When subtracting fractions, first we want to determine whether the denominators are like or unlike. You might also say common or uncommon denominators. Are the denominators the same or alike?**

Students No.

Teacher **How do you know the denominators are not alike?**

Students We have a denominator of 8 and a denominator of 4. Those are not the same.

Teacher **The denominators are not the same. To subtract, we should subtract the parts of the subtrahend from the parts of the minuend. When the denominators are unlike, the parts or numerators do not represent the same value. So, we will work to make the fractions have like denominators. Why do we want to subtract fractions with like denominators?**

Students So we can subtract the parts or numerators of the fractions.

Teacher **To do this, let's write the first five multiples of each denominator. The minuend has a denominator of 8, so let's write the first five multiples of 8.** (Write multiples as 8, 16, 24, 32, 40.)

Teacher **What are the multiples of 8? Say them with me.**

Students 8, 16, 24, 32, 40.

Teacher **The subtrahend has a denominator of 4, so let's write the first five multiples of 4.** (Write multiples as 4, 8, 12, 16, 20.)

Teacher **What are the multiples of 4? Say them with me.**

Students 4, 8, 12, 16, 20.

Teacher **Great. Let's determine the least common multiple of the two fractions. What is the multiple with the least value that you see on both lists of multiples?**

Students 8.

Teacher **So, 8 is the least common multiple. What is 8?**

Students The least common multiple.

Teacher **Sometimes we call the least common multiple the LCM. What do we call the least common multiple?**

Students LCM.

Teacher **The least common multiple, or LCM, helps us determine the common denominator for the two fractions. What does the LCM help with?**

Students Finding a common denominator for the two fractions.

Teacher **The minuend has a denominator of 8, which is the original denominator. We don't need to convert this fraction. What do we need to do?**

Students Nothing.

Teacher **What's the subtrahend?**

Students $\frac{1}{4}$.

Teacher **The subtrahend has a denominator of 4, which is not the original denominator. We need to convert the fraction from a denominator of 4 to a denominator of 8. What do we need to do?**

Students Convert the fraction from a denominator of 4 to a denominator of 8.

Teacher	**To convert the fraction to a denominator of 8, I determine how many groups of 4 I need to make 8. I see I need to make 1, 2 groups of 4.** (Point to the multiples of 4 and 8.) **How many groups?**
Students	2.
Teacher	**Let's make 2 groups of the fraction $\frac{1}{4}$ with the two-color counters. We already have one group of $\frac{1}{4}$. Let's make a second group** (show 1 red counter and 3 yellow counters). **Our new fraction is $\frac{2}{8}$. Is $\frac{2}{8}$ equivalent to $\frac{1}{4}$?**
Students	Yes. The fractions are equivalent.
Teacher	**Now that we have common denominators, we want to subtract the subtrahend parts or numerator from the minuend parts or numerator. That means we need to subtract 2 one-eighth parts from 7 one-eighth parts. What do we subtract?**
Students	We subtract the parts or numerators of the fractions.
Teacher	**Let's subtract the parts or numerators. With the two-color counters, we leave the subtrahend set alone. We subtract the 2 one-eighth parts by turning over the parts or numerators of the minuend. How many parts do we subtract?**
Students	2.
Teacher	**We subtract the 2 one-eighth parts. We now have 1, 2, 3, 4, 5 one-eighth parts. How many parts?**
Students	5.
Teacher	**When you have $\frac{7}{8}$ minus $\frac{2}{8}$, the difference is $\frac{5}{8}$. What's the difference?**
Students	$\frac{5}{8}$.
Teacher	**$\frac{7}{8}$ minus $\frac{2}{8}$ equals $\frac{5}{8}$. Let's say that together.**
Students	$\frac{7}{8}$ minus $\frac{2}{8}$ equals $\frac{5}{8}$.
Teacher	**If you have a set of $\frac{7}{8}$ and subtract a set of $\frac{1}{4}$, the difference is $\frac{5}{8}$. $\frac{7}{8}$ minus $\frac{2}{8}$ equals $\frac{5}{8}$. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The amount between the minuend and subtrahend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We showed the minuend and subtrahend. We used the LCM to determine the common denominator. Then, we subtracted the parts of the subtrahend from the parts of the minuend to learn the difference.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

(3) Subtraction of Decimals with Traditional Algorithm

Routine

Materials:

- Module 9 Problem Sets
- Module 9 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or money
  - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with our number line.** (Show number line.)
Teacher	**When we use the Base-10 blocks with decimals, we can shift the meaning of each type of block. Today, let's use the flats to represent ones. What do the flats represent?**
Students	Ones.
Teacher	**We'll use the rods to represent tenths. What do the rods represent?**
Students	Tenths.
Teacher	**How can we use the rods to represent tenths?**
Students	1 rod equals 1 tenth.
Teacher	**What do you notice about the relationship between the rods and the flat?**
Students	There are 10 tenths in 1 in the same way there are 10 rods in 1 flat.
Teacher	**With our Base-10 blocks, the units represent hundredths. What do the units represent?**
Students	Hundredths.
Teacher	**What do you notice about the relationship between the units and the rods?**
Students	There are 10 hundredths in 1 tenth in the same way there are 10 units in 1 rod.
Teacher	**Our minuend is __. What's our minuend?**
Students	__.
Teacher	**Let's show the minuend by showing __ ones, __ tenths, and __ hundredths.** (Show with Base-10 blocks.)
Teacher	**How many?**
Students	__.
Teacher	**Our subtrahend is __. What's our subtrahend?**

Students ___.

Teacher **Let's show the subtrahend by showing __ ones, __ tenths, and __ hundredths.**
(Show with Base-10 blocks.)

Teacher **How many?**

Students ___.

Teacher **So, we have __ minus __. Let's subtract by separating. What does separating mean?**

Students To take away.

Teacher **Yes. Let's separate or take away. First, let's subtract the least place value. That means the place value with the least or smallest value. What's the least place value in this problem?**

Students Hundredths.

Teacher **Let's subtract the hundredths.**
(Subtract the subtrahend hundredths from the minuend hundredths.)

Teacher **Let's separate __ hundredths from __ hundredths. Do we have enough minuend hundredths to separate the __ subtrahend hundredths?**

Students Yes.

Teacher **If we don't have enough hundredths, we have to regroup. Do we have to regroup?**

Students No.

Teacher **So, let's separate the subtrahend hundredths from the minuend hundredths.**
(Remove hundredths.)

Teacher **How many hundredths are remaining?**

Students ___.

Teacher **Yes! There are __ hundredths remaining. We leave the remaining hundredths here. Now, let's subtract the tenths. What should we subtract?**

Students The tenths.

Teacher **Let's separate __ tenths from __ tenths. Do we have enough minuend tenths to separate the __ subtrahend tenths?**

Students No.

Teacher **That means we have to regroup. To regroup, we exchange 1 one for 10 tenths. How do we regroup?**

Students We exchange 1 one for 10 tenths.
(Show regrouping.)

Teacher **Now, we have __ tenths and can subtract __ tenths. Let's separate the subtrahend tenths from the minuend tenths.**
(Remove tenths.)

Teacher **How many tenths are remaining?**

Students ___.

Teacher **There are __ tenths remaining. We leave the remaining tenths here. Now, let's subtract the ones. What should we subtract?**

Students The ones.

Teacher **Let's separate __ ones from __ ones. Do we have enough minuend ones to separate the subtrahend ones?**

Students	Yes.
Teacher	**We don't have to regroup. Let's subtract the ones.** (Remove ones.)
Teacher	**How many ones are remaining?**
Students	__.
Teacher	**So, let's count the ones, tenths, and hundredths to learn the difference. Ready?** (Count the ones, then tenths, then hundredths.)
Teacher	**That means __ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**Let's say it together again.**
Students	__ minus __ equals __.
Teacher	**So, if you have a set of __ and subtract a set of __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The amount between the minuend and subtrahend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We subtracted the hundredths, then the tenths, then the ones to learn the difference.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point)**. The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the different place value columns. Let's draw a vertical line between the ones column and the tenths column and another vertical line between the tenths column and the hundredths column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by subtracting the hundredths. What should we subtract first?**

Students The hundredths.

Teacher **Which hundredths do we subtract?**

Students __ minus __.

Teacher **Do you have enough minuend hundredths to subtract?**

Students No.

Teacher **So, we have to regroup. To regroup, we regroup/trade/exchange 1 tenth for 10 hundredths. I subtract 1 tenth from the tenths column. __ minus 1 equals __. I like to cross out the __ and write a __ in the tenths column.**
(Show subtraction of 1 tenth.)

Teacher **Now, I imagine regrouping this 1 tenth into 10 hundredths. If I have 10 hundredths and add these hundredths to the __ hundredths, how many hundredths do I have now?**

Students __.

Teacher **I like to show the __ hundredths by crossing out the __ and writing __ in the hundredths column.**
(Show addition of 10 hundredths.)

Teacher **Now, let's subtract the hundredths. What's __ minus __?**
(If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)

Students __.

Teacher **Great. There are __ hundredths. Let's write __ below the equal line.**
(Write hundredths.)

Teacher **Now, let's subtract the tenths. Which tenths do we subtract?**

Students __ minus __.

Teacher **Do you have enough tenths to subtract __ tenths?**

Students Yes.

Teacher **You have enough tenths to subtract or take away __ tenths. We don't have to regroup. What's __ minus __?**

Students __.

Teacher **There are __ tenths. Let's write __ below the equal line.**
(Write tenths.)

Teacher **Now, let's subtract the ones. Which ones do we subtract?**

Students __ minus __.

Teacher **Do you have enough ones to subtract __ ones?**

Students Yes.

Teacher **You have enough ones to subtract. You don't have to regroup. What's __ minus __?**

Students __.

Teacher **Let's write __ below the equal line.**

Students __.

Teacher **So, let's look at the problem. What's __ minus __?**

Students __.

Teacher	**That's right. __ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**So, if you have a set of __ and subtract a set of __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We subtracted the hundredths but we didn't have enough hundredths so we regrouped 1 tenth for 10 hundredths. Then, we subtracted the tenths. Then, we subtracted the ones. We figured out the difference between __ and __.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

Example

$$\begin{array}{r} 3.25 \\ -\ \underline{2.89} \\ 0.36 \end{array}$$

EXAMPLE WITH MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**When we use the Base-10 blocks with decimals, we can shift the meaning of each type of block. Today, let's use the flats to represent ones. What do the flats represent?**
Students	Ones.
Teacher	**We'll use the rods to represent tenths. What do the rods represent?**
Students	Tenths.
Teacher	**How can we use the rods to represent tenths? What do you notice about the relationship between the rods and the flat?**

Students	There are 10 tenths in 1 in the same way there are 10 rods in 1 flat.
Teacher	**With our Base-10 blocks, the units represent hundredths. What do the units represent?**
Students	Hundredths.
Teacher	**What do you notice about the relationship between the units and the rods?**
Students	There are 10 hundredths in 1 tenth in the same way there are 10 units in 1 rod.
Teacher	**Our minuend is 3 and 25 hundredths. What's our minuend?**
Students	3 and 25 hundredths.
Teacher	**Let's show the minuend by showing 3 ones, 2 tenths, and 5 hundredths.** (Show with Base-10 blocks.)
Teacher	**How many?**
Students	3 and 25 hundredths.
Teacher	**Our subtrahend is 2 and 89 hundredths. What's our subtrahend?**
Students	2 and 89 hundredths.
Teacher	**Instead of showing the subtrahend, let's subtract the subtrahend from the minuend. What should we do?**
Students	Subtract the subtrahend from the minuend.
Teacher	**Let's start by subtracting the least place value. What's the least place value in this problem?**
Students	Hundredths.
Teacher	**How many hundredths do we subtract?**
Students	9.
Teacher	**We need to subtract 9 hundredths. How many hundredths are in the minuend?**
Students	5.
Teacher	**Do you have enough hundredths to subtract 9 hundredths?**
Students	No.
Teacher	**So, let's regroup. Let's regroup/trade/exchange 1 tenths for 10 hundredths.** (Show 1 tenth as equivalent to 10 hundredths.)
Teacher	**I place the 10 hundredths in the hundredths column.** (Place 10 hundredths in hundredths column.)
Teacher	**Now we have 15 hundredths. How many hundredths?**
Students	15.
Teacher	**Let's subtract the hundredths. That means we subtract 9 hundredths from 15 hundredths.** (Move hundredths.)
Teacher	**How many hundredths remaining?**
Students	6 hundredths.
Teacher	**Let's subtract the tenths. We have 1 tenth in the minuend and we need to subtract 8 tenths of the subtrahend. Do we have enough tenths to subtract 8 tenths?**
Students	No.
Teacher	**What do we have to do?**
Students	Regroup.

Teacher	**We need to regroup 1 one for 10 tenths. Let's regroup/trade/exchange 1 one for 10 tenths.** (Show 1 one as equivalent to 10 tenths.)
Teacher	**I place the 10 tenths in the tenths column.** (Place 10 tenths in the tenths column.)
Teacher	**Now we have 11 tenths. How many tenths?**
Students	11.
Teacher	**Let's subtract the tenths. We subtract 8 tenths from 11 tenths.** (Move tenths.)
Teacher	**How many tenths remaining?**
Students	3 tenths.
Teacher	**Now, let's subtract the ones. We have 2 ones in the minuend and 2 ones in the subtrahend. Do we have enough ones to subtract?**
Students	Yes.
Teacher	**What's 2 minus 2?**
Students	0.
Teacher	**So, let's count the ones, tenths, and hundredths to learn the difference. Ready?** (Count the ones, then tenths, then hundredths.)
Teacher	**That means 3 and 25 hundredths minus 2 and 89 hundredths equals 36 hundredths. Let's say that together.**
Students	3 and 25 hundredths minus 2 and 89 hundredths equals 36 hundredths.
Teacher	**Let's say it together again.**
Students	3 and 25 hundredths minus 2 and 89 hundredths equals 36 hundredths.
Teacher	**Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The amount between the minuend and subtrahend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We subtracted the hundredths but first we had to regroup. Then, we subtracted the tenths but we also had to regroup. Then, we subtracted the ones. The difference between 3 and 25 hundredths and 2 and 89 hundredths is 36 hundredths.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

(4) Subtraction of Decimals with Adding Up Algorithm

Routine

Materials:

- Module 9 Problem Sets
- Module 9 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or money
    - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Today, let's think about subtraction as the difference between two numbers. How can we interpret subtraction?**
Students	The difference between two numbers.
Teacher	**So, in this problem, subtraction is the difference between what two numbers?**
Students	__ and __.
Teacher	**Let's figure out the difference between __ and __. Let's do this with our Base-10 blocks.** (Show Base-10 blocks.)
Teacher	**When we think about subtraction as the difference between two numbers, let's start with our subtrahend. What's the subtrahend in this problem?**
Students	__.
Teacher	**Let's show the subtrahend with our Base-10 blocks. How many ones?**
Students	__.
Teacher	**How many tenths?**
Students	__.
Teacher	**How many hundredths?**
Students	__. (Show subtrahend with Base-10 blocks.)
Teacher	**Now, let's think about what we could add to the subtrahend to reach the minuend, __. I see that I could add __ hundredths to get to the nearest tenth. I'll add the hundredths over here so I don't confuse these hundredths with the subtrahend hundredths.** (Add hundredths in separate pile.)

Teacher	**Now, what else could we add to reach the minuend, __? I see that I could add __ tenths to get very close to the minuend of __. I'll add the tenths over here so I don't confuse these tenths with the subtrahend tenths.** (Add tenths.)
Teacher	**Have we reached the minuend yet?**
Students	No.
Teacher	**What could we add to reach the minuend?**
Students	__.
Teacher	**I could add __ ones to reach the minuend. Let's add the ones over here so I don't confuse these ones with the subtrahend ones.** (Add ones.)
Teacher	**So, the difference between __ and __ is: __, __, __, … What's the difference?**
Students	__.
Teacher	**That means __ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**Let's say it together again.**
Students	__ minus __ equals __.
Teacher	**With this strategy, called adding up, you figure out the difference between __ and __ by adding up. You add up to find the difference between __ and __. How do you find the difference?**
Students	Adding up from __ to __.
Teacher	**Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**How could you explain adding up to a friend?**
Students	You start with the subtrahend. You keep adding until you reach the minuend. You add up to find the difference between the minuend and subtrahend.

Example

5.17	2.99	
– 2.99	3.00	+.01
	5.00	+2.00
	5.17	+.17
		2.18

EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Today, let's think about subtraction as the difference between two numbers. How can we interpret subtraction?**
Students	The difference between two numbers.
Teacher	**So, in this problem, subtraction is the difference between what two numbers?**
Students	5.17 and 2.99.
Teacher	**Let's figure out the difference between 5.17 and 2.99.**
Teacher	**When we think about subtraction as the difference between two numbers, let's start with our subtrahend. What's the subtrahend in this problem?**
Students	2.99.
Teacher	**Let's write the subtrahend next to the problem. What should we write?**
Students	2.99.
Teacher	**Now, let's think about what we could add to 2.99 to reach the minuend, 5.17. I see that I could add 1 hundredth to get to the nearest tenth. I'll write +.01 over here to show I wanted to add 1 hundredth.** (Write +.01.)
Teacher	**If I added .01 to 2.99, what's the sum?**
Students	3.00.
Teacher	**Let's write 3.00 below 2.99 to remember we're now at 3.00.** (Write 3.00 below 2.99.)
Teacher	**Let's figure out what we could add to 3.00 to reach the minuend, 5.17. Could we add 2 more to get to 5?**
Students	Yes.
Teacher	**Let's write +2.00 to show we wanted to add 2 ones.** (Write +2.00 below +.01.)
Teacher	**If we added 2 to 3, what's the sum?**
Students	5.
Teacher	**Let's write 5.00 below 3.00 to remember we're now at 5.00.** (Write 5.00 below 3.00.)
Teacher	**Let's keep going. What could we add to 5.00 to reach the minuend?**
Students	.17.
Teacher	**Great idea. Let's write +.17 to show we wanted to add .17.** (Write +.17.)
Teacher	**If I added .17 to 5.00, what's the sum?**
Students	5.17.
Teacher	**Let's write 5.17 below 5.00 to remember we're now at 5.17.**

	(Write 5.17 below 5.00.)
Teacher	**Did we reach the minuend?**
Students	Yes!
Teacher	**Now, we add +.01 and +2.00 and +.17 to determine the difference. How could we add these numbers?**
Students	2.00 + .17 + .01 (or other responses).
Teacher	**So, the difference is 2.18. What's the difference?**
Students	2.18.
Teacher	**That means 5.17 minus 2.99 equals 2.18. Let's say that together.**
Students	5.17 minus 2.99 equals 2.18.
Teacher	**Let's say it together again.**
Students	5.17 minus 2.99 equals 2.18.
Teacher	**With this strategy, called adding up, you figure out the difference between 5.17 and 2.99 by adding up. How do you find the difference?**
Students	Adding up from 2.99 to 5.17.
Teacher	**Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**How could you explain adding up to a friend?**
Students	You start with the subtrahend. You keep adding until you reach the minuend. You add up to find the difference between the minuend and subtrahend.

D. Problems for Use During Instruction

See Module 9 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 9 Vocabulary Cards.

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Module 9:
Subtraction of Rational Numbers

Problem Sets

A. Proper fractions with like denominators and sums <1 (20)
B. Improper fractions with like denominators and sums >1 (10)
C. Mixed numbers with like denominators and sums >1 (10)
D. Proper fractions with unlike denominator and sums <1 (20)
E. Improper fractions with unlike denominator and sums >1 (10)
F. Mixed numbers with unlike denominator and sums >1 (10)

G. Decimals with tenths; no regrouping (20)
H. Decimals with tenths; regrouping (20)
I. Decimals with hundredths; no regrouping (20)
J. Decimals with hundredths; regrouping (20)
K. Decimals with tenths and hundredths; mix of regrouping (20)

A.

$$\frac{4}{5} - \frac{1}{5} =$$

A.

$$\frac{6}{10} - \frac{3}{10} =$$

A.

$$\frac{3}{6} - \frac{1}{6} =$$

A.

$$\frac{2}{4} - \frac{1}{4} =$$

A.

$$\frac{2}{3} - \frac{1}{3} =$$

A.

$$\frac{4}{6} - \frac{2}{6} =$$

A.

$$\frac{6}{8} - \frac{1}{8} =$$

A. $\frac{4}{10} - \frac{1}{10} =$

A.

$$\frac{6}{12} - \frac{4}{12} =$$

A.

$$\frac{4}{10} - \frac{1}{10} =$$

A.

$$\frac{9}{12} - \frac{4}{12} =$$

A.

$$\frac{3}{5} - \frac{1}{5} =$$

A.

$$\frac{5}{6} - \frac{1}{6} =$$

A.

$$\frac{4}{7} - \frac{1}{7} =$$

A.

$$\frac{6}{9} - \frac{2}{9} =$$

A.

$$\frac{6}{10} - \frac{1}{10} =$$

A.

$$\frac{3}{4} - \frac{1}{4} =$$

A.

$$\frac{5}{6} - \frac{3}{6} =$$

A.

$$\frac{6}{7} - \frac{3}{7} =$$

A.

$$\frac{6}{8} - \frac{2}{8} =$$

B.

$$\frac{10}{5} - \frac{4}{5} =$$

B. $$\frac{16}{12} - \frac{3}{12} =$$

B. $\frac{11}{6} - \frac{3}{6} =$

B. $$\frac{8}{4} - \frac{1}{4} =$$

B.

$$\frac{8}{3} - \frac{2}{3} =$$

B.

$$\frac{9}{6} - \frac{1}{6} =$$

B. $\frac{14}{8} - \frac{5}{8} =$

B.

$$\frac{13}{10} - \frac{1}{10} =$$

B. $\frac{17}{12} - \frac{4}{12} =$

B.

$$\frac{18}{10} - \frac{4}{10} =$$

c.

$$7\frac{7}{8} - 4\frac{3}{8} =$$

c.

$$2\frac{3}{5} - 1\frac{3}{5} =$$

c.

$$2\frac{5}{6} - 1\frac{2}{6} =$$

c.

$$2\frac{6}{10} - 1\frac{1}{10} =$$

c.

$$2\frac{6}{9} - 1\frac{4}{9} =$$

c.

$$7\frac{3}{5} - 1\frac{4}{5} =$$

c.

$$7\frac{3}{4} - 4\frac{1}{4} =$$

c.

$$7\frac{4}{6} - 2\frac{4}{6} =$$

c.

$$2\frac{5}{8} - 1\frac{4}{8} =$$

c.

$$2\frac{6}{8} - 1\frac{5}{8} =$$

D.

$$\frac{3}{5} - \frac{1}{3} =$$

D.

$$\frac{5}{6} - \frac{1}{2} =$$

D.

$$\frac{2}{4} - \frac{1}{12} =$$

D.

$$\frac{3}{5} - \frac{1}{10} =$$

D.

$$\frac{1}{3} - \frac{1}{6} =$$

D.

$$\frac{2}{5} - \frac{1}{10} =$$

D.

$$\frac{4}{8} - \frac{2}{16} =$$

D.

$$\frac{3}{6} - \frac{1}{3} =$$

D.

$$\frac{2}{3} - \frac{1}{2} =$$

D.

$$\frac{6}{8} - \frac{2}{4} =$$

D.

$$\frac{9}{10} - \frac{2}{5} =$$

D. $$\frac{12}{5} - \frac{3}{10} =$$

D.

$$\frac{2}{3} - \frac{2}{5} =$$

D.

$$\frac{1}{3} - \frac{1}{4} =$$

D.

$$\frac{2}{4} - \frac{2}{5} =$$

D.

$$\frac{1}{2} - \frac{1}{5} =$$

D.

$$\frac{2}{4} - \frac{2}{12} =$$

D. $$\frac{7}{9} - \frac{1}{3} =$$

D.

$$\frac{3}{4} - \frac{5}{8} =$$

D.

$$\frac{2}{3} - \frac{2}{12} =$$

E. $\frac{15}{3} - \frac{4}{2} =$

E.

$$\frac{5}{4} - \frac{1}{5} =$$

E.

$$\frac{17}{4} - \frac{5}{2} =$$

E. $\frac{16}{5} - \frac{5}{3} =$

E. $\frac{11}{4} - \frac{10}{8} =$

E.

$$\frac{19}{2} - \frac{8}{6} =$$

E.

$$\frac{7}{4} - \frac{5}{8} =$$

E. $$\frac{30}{3} - \frac{1}{3} =$$

E. $$\frac{13}{5} - \frac{2}{10} =$$

E. $$\frac{19}{15} - \frac{4}{5} =$$

F.

$$2\frac{1}{3} - 1\frac{2}{9} =$$

F.

$$7\frac{1}{10} - 1\frac{7}{8} =$$

F.

$$7\frac{1}{2} - 3\frac{1}{5} =$$

F.

$$2\frac{9}{12} - 1\frac{2}{4} =$$

F.

$$7\frac{1}{3} - 2\frac{7}{12} =$$

F.

$$2\frac{4}{10} - 1\frac{2}{5} =$$

F.

$$7\frac{3}{6} - 2\frac{5}{12} =$$

F.

$$2\frac{2}{3} - 1\frac{1}{9} =$$

F.

$$7\frac{1}{2} - 5\frac{5}{6} =$$

F.

$$2\frac{5}{6} - 1\frac{5}{12} =$$

G.

$$
\begin{array}{r}
9.9 \\
-\ 2.1 \\
\hline
\end{array}
$$

G.

$$
\begin{array}{r}
8.8 \\
-\ 3.2 \\
\hline
\end{array}
$$

G.

$$\begin{array}{r} 6.7 \\ -\ 5.5 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 5.3 \\ -\ 4.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 9.8 \\ -\ 5.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 8.6 \\ -\ 2.3 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 0.3 \\ -\ 0.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 2.2 \\ -\ 1.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 3.3 \\ -\ 0.3 \\ \hline \end{array}$$

G.

$$
\begin{array}{r}
4.5 \\
-\ 2.2 \\
\hline
\end{array}
$$

G.

$$
\begin{array}{r}
4.1 \\
-\ 0.1 \\
\hline
\end{array}
$$

G.

$$\begin{array}{r} 9.6 \\ -\ 3.3 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 2.8 \\ -\ 0.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 4.6 \\ -\ 1.5 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 6.3 \\ -\ 2.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 3.8 \\ -\ 1.5 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 5.7 \\ -\ 4.1 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 9.4 \\ -\ 0.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 6.7 \\ -\ 1.2 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 0.7 \\ -\ 0.2 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 8.4 \\ -\ 2.9 \\ \hline \end{array}$$

H.

$$
\begin{array}{r}
5.3 \\
-\ 3.7 \\
\hline
\end{array}
$$

H.

$$\begin{array}{r} 8.5 \\ -\ 4.8 \\ \hline \end{array}$$

H.

$$
\begin{array}{r} 8.6 \\ -\ 5.9 \\ \hline \end{array}
$$

H.

$$\begin{array}{r} 6.5 \\ -\ 4.6 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 7.4 \\ -\ 5.7 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 8.3 \\ -\quad 5.6 \\ \hline \end{array}$$

H.

$$
\begin{array}{r} 6.4 \\ -\ 1.9 \\ \hline \end{array}
$$

H.

$$
\begin{array}{r}
9.5 \\
-\ \ 4.6 \\
\hline
\end{array}
$$

H.

$$\begin{array}{r} 7.2 \\ -\ 0.7 \\ \hline \end{array}$$

H.

$$
\begin{array}{r} 5.5 \\ -\ 3.6 \\ \hline \end{array}
$$

H.

$$\begin{array}{r} 4.1 \\ -\ 2.9 \\ \hline \end{array}$$

H.

$$
\begin{array}{r} 8.6 \\ -\ 4.7 \\ \hline \end{array}
$$

H.

$$\begin{array}{r} 6.1 \\ -\ 3.8 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 8.6 \\ -\ 1.8 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 7.2 \\ -\ 2.8 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 6.5 \\ -\ 0.6 \\ \hline \end{array}$$

H.

$$
\begin{array}{r} 7.2 \\ -\quad 4.8 \\ \hline \end{array}
$$

H.

$$
\begin{array}{r} 8.2 \\ -\ 2.9 \\ \hline \end{array}
$$

H.

$$\begin{array}{r} 4.2 \\ -\ 0.8 \\ \hline \end{array}$$

1.

$$
\begin{array}{r} 0.73 \\ -\quad 0.21 \\ \hline \end{array}
$$

1.

$$
\begin{array}{r}
3.46 \\
-\quad 1.32 \\
\hline
\end{array}
$$

1.

$$
\begin{array}{r} 6.58 \\ -\quad 2.11 \\ \hline \end{array}
$$

1.

$$
\begin{array}{r}
9.82 \\
-\quad 0.01 \\
\hline
\end{array}
$$

1.

$$\begin{array}{r} 8.34 \\ -\ 0.22 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 2.59 \\ -\quad 1.46 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 2.61 \\ -\quad 1.30 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 7.47 \\ -\quad 2.31 \\ \hline \end{array}$$

1.

$$
\begin{array}{r} 9.63 \\ -\quad 0.60 \\ \hline \end{array}
$$

1.

$$\begin{array}{r} 12.46 \\ -\quad 1.10 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 26.24 \\ -\quad 3.03 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 4.71 \\ -\quad 1.10 \\ \hline \end{array}$$

1.

$$
\begin{array}{r} 4.35 \\ -\quad 2.22 \\ \hline \end{array}
$$

1.

$$
\begin{array}{r}
1.88 \\
-\quad 0.01 \\
\hline
\end{array}
$$

1.

$$\begin{array}{r} 3.63 \\ -\ 1.21 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 10.26 \\ -\ 10.13 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 9.44 \\ -\quad 2.34 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 5.62 \\ -\quad 1.20 \\ \hline \end{array}$$

1.

$$\begin{array}{r} 6.48 \\ -\quad 4.01 \\ \hline \end{array}$$

1.

$$
\begin{array}{r} 10.55 \\ -\quad 0.33 \\ \hline \end{array}
$$

J.

$$\begin{array}{r} 3.56 \\ -\quad 2.47 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 5.24 \\ -\quad 1.37 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 5.45 \\ -\quad 3.78 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 6.67 \\ -\quad 2.29 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 2.14 \\ -\quad 1.47 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 4.23 \\ -\quad 2.25 \\ \hline \end{array}$$

J.

$$
\begin{array}{r} 4.71 \\ -\quad 3.89 \\ \hline \end{array}
$$

J.

$$
\begin{array}{r} 3.52 \\ -\quad 1.77 \\ \hline \end{array}
$$

J.

$$\begin{array}{r} 6.84 \\ -\quad 2.16 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 14.80 \\ -\ 6.96 \\ \hline \end{array}$$

J.

$$
\begin{array}{r} 7.83 \\ -\quad 6.99 \\ \hline \end{array}
$$

J.

$$
\begin{array}{r}
9.75 \\
-\quad 8.80 \\
\hline
\end{array}
$$

J.

$$\begin{array}{r} 46.80 \\ -\ 12.93 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 3.14 \\ -\quad 1.99 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 7.21 \\ -\quad 4.66 \\ \hline \end{array}$$

J.

$$\begin{array}{r} 5.44 \\ -\quad 2.08 \\ \hline \end{array}$$

J.

$$
\begin{array}{r} 9.66 \\ -\quad 1.67 \\ \hline \end{array}
$$

J.

$$
\begin{array}{r}
8.33 \\
-\quad 1.92 \\
\hline
\end{array}
$$

J.

$$
\begin{array}{r} 42.12 \\ -\ 10.09 \\ \hline \end{array}
$$

J.

$$\begin{array}{r} 6.81 \\ -\quad 2.33 \\ \hline \end{array}$$

K.

$$
\begin{array}{r} 30.15 \\ -\quad 2.6 \\ \hline \end{array}
$$

K.

$$\begin{array}{r} 2.5 \\ -\quad 1.49 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 14.58 \\ -\ 1.4 \\ \hline \end{array}$$

K.

$$
\begin{array}{r} 10.2 \\ -\quad 5.73 \\ \hline \end{array}
$$

K.

$$
\begin{array}{r}
5.4 \\
-\quad .54 \\
\hline
\end{array}
$$

K.

$$
\begin{array}{r} 8.3 \\ -\quad .91 \\ \hline \end{array}
$$

K.

$$\begin{array}{r} 4.6 \\ -\quad .64 \\ \hline \end{array}$$

K.

$$
\begin{array}{r}
9.38 \\
-\quad .19 \\
\hline
\end{array}
$$

K.

$$\begin{array}{r} 10.21 \\ -\quad 5.6 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 3.9 \\ -\quad 1.01 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 17.72 \\ -\ 12.58 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 42.1 \\ -\ 17.96 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 9.3 \\ -\quad 6.31 \\ \hline \end{array}$$

K.

$$
\begin{array}{r}
9.0 \\
-\quad 8.12 \\
\hline
\end{array}
$$

K.

$$
\begin{array}{r}
9.17 \\
-\ 2.7 \\
\hline
\end{array}
$$

K.

$$\begin{array}{r} 3.46 \\ -\quad 1.6 \\ \hline \end{array}$$

K.

$$\begin{array}{r} 9.9 \\ -\quad 4.23 \\ \hline \end{array}$$

K.

$$
\begin{array}{r} 15.5 \\ -\ 12.22 \\ \hline \end{array}
$$

K.

$$
\begin{array}{r}
17.5 \\
-\quad 8.83 \\
\hline
\end{array}
$$

K.

$$
\begin{array}{r} 9.2 \\ -\quad 6.75 \\ \hline \end{array}
$$

Module 9:

Subtraction of Rational Numbers

Vocabulary Cards

algorithm
compare
computation
decimal
denominator
difference
equal sign
equivalent
fraction
hundredths
improper fraction
least common multiple
minuend
minus sign
mixed number
multiple
numerator
ones
regroup/trade/exchange
separate
subtract/subtraction
subtrahend
tenths

algorithm

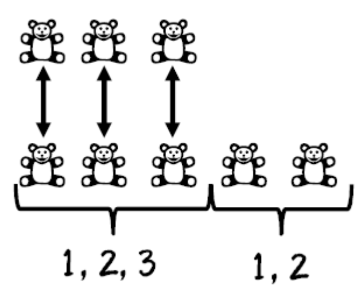
A procedure or description of steps that can be used to solve a problem.

---

compare

To find the difference between two sets.

$$5 - 3 = 2$$



computation

The action used to solve a problem.

---

decimal

A number based on powers of ten.

34.107

tens ones tenths hundredths thousandths

denominator

The term in a fraction that tells the number of equal parts in a whole.

2 / 3 $\quad \frac{2}{3}$ **In these fractions, 3 is the denominator.**

---

difference

The result of subtracting one number from another number.

6 – 4 = 2

2 is the difference

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$12 - 8 = 4$$

= is the equal sign

---

equivalent

Two numbers that have the same value.

$$\frac{1}{4} = \frac{2}{8} \qquad \frac{2}{3} = \frac{8}{12}$$

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \qquad \frac{10}{12} \qquad \frac{8}{3}$$

---

hundredths

The digit in representing $\frac{1}{100}$.

In the number 4.23, 3 is in the hundredths place.

improper fraction

Any fraction in which the numerator is greater than the denominator.

$$\frac{9}{4} \qquad \frac{17}{12} \qquad \frac{10}{3}$$

---

least common multiple

The common multiple with the least value.

6: 6, 12, 18, 24, 30

8: 8, 16, 24, 32, 40

With multiples of 6 and 8, the least common multiple is 24.

minuend

The number from which another number is subtracted.

9 – 4 = 5

9 is the minuend

---

minus sign

The symbol that tells you to subtract.

9 – 4 = 5

– is the minus sign

mixed number

A whole number and a fraction combined.

$$1\frac{1}{6} \qquad 4\frac{5}{12} \qquad 12\frac{4}{3}$$

---

multiple

The product of a number and any integer.

4: 4, 8, 12, 16, 20

numerator

The term in a fraction that tells how many parts of a fraction.

2 / 3 $\frac{2}{3}$ **In these fractions, 2 is the numerator.**

---

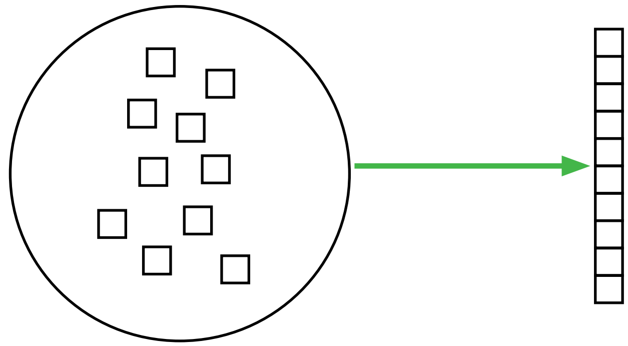
ones

The digit representing 1.

In the number 4.23, 4 is in the ones place.

regroup/trade/exchange

The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



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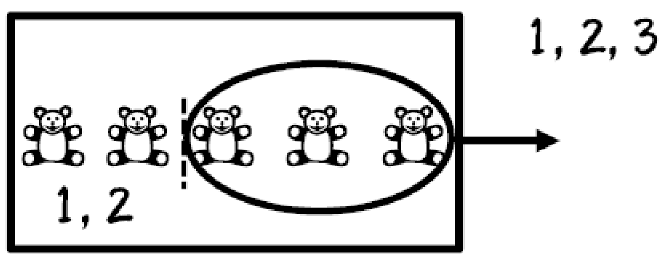
separate

To start with a set and take away from that set.

5 – 3 = 2

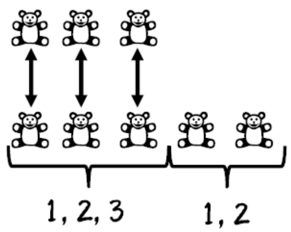


subtract/subtraction

To compare two sets or to separate from a set.

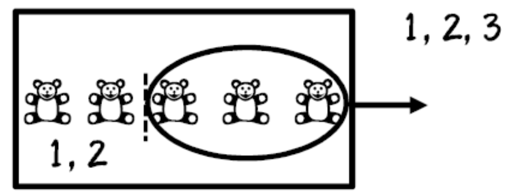
To compare two sets

5 – 3 = 2



To separate from a set

5 – 3 = 2



---

subtrahend

The number to be subtracted.

9 – 4 = 5

4 is the subtrahend

tenths

The digit in representing $\frac{1}{10}$.

In the number 4.23**, 2 is in the tenths place.**

Instructional Routines for Mathematics Intervention

MODULE 10

Concepts of Multiplication

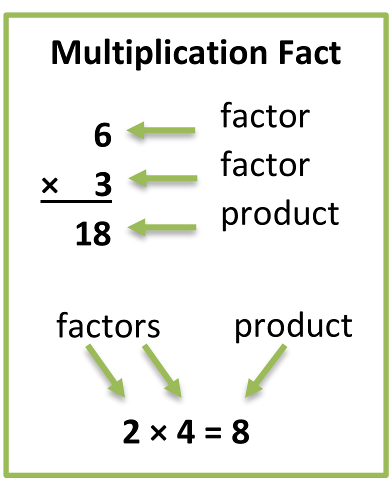
Module 10: Concepts of Multiplication
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
area	The number of square units that covers a closed figure.
array	A set of objects, pictures, or numbers arranged in columns and rows.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
factor	A number that you multiply with another number to get the product.
multiply/multiplication	The process of adding a number to itself a number of times.
multiplication sign	The symbol that tells you to multiply.
partial products	The product of parts of each factor.
product	The result of multiplying two or more factors.

B. Background Information

Students need to learn two concepts of multiplication: (1) multiplication as equal groups and (2) multiplication as comparison. Typically, students first learn about multiplying as equal groups. Then, students learn about multiplying as comparison.



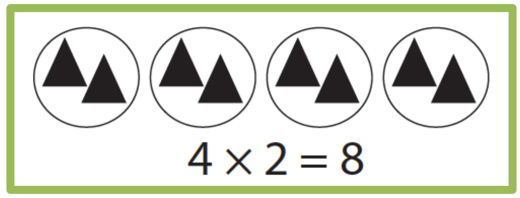
For learning the concepts of multiplication, we recommend using *mathematics facts*. We define a multiplication mathematics fact as single-digit factors multiplied for a single- or double-digit product. You may present multiplication facts vertically or horizontally.

C. Routines and Examples

(1) Multiplication as Equal Groups

Routine

Materials:



- [Module 10 Multiplication Problems]
- [Module 10 Vocabulary Cards]
    - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., cubes, clips) and any container (e.g., plates, cups)

Teacher	**Let's work on multiplication. Today, let's think about multiplication as equal groups. What does equal mean?**
Students	The same.
Teacher	**What is a group?**
Students	A collection of items.
Teacher	**So, with equal groups, we'll have the same number in each group. What's an equal group?**
Students	Same number in each group.
Teacher	**When we create equal groups, we'll put the same number in each group. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**We'll multiply by creating equal groups. In a multiplication problem, we'll use the first factor to tell us the number of groups. What will the first factor tell us?**
Students	The number of groups.
Teacher	**And we'll use the second factor to tell us how many in each group. What will the second factor tell us?**
Students	How many in each group.
Teacher	**Great. Let's do this problem.** (Move clips to workspace.)
Teacher	**Our first factor is __. What's our first factor?**
Students	__.
Teacher	**Let's show this factor by showing __ groups. We'll show the groups with the plates.** (Use plates to show groups.)
Teacher	**How many groups?**

Students __.

Teacher **Our second factor is __. What's our second factor?**

Students __.

Teacher **Let's show the second factor by placing __ clips in each group. Remember we need to show the same, or equal, number of clips in each group.**
(Show clips in groups.)

Teacher **How many clips in each group?**

Students __.

Teacher **So, we have __ groups of __. Let's multiply by counting all the clips. How could we count the clips?**

Students Count all the clips or count by groups.

Teacher **Yes. Let's count by groups to learn the product. We have __ groups of __ so that's __, __, __, …**
(Count clips by groups – skip count.)

Teacher **How many clips are there altogether?**

Students __.

Teacher **Yes! There are __ clips. So, __ times __ equals __. Let's say that together.**

Students __ times __ equals __.

Teacher **Let's say it together again.**

Students __ times __ equals __.

Teacher **So, if you have __ groups with an equal number of __ in each group, the product is __. __ times __ equals __. Let's review. What's a factor?**

Students One of the numbers multiplied in a multiplication problem.

Teacher **What's a product?**

Students The result of multiplying factors.

Teacher **What does it mean to make equal groups?**

Students To have groups with an equal number in each group.

Teacher **How could you explain multiplying to a friend?**

Students We started groups and placed the same number of clips in each group. The product was the total number of clips.

Example

$$\begin{array}{r} 4 \\ \times\ \ 3 \\ \hline 12 \end{array}$$

Teacher **Let's work on multiplication. Today, let's think about multiplication with equal groups. What does it mean to make equal groups?**

Students Show groups with an equal number in each group.

Teacher **When we make equal groups, we show groups with an equal number in each group. Look at this problem.**

(Show problem.)

Teacher **First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**

Students To multiply.

Teacher **We'll multiply by making equal groups. Let's show the first factor with our groups or plates. How will we show the first factor?**

Students As the groups with our plates.

Teacher **Our first factor is 4. What's our first factor?**

Students 4.

Teacher **Let's show this factor by showing 4 groups.**
(Show 4 groups by showing 4 plates.)

Teacher **How many groups?**

Students 4.

Teacher **Our second factor is 3. What's our second factor?**

Students 3.

Teacher **Let's show the second factor by 3 placing cubes in each group.**
(Place 3 cubes in each group.)

Teacher **How many cubes in each group?**

Students 3.

Teacher **So, we have 4 groups of 3 or 4 times 3. Let's multiply to learn the product. Let's count the cubes. How could we count?**

Students We could count all of the cubes.

Teacher **We could count all the cubes. Let's do that together.**

Students 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Teacher **We could also skip count the cubes in groups. Let's do that together.**

Students 3, 6, 9, 12.

Teacher **What's the product?**

Students 12.

Teacher **How many cubes are there altogether?**

Students 12.

Teacher **Yes! There are 12 cubes. So, 4 groups of 3 equals 12. Let's say that together.**

Students 4 groups of 3 equals 12.

Teacher **We could also say 4 times 3 equals 12. Let's say that together.**

Students 4 times 3 equals 12.

Teacher **Let's say it together again.**

Students 4 times 3 equals 12.

Teacher **So, if you have 4 groups with 3 in each group, the product is 12. 4 times 3 equals 12. Let's review. What's a factor?**

Students One of the numbers multiplied in a multiplication problem.

Teacher **What's a product?**

Students The total when you multiply groups with an equal number in each group.

Teacher **What does it mean to make equal groups?**

Students To have groups with an equal number in each group.

Teacher **How could you explain multiplying to a friend?**

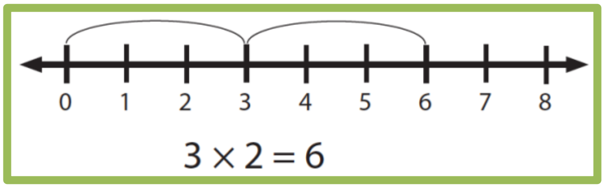
Students	We started groups and placed the same number of cubes in each group. The product was the total number of cubes.

(2) Multiplication as Comparison

Routine

Materials:

- Module 10 Problems
- Module 10 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- Number line



Teacher	**Let's work on multiplication. Today, let's think about multiplication as comparison. What does comparison mean?**
Students	To look at one set compared to another set.
Teacher	**In subtraction, we compare two sets by determining the difference between two numbers. In multiplication, we compare two sets by multiplying a first set and a second set. How do we compare in multiplication?**
Students	Multiply two sets together.
Teacher	**When we multiply by comparison, we have a set. Imagine you have a set of 5 apples. Your friend has 4 times as many apples as you. We multiply to figure out how many apples your friend has. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Today we'll multiply by comparison, but there are other ways to multiply – like using equal groups. Let's start by getting out our number line.** (Move number line to workspace.)
Teacher	**Our first factor is __. What's our first factor?**
Students	__.
Teacher	**The first factor is our set. What's the first factor?**
Students	Our set.
Teacher	**Our second factor is __. What's our second factor?**
Students	__.
Teacher	**The second factor tells us how many times to multiply the first factor. What does the second factor tell us?**
Students	How many times to multiply the set.
Teacher	**So, let's show the first set on the number line. We have a set of __ so let's count out __.** (Count first set on number line.)
Teacher	**Now, let's multiply that set __ times. How many times?**
Students	__.

Teacher	**To multiply, let's count the original set __ times. Watch me: __, __, __, …** (Count sets by multiplying.)
Teacher	**The product is the last number we said. We counted to __. What's the product?**
Students	__.
Teacher	**How many altogether?**
Students	__.
Teacher	**Yes! There are __. So, __ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Let's say it together again.**
Students	__ times __ equals __.
Teacher	**So, if you have a set of __ and multiply that set __ times, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	One of the numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The total when you multiply groups with an equal number in each group.
Teacher	**What does it mean to multiply by comparison?**
Students	To have a set and multiply the set a number of times.
Teacher	**How could you explain multiplying to a friend?**
Students	We started a set and counted the set a number of times on the number line. The product was the total.

Example

$$\begin{array}{r} 4 \\ \times\ \ 3 \\ \hline 12 \end{array}$$

Teacher	**Let's work on multiplication. Today, let's think about multiplication by comparison. What does it mean to compare?**
Students	To have a set and compare that set a number of times.
Teacher	**When we compare, we start with a set and multiply that set a number of times. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Today we'll multiply by comparison, but there are other ways to multiply – like with equal groups. Let's start by getting out our number line. Let's do this together.** (Move number line to workspace.)
Teacher	**Our first factor is 4. What's our first factor?**

Students	4.
Teacher	**Our second factor is 3. What's our second factor?**
Students	3.
Teacher	**That means we're going to multiply the set of four 3 times. What does our problem mean?**
Students	Multiply the set of 4 3 times.
Teacher	**Ready? Let's use the number line to count the set of four 3 times. I show one set of 4** (place finger on 4), **a second set of 4** (place finger on 8), **and a third set of 4** (place finger on 12). **What's the last number we said?**
Students	12.
Teacher	**The product is the last number we said. We counted 12. What's the product?**
Students	12.
Teacher	**So, 4 times 3 equals 12. Let's say that together.**
Students	4 times 3 equals 12.
Teacher	**Let's say it together again.**
Students	4 times 3 equals 12.
Teacher	**So, if you have a set of 4 and multiply that set 3 times, the product is 12. 4 times 3 equals 12. Let's review. What's a factor?**
Students	One of the numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The total when you multiply groups with an equal number in each group.
Teacher	**What does it mean to multiply by comparison?**
Students	To have a set and multiply the set a number of times.
Teacher	**How could you explain multiplying to a friend?**
Students	We started 4 and multiplied four 3 times. 4 times 3 equals 12.

D. Problems for Use During Instruction

See Module 10 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 10 Vocabulary Cards.

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Module 10:

Concepts of Multiplication

Problem Sets

$$
\begin{array}{r} 1 \\ \times \quad 0 \\ \hline \end{array}
$$

$$\begin{array}{r} 5 \\ \times \quad 4 \\ \hline \end{array}$$

$$
\begin{array}{r}
7 \\
\times \quad 5 \\
\hline
\end{array}
$$

$$\begin{array}{r} 2 \\ \times \quad 1 \\ \hline \end{array}$$

$$
\begin{array}{r} 7 \\ \times \quad 6 \\ \hline \end{array}
$$

$$\begin{array}{r} 4 \\ \times \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times \quad 5 \\ \hline \end{array}$$

$$
\begin{array}{r} 9 \\ \times \quad 9 \\ \hline \end{array}
$$

$$\begin{array}{r} 2 \\ \times \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times \quad 8 \\ \hline \end{array}$$

$$
\begin{array}{r}
4 \\
\times \quad 4 \\
\hline
\end{array}
$$

$$\begin{array}{r} 5 \\ \times \quad 8 \\ \hline \end{array}$$

$$
\begin{array}{r} 4 \\ \times \quad 7 \\ \hline \end{array}
$$

$$
\begin{array}{r} 3 \\ \times \quad 4 \\ \hline \end{array}
$$

$$\begin{array}{r} 3 \\ \times \quad 2 \\ \hline \end{array}$$

$$
\begin{array}{r}
4 \\
\times \quad 3 \\
\hline
\end{array}
$$

$$\begin{array}{r} 9 \\ \times \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times \quad 4 \\ \hline \end{array}$$

$$
\begin{array}{r}
8 \\
\times \quad 7 \\
\hline
\end{array}
$$

$$\begin{array}{r} 5 \\ \times \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \times \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \times \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times \quad 4 \\ \hline \end{array}$$

$$
\begin{array}{r}
3 \\
\times \quad 1 \\
\hline
\end{array}
$$

$$\begin{array}{r} 9 \\ \times \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \times \quad 1 \\ \hline \end{array}$$

$$
\begin{array}{r} 6 \\ \times \quad 5 \\ \hline \end{array}
$$

$$\begin{array}{r} 8 \\ \times \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \times \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ \times \quad 4 \\ \hline \end{array}$$

$$
\begin{array}{r}
2 \\
\times \quad 3 \\
\hline
\end{array}
$$

$$\begin{array}{r} 8 \\ \times \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ \times \quad 2 \\ \hline \end{array}$$

$$
\begin{array}{r} 0 \\ \times \quad 9 \\ \hline \end{array}
$$

$$\begin{array}{r} 6 \\ \times \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \times \quad 2 \\ \hline \end{array}$$

$$
\begin{array}{r}
8 \\
\times \quad 9 \\
\hline
\end{array}
$$

$$\begin{array}{r} 1 \\ \times \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \times \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \times \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times \quad 8 \\ \hline \end{array}$$

$$
\begin{array}{r}
1 \\
\times \quad 1 \\
\hline
\end{array}
$$

$$\begin{array}{r} 5 \\ \times \quad 6 \\ \hline \end{array}$$

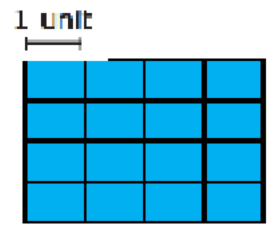
Module 10:
Concepts of Multiplication

Vocabulary Cards

area
array
equal groups
equal sign
factor
multiply/multiplication
multiplication sign
partial products
product

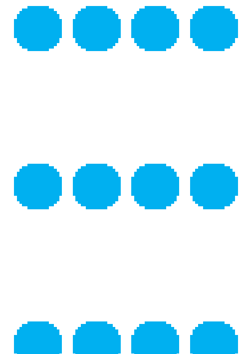
area

The number of square units that covers a closed figure.



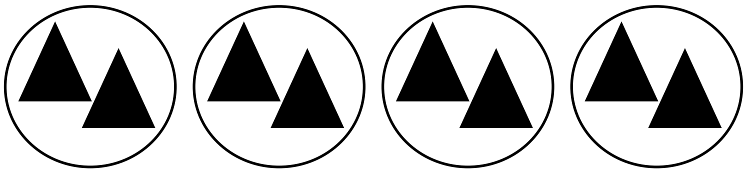
array

A set of objects, pictures, or numbers arranged in columns and rows.



equal groups

Groups with the same number of objects or items in each group.



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equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$2 \times 8 = 16$$

= is the equal sign

factor

A number you multiply with another number to get the product.

$$2 \times 8 = 16$$

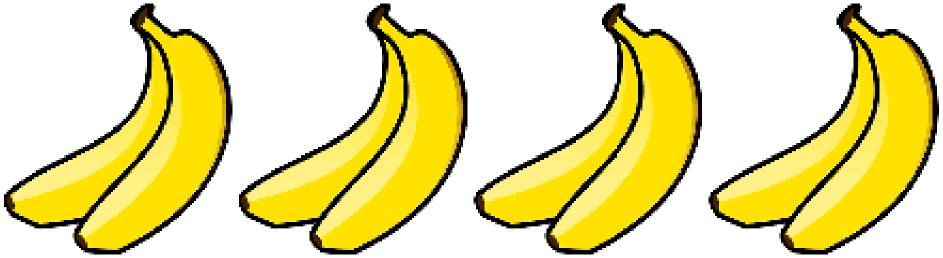
2 and 8 are the factors

---

multiply/multiplication

The process of adding a number to itself a number of times.

$$4 \times 2 = 8$$



multiplication sign

The symbol that tells you to multiply.

$$2 \times 8 = 16$$

× is the multiplication sign

---

partial products

The product of parts of each factor.

$$
\begin{array}{r l}
13 & \\
\times\ \ 45 & \\
\hline
400 & (40 \times 10) \\
120 & (40 \times 3) \\
50 & (10 \times 5) \\
+\ \ 15 & (5 \times 3) \\
\hline
585 &
\end{array}
$$

product

The result of multiplying two or more factors.

$$2 \times 8 = 16$$

16 is the product

Instructional Routines for Mathematics Intervention

MODULE 11

Multiplication of Whole Numbers

Module 11: Multiplication of Whole Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A procedure or description of steps that can be used to solve a problem.
area	The number of square units that covers a closed figure.
array	A set of objects, pictures, or numbers arranged in columns and rows.
commutative property of multiplication	Two factors can be multiplied in any order.
computation	The action used to solve a problem.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
factor	A number that you multiply with another number to get the product.
hundreds column	The column with digits in the hundreds place.
multiply/multiplication	The process of adding a number to itself a number of times.
multiplication sign	The symbol that tells you to multiply.
ones column	The column with digits in the ones place.
partial products	The product of parts of each factor.
product	The result of multiplying two or more factors.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
tens column	The column with digits in the tens place.

B. Background Information

Background Information:
If your focus is on the conceptual understanding of multiplication, see *Module 10: Concepts of Multiplication*. This module, *Module 11*, focuses on computation with multiplication of whole numbers. As you focus on computation, continue to emphasize multiplication as equal groups and multiplication as comparison because students will see these concepts within word problems.

For learning computation with multiplication, we recommend presenting problems vertically. Some students may require explicit instruction on translating a horizontal problem (e.g., 12 × 27) to the vertical presentation (see below). Depending upon the algorithm, leave enough space above or below the problem for students to complete their written work.

Every student should develop efficiency with a multiplication computation strategy. In the following sections, we provide examples of (1) multiplication with traditional algorithm, (2) multiplication with partial products algorithm, and (3) multiplication with array (or area model). Teachers should help students develop competency with at least one algorithm.

Computation with Multiplication

14	← factor
× 32	← factor
448	← product

C. Routines and Examples

(1) Multiplication with Traditional Algorithm

Routine

Materials:

- Module 11 Problem Sets
- Module 11 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
    - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT × 1-DIGIT: ROUTINE WITH MANIPULATIVES

(Only use manipulatives with simpler problems)

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)

Teacher	**With our Base-10 blocks, each cube represents one thousand. What do the cubes represent?**
Students	Thousands.
Teacher	**The flats represent hundreds. What do the flats represent?**
Students	Hundreds.
Teacher	**The rods represent tens. What do the rods represent?**
Students	Tens.
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**
Students	Ones.
Teacher	**Our first factor is __. What's our first factor?**
Students	__.
Teacher	**Our second factor is __. What's our second factor?**
Students	__.
Teacher	**Let's solve this problem using multiplication as equal groups. What does equal groups mean?**
Students	We have groups with an equal number in each group.
Teacher	**In this problem, we have __ groups of __. What do we have?**
Students	__ groups of __.
Teacher	**If we want to use the Base-10 blocks, I want to use the commutative property. The commutative property says that, in multiplication, the order of the factors does not matter. We could multiply __** (first factor) **times __** (second factor) **or __** (second factor) **times __** (first factor). **The product will be the same. What's the commutative property?**
Students	In multiplication, the order of factors does not matter.
Teacher	**So, in this problem. I want to interpret this as __** (1-digit number) **groups of __** (2-digit number). **It will be easier to set up the problem. So, we have __ groups of __. What do we have?**
Students	__ groups of __.
Teacher	**Let's use the Base-10 blocks to make __ groups of __. I'll make one group at a time.** (Use Base-10 blocks to show groups with an equal number in each group.)
Teacher	**Now, let's combine all the groups to learn the product. Let's put together all the ones.** (Put together ones.)
Teacher	**If we have more than 9 ones we have to regroup. Do we have more than 9 ones?**
Students	*OPTION 1:* No. We don't have to regroup. *OPTION 2:* Yes. We have to regroup.
Teacher	***OPTION 2:*** **How do we group?**
Students	Regroup/trade/exchange 10 ones for 1 ten.
Teacher	**Let's exchange 10 ones for 1 ten. We'll leave the remaining ones and place the 1 ten with the tens.** (Regroup.)

Teacher **Now, let's combine the tens.**
(Put together tens.)

Teacher **If we have more than 9 tens we have to regroup. Do we have more than 9 tens?**

Students *OPTION 1:* No. We don't have to regroup.
OPTION 2: Yes. We have to regroup.

Teacher ***OPTION 2:*** **How do we group?**

Students Regroup/trade/exchange 10 tens for 1 hundred.

Teacher **Let's exchange 10 tens for 1 hundred. We'll leave the remaining tens and place the 1 hundred with the hundreds.**
(Regroup.)

Teacher **Now, let's combine the hundreds.**
(Put together hundreds.)

Teacher **If we have more than 9 hundreds we have to regroup. Do we have more than 9 hundreds?**

Students *OPTION 1:* No. We don't have to regroup.
OPTION 2: Yes. We have to regroup.

Teacher ***OPTION 2:*** **How do we group?**

Students Regroup/trade/exchange 10 hundreds for 1 thousand.

Teacher **Let's exchange 10 hundreds for 1 thousand. We'll leave the remaining hundreds and place the 1 thousand with the thousands.**
(Regroup.)

Teacher **Let's count to determine the product.**
(Count the thousands, hundreds, tens, and ones.)

Teacher **That means __ times __ equals __. Let's say that together.**

Students __ times __ equals __.

Teacher **Let's say it together again.**

Students __ times __ equals __.

Teacher **So, if you have __ groups of __ and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**

Students The numbers multiplied in a multiplication problem.

Teacher **What's a product?**

Students The result of multiplying factors.

Teacher **What does it mean to make equal groups?**

Students To have groups with an equal number in each group.

Teacher **How could you explain multiplying to a friend?**

Students We used Base-10 blocks to make groups with the same number in each group. The product was the total number of blocks.

2-DIGIT × 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on multiplication. What does it mean to multiply?**

Students To make equal groups or to compare.

Teacher **Multiplication means to make equal groups or to compare. Look at this problem.**
(Show problem.)

Teacher **First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**

Students To multiply.

Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.**
(Draw vertical lines to separate place value columns.)

Teacher **Now, we start by multiplying the ones of the second factor. This means we'll write these products starting in the ones column below the equal line. Where will we write the products?**

Students Below the equal line.

Teacher **We first multiply the ones of the second factor times the ones of the first factor. What should we multiply first?**

Students The ones of the second factor times the ones of the first factor.

Teacher **Which ones do we multiply?**

Students __ times __.

Teacher **What's __ times __?**
(If a student has difficulty with multiplication, use a multiplication table or other resource.)

Students __.

Teacher **__ times __ equals __. Let's write __ below the equal line in the ones column.**
IF REGROUPING: **Our product is greater than 9, so we have to regroup. That means we write the ones in the ones place and regroup the tens.**
(Write product.)

Teacher **Now, we multiply the ones of the second factor times the tens of the first factor. What do we multiply?**

Students The ones of the second factor times the tens of the first factor.

Teacher **So, what do we multiply?**

Students __ times __.

Teacher **What's __ times __?**

Students __.

Teacher IF REGROUPING: **Remember, we regrouped __ from when we multiplied the ones of the second factor by the ones of the first factor. Now, we add that regrouped amount to our product of __ times __. So, what's __ plus __?**

Students __.

Teacher **Let's write __ below the equal line in the tens column.**
(Write product.)

Teacher	**So, we multiplied the ones of the second factor times the ones of the first factor then the ones of the second factor times the tens of the first factor. Who can describe what we multiplied so far?**
Students	We multiplied the ones of the second factor times the ones of the first factor then times the tens of the first factor.
Teacher	**We've multiplied the ones of the second factor. Now, it's time to multiply the tens of the second factor. What will we multiply now?**
Students	The tens of the second factor.
Teacher	**When writing the products of multiplying the tens of the second factor, we'll write them below this first line of products. Because we're now multiplying by ten, we will write our products starting in the tens column. I like to place an X or zero in the ones column below the equal line to remember to start writing my products in the tens column.** (Write X or 0.)
Teacher	**Now, let's multiply the tens of the second factor times the ones of the first factor. What should we multiply?**
Students	The tens of the second factor times the ones of the first factor.
Teacher	**What numbers do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)
Students	__.
Teacher	**__ times __ equals __. Let's write __ below the equal line in the tens column.** IF REGROUPING: **Our product is greater than 9, so we have to regroup. That means we write the ones and regroup the tens above the problem.** (Write product.)
Teacher	**Now, we multiply the tens of the second factor times the tens of the first factor. What do we multiply?**
Students	The tens of the second factor times the tens of the first factor.
Teacher	**So, what do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	IF REGROUPING: **Remember, we regrouped __ from when we multiplied the tens of the second factor by the ones of the first factor. Now, we add that regrouped amount to our product of __ times __. So, what's __ plus __?**
Students	__.
Teacher	**Let's write __ below the equal line.** (Write product.)

Teacher	**So, we multiplied the tens of the second factor times the ones of the first factor and then the tens of the second factor times the tens of the first factor. Who can describe what we multiplied?**
Students	We multiplied the tens of the second factor times the ones of the first factor then times the tens of the first factor.
Teacher	**Now, we did all the multiplication but we are not finished! We call these numbers here** (point to numbers under equal line) **our partial products. We have to add the partial products together to determine the final product. Let's draw another equal line and write in a plus sign. What should we draw?**
Students	An equal line and plus sign.
	(Write equal line and plus sign.)
Teacher	**So, let's add __ plus __. What's __ plus __?** (If students need help with addition of whole numbers, see Module 5.)
Students	__.
Teacher	**Yes. So, I write __ under the equal line.**
	(Write final product.)
Teacher	**That means __ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Let's say it together again.**
Students	__ times __ equals __.
Teacher	**So, if you have __ and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	One of the numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to make equal groups?**
Students	To have groups with an equal number in each group.
Teacher	**How could you explain multiplication of double-digit numbers to a friend?**
Students	We multiplied the ones of the second factor times the ones and tens of the first factor. Then, we multiplied the tens of the second factor times the ones and tens of the first factor. Finally, we added the partial products to determine the final product.

Example

$$\begin{array}{r} 13 \\ \times\ \ 45 \\ \hline 585 \end{array}$$

2-DIGIT × 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.

Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by multiplying the ones of the second factor. This means we'll write these products starting in the ones column below the equal line. Where will we write the products?**
Students	Below the equal line.
Teacher	**We first multiply the ones of the second factor times the ones of the first factor. What should we multiply first?**
Students	The ones of the second factor times the ones of the first factor.
Teacher	**What are the ones of the second factor?**
Students	5.
Teacher	**What are the ones of the first factor?**
Students	3.
Teacher	**So, we'll multiply 5 times 3. What do we multiply?**
Students	5 times 3.
Teacher	**What's 5 times 3?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)
Students	15.
Teacher	**5 times 3 equals 15. 15 is greater than 9, so we have to regroup. That means we write the 5 of 15 in the ones place below the equal line. We write the 1 of 15 above the tens column.** (Write product.)
Teacher	**Now, we multiply the ones of the second factor times the tens of the first factor. What do we multiply?**
Students	The ones of the second factor times the tens of the first factor.
Teacher	**So, what do we multiply?**
Students	5 times 1.
Teacher	**What's 5 times 1?**
Students	5.
Teacher	**Is the product greater than 9?**
Students	No.
Teacher	**Do we have to regroup?**
Students	No.

Teacher	**But we do have to remember to add the regrouped amount to our product. That means we'll add 5 plus 1. What's 5 plus 1?**
Students	6.
Teacher	**Let's write 6 below the equal line in the tens column.** (Write product.)
Teacher	**So, we multiplied the ones of the second factor times the ones then the tens. Who can describe what we multiplied so far?**
Students	We multiplied 5 times 3. Then we multiplied 5 times 1.
Teacher	**We've multiplied the ones of the second factor. Now, it's time to multiply the tens of the second factor. What will we multiply now?**
Students	The tens of the second factor.
Teacher	**When writing the products of multiplying the tens of the second factor, we'll write them below this first line of products. Because we're now multiplying by ten, we will write our products starting in the tens column. I like to place an X or zero in the ones column below the equal line to remember to start writing my products in the tens column.** (Write X or 0.)
Teacher	**Now, let's multiply the tens of the second factor times the ones of the first factor. What should we multiply?**
Students	The tens of the second factor times the ones of the first factor.
Teacher	**What are the tens of the second factor?**
Students	4.
Teacher	**What are the ones of the first factor?**
Students	3.
Teacher	**So, we'll multiply 4 times 3. What do we multiply?**
Students	4 times 3.
Teacher	**What's 4 times 3?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)
Students	12.
Teacher	**4 times 3 equals 12. 12 is greater than 9, so we have to regroup. That means we write the 2 of 12 in the tens place below the equal line. We write the 1 of 12 above the hundreds column.** (Write product.)
Teacher	**Now, we multiply the tens of the second factor times the tens of the first factor. What do we multiply?**
Students	The tens of the second factor times the tens of the first factor.
Teacher	**So, what do we multiply?**
Students	4 times 1.
Teacher	**What's 4 times 1?**
Students	4.
Teacher	**Is the product greater than 9?**
Students	No.
Teacher	**Do we have to regroup?**

Students	No.
Teacher	**But we do have to remember to add the regrouped amount to our product. That means we'll add 4 plus 1. What's 4 plus 1?**
Students	5.
Teacher	**Let's write 5 below the equal line in the tens column.** (Write product.)
Students	__.
Teacher	**Let's write __ below the equal line.** (Write product.)
Teacher	**So, we multiplied the tens of the second factor times the ones of the first factor then the tens of the first factor. Who can describe what we multiplied?**
Students	We multiplied 4 times 3 then 4 times 1.
Teacher	**We did the multiplication. Are we finished?**
Students	No!
Teacher	**We are not finished! We call these numbers here** (point to numbers under equal line) **our partial products. We have to add the partial products together to determine the final product. Let's draw another equal line and write in a plus sign. What should we draw?**
Students	An equal line and plus sign. (Write equal line and plus sign.)
Teacher	**So, let's add 65 plus 520. What's 65 plus 520?** (If students need help with addition of whole numbers, see Module 5.)
Students	585.
Teacher	**Yes. So, I write 585 under the equal line.** (Write final product.)
Teacher	**That means 13 times 45 equals 585. Let's say that together.**
Students	13 times 45 equals 585.
Teacher	**Let's say it together again.**
Students	13 times 45 equals 585.
Teacher	**So, if you have 13 and multiply by 45, the product is 585. Let's review. What's a factor?**
Students	One of the numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to make equal groups?**
Students	To have groups with an equal number in each group.
Teacher	**How could you explain multiplication of double-digit numbers to a friend?**
Students	We multiplied the ones of the second factor first. That meant we multiplied 5 times 3 then 5 times 1. Then, we multiplied the tens of the second factor. We multiplied 4 times 3 then 4 times 1. Finally, we added the partial products of 65 and 520 to determine the product of 585.

(2) Multiplication with Partial Products Algorithm*

*For clarity, read **Example** before using **Routines**.

Routine

Materials:

- [Module 11 Problem Sets]
- [Module 11 Vocabulary Cards]
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
    - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT × 1-DIGIT: ROUTINE WITH MANIPULATIVES

(Only use manipulatives with simpler problems)

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**With our Base-10 blocks, the flats represent hundreds. What do the flats represent?**
Students	Hundreds.
Teacher	**The rods represent tens. What do the rods represent?**
Students	Tens.
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**
Students	Ones.
Teacher	**Our first factor is __. What's our first factor?**
Students	__.
Teacher	**Our second factor is __. What's our second factor?**
Students	__.
Teacher	**Let's solve this problem using multiplication as equal groups. What does equal groups mean?**
Students	We have groups with an equal number in each group.
Teacher	**We will use the partial products strategy to solve this problem. Say partial products.**
Students	Partial products.

Teacher	**With the partial products strategy, we do the multiplication for each factor then we add the partial products together for a final product. With the partial products strategy, we work from the greatest place value to the least place value. How do we work?**
Students	From the greatest place value to the least place value.
Teacher	**In this problem, what is the greatest place value?**
Students	Tens.
Teacher	**The tens are the greatest place value, so we'll start by multiplying the ones of the second factor by the tens of the first factor. Where do we start?**
Students	By multiplying the ones of the second factor times the tens of the first factor.
Teacher	**First, let's multiply the ones of the second factor times the tens of the first factor. What are the tens of the first factor?**
Students	__.
Teacher	**We have __ tens. __ tens is the same as what?**
Students	__.
Teacher	**So, we multiply __ times __. Let's use the Base-10 blocks to make __ groups of __. I'll make one group at a time.** (Use Base-10 blocks to show groups with an equal number in each group.)
Teacher	**These Base-10 blocks are one of our partial products. Now, let's multiply the ones of the second factor times the ones of the first factor. What are the ones of the second factor?**
Students	__.
Teacher	**Let's then multiply __ times __. Let's use the Base-10 blocks to make __ groups of __. I'll make one group at a time.** (Use Base-10 blocks to show groups with an equal number in each group.)
Teacher	**This group of Base-10 blocks is another partial product. Now, let's add all the partial products, or Base-10 blocks, to determine the final product.** (Count the hundreds, tens, and ones.)
Teacher	**That means __ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Let's say it together again.**
Students	__ times __ equals __.
Teacher	**So, if you have __ groups of __ and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to use the partial products strategy?**
Students	We multiplied each factor for a partial product. Then, we added the partial products to determine the final product.
Teacher	**How could you explain multiplying to a friend?**
Students	We multiplied the ones of the second factor times the tens of the first factor. Then, we multiplied the ones of the second factor times the ones of the first factor. We added the partial products to determine the final product.

2-DIGIT × 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.** (Draw vertical lines to separate place value columns.)
Teacher	**Let's use the partial products strategy. What strategy?**
Students	Partial products.
Teacher	**With the partial products strategy, we do the multiplication for each factor then we add the partial products together for a final product. With the partial products strategy, we work from the greatest place value to the least place value. How do we work?**
Students	From the greatest place value to the least place value.
Teacher	**First, we'll multiply the tens of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the tens of the second factor?**
Students	__.
Teacher	**We have __ tens in the second factor. __ tens is the same as what?**
Students	__.
Teacher	**Look at the first factor. What are the tens of the first factor?**
Students	__.
Teacher	**We have __ tens in the first factor. __ tens is the same as what?**
Students	__.
	So, let's multiply __ times __. What's __ times __?
Students	__.
Teacher	**__ times __ equals __. Let's write __ below the equal line.** (Write product.)
Teacher	**__ is our first partial product. Now, let's multiply the tens of the second factor times the ones of the first factor? What do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.

Teacher	**Let's write __ below the equal line. We'll write this second partial product under the first partial product.** (Write product.)
Teacher	**Now, let's multiply the ones of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the ones of the second factor?**
Students	__.
Teacher	**We have __ ones in the second factor. Look at the first factor. What are the tens of the first factor?**
Students	__.
Teacher	**We have __ tens in the first factor. __ tens is the same as what?**
Students	__.
	So, let's multiply __ times __. What's __ times __?
Students	__.
Teacher	**__ times __ equals __. Let's write __ below the equal line under our other partial products.** (Write product.)
Teacher	**Finally, let's multiply the ones of the second factor times the ones of the first factor. What do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**Let's write __ below the equal line under our other partial products.** (Write product.)
Teacher	**To determine the final product, we add all the partial products together. I'll write a plus sign and another equal line.** (Write plus sign and equal line.)
Teacher	**So, what's __ plus __ plus __ plus __?** (For assistance with the partial sums algorithm for addition, see Module 5.)
Students	__.
Teacher	**__ is our final product. Let's write __ under the equal line.**
Students	(Write product.)
Teacher	**That means __ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Let's say it together again.**
Students	__ times __ equals __.
Teacher	**So, if you have __ groups and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to use the partial products strategy?**
Students	We multiplied each factor for a partial product. That means we multiplied the tens of the second factor times the tens of the first factor then the ones of the

first factor. We also multiplied the ones of the second factor times the tens of the first factor and ones of the first factor. Then, we added the partial products to determine the final product.

Teacher **How could you explain multiplying to a friend?**

Students We multiplied the tens of the second factor times the tens and ones of the first factor. Then, we multiplied the ones of the second factor times the tens and ones of the first factor. We added the partial products to determine the final product.

Example

13
× 45
400
120
50
+ 15
585

2-DIGIT × 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher **Let's work on multiplication. What does it mean to multiply?**

Students To make equal groups or to compare.

Teacher **Multiplication means to make equal groups or to compare. Look at this problem.**

(Show problem.)

Teacher **First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**

Students To multiply.

Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.**

(Draw vertical lines to separate place value columns.)

Teacher **Let's use the partial products strategy. What strategy?**

Students Partial products.

Teacher **With the partial products strategy, we do the multiplication for each factor then we add the partial products together for a final product. With the partial products strategy, we work from the greatest place value to the least place value. How do we work?**

Students From the greatest place value to the least place value.

Teacher	**First, we'll multiply the tens of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the tens of the second factor?**
Students	4.
Teacher	**We have 4 tens in the second factor. 4 tens is the same as what?**
Students	40.
Teacher	**Look at the first factor. What are the tens of the first factor?**
Students	1.
Teacher	**We have 1 ten in the first factor. 1 ten is the same as what?**
Students	10.
	So, let's multiply 40 times 10. What's 40 times 10?
Students	400.
Teacher	**40 times 10 equals 400. Let's write 400 below the equal line.** (Write product.)
Teacher	**400 is our first partial product. Now, let's multiply the tens of the second factor times the ones of the first factor? What do we multiply?**
Students	40 times 3.
Teacher	**What's 40 times 3?**
Students	120.
Teacher	**Let's write 120 below the equal line. We'll write this partial product under the first partial product.** (Write product.)
Teacher	**Now, let's multiply the ones of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the ones of the second factor?**
Students	5.
Teacher	**We have 5 ones in the second factor. Look at the first factor. What are the tens of the first factor?**
Students	1.
Teacher	**We have 1 ten in the first factor. 1 ten is the same as what?**
Students	10.
	So, let's multiply 5 times 10. What's 5 times 10?
Students	50.
Teacher	**5 times 10 equals 50. Let's write 50 below the equal line under our other partial products.** (Write product.)
Teacher	**Finally, let's multiply the ones of the second factor times the ones of the first factor. What do we multiply?**
Students	5 times 3.
Teacher	**What's 5 times 3?**
Students	15.
Teacher	**Let's write 15 below the equal line under our other partial products.** (Write product.)

Teacher	**To determine the final product, we add all the partial products together. I'll write a plus sign and another equal line.** (Write plus sign and equal line.)
Teacher	**I like to add in steps. What's 400 plus 120?**
Students	520.
Teacher	**What's 520 plus 50?**
Students	570.
Teacher	**What's 570 plus 15?**
Students	585.
Teacher	**585 is our final product. Let's write 585 under the equal line.**
Students	(Write product.)
Teacher	**That means 13 times 45 equals 585. Let's say that together.**
Students	13 times 45 equals 585.
Teacher	**Let's say it together again.**
Students	13 times 45 equals 585.
Teacher	**So, if you have 13 and multiply by 45, the product is 585. 13 times 45 equals 585. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to use the partial products strategy?**
Students	We multiplied each factor for a partial product. Then, we added the partial products to determine the final product.
Teacher	**How could you explain multiplying to a friend?**
Students	We multiply 40 times 10, then 40 times 3. Then, we multiplied 5 times 10 and 5 times 3. We added the partial products for a final product of 585.

(3) Multiplication with Array (Area Model)

*For clarity, read **Example** before using **Routine**.

Routine

Materials:

- Module 11 Problem Sets
- Module 11 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching

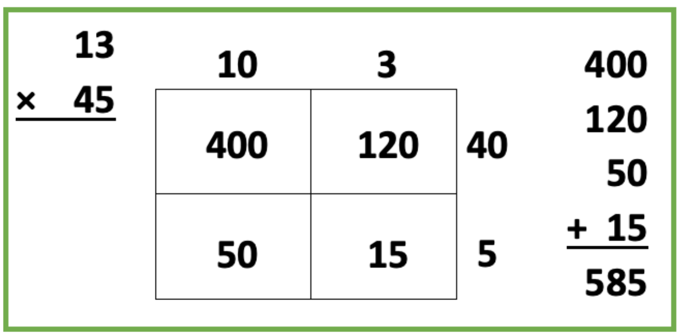
2-DIGIT × 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem using the array model. We'll create an array or rectangular area with our multiplication problem. The array model is similar to the partial products model. Let's get started. First, I have to draw rectangular array. What do I have to draw?**
Students	Rectangular array.
Teacher	**My array includes the place value of each factor. How many digits in the first factor?**
Students	__.
Teacher	**So, that's a __-digit factor. How many digits in the second factor?**
Students	__.
Teacher	**So, that's a __-digit factor. Our array should have __ columns for the first factor and __ rows for the second factor. Let's draw an array with __ columns and __ rows.** (Draw array.)
Teacher	**Now, I write the first factor in expanded form. What does expanded form mean?**
Students	To write the number in tens and ones.
Teacher	**How many tens are in the first factor?**
Students	__.
Teacher	**__ tens is the same as __. So the expanded form of __ would be __ plus __. Let's write __ and __ above the columns.** (Write first factor in expanded form.)
Teacher	**Now, I write the second factor in expanded form on the right side of the array. What does expanded form mean?**

Students	To write the number in tens and ones.
Teacher	**How many tens are in the second factor?**
Students	__.
Teacher	**__ tens is the same as __. So the expanded form of __ would be __ plus __. Let's write __ and __ next to the row on the right side.** (Write second factor in expanded form.)
Teacher	**Now that we have set up the problem, let's multiply. I like to multiply the second factor times the first factor but any order is okay – the commutative property helps us with that! Let's multiply __** (tens on row) **times __** (tens on column.) **What's __ times __?**
Students	__.
Teacher	**__ times __ equals __. Let's write __ in the part of the array in which the row and column meet.** (Write product.)
Teacher	**__ is a partial product. Now, let's multiply the tens of the second factor times the ones of the first factor. What do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**Let's write __ in the part of the array in which the row and column meet.** (Write product.)
Teacher	**Now, let's multiply the ones of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the ones of the second factor?**
Students	__.
Teacher	**We have __ ones in the second factor. Look at the first factor. What are the tens of the first factor?**
Students	__.
Teacher	**We have __ tens in the first factor. __ tens is the same as what?**
Students	__.
	So, let's multiply __ times __. What's __ times __?
Students	__.
Teacher	**__ times __ equals __. Let's write __ in the part of the array in which the row and column meet.** (Write product.)
Teacher	**Finally, let's multiply the ones of the second factor times the ones of the first factor. What do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**Let's write __ in the part of the array in which the row and column meet.** (Write product.)
Teacher	**To determine the final product, we add all the partial products together. I'll write all the partial products from greatest to least.**

	(Rewrite partial products.)
Teacher	**So, what's __ plus __ plus __ plus __?** (For assistance with the partial sums algorithm for addition, see Module 5.)
Students	__.
Teacher	**__ is our final product. Let's write __ under the equal line.**
Students	(Write product.)
Teacher	**That means __ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Let's say it together again.**
Students	__ times __ equals __.
Teacher	**So, if you have __ groups and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to use an array?**
Students	We determined the expanded form for each factor. Then, we multiplied each factor for a partial product. Finally, we added the partial products to determine the final product.

Example



2-DIGIT × 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem using the array model. We'll create an array or rectangular area with our multiplication problem. The array model is similar to the partial products model. Let's get started. First, I have to draw rectangular array. What do I have to draw?**

Students Rectangular array.

Teacher My array includes the place value of each factor. How many digits in the first factor?

Students 2.

Teacher So, that's a 2-digit factor. How many digits in the second factor?

Students 2.

Teacher So, that's a 2-digit factor. Our array should have 2 columns for the first factor and 2 rows for the second factor. Let's draw an array with 2 columns and 2 rows.

(Draw array.)

Teacher Now, I write the first factor in expanded form. What does expanded form mean?

Students To write the number in tens and ones.

Teacher How many tens are in the first factor?

Students 1.

Teacher 1 ten is the same as 10. So, the expanded form of 13 would be 10 plus 3. Let's write 10 and 3 above the columns.

(Write first factor in expanded form.)

Teacher Now, I write the second factor in expanded form on the right side of the array. What does expanded form mean?

Students To write the number in tens and ones.

Teacher How many tens are in the second factor?

Students 4.

Teacher 4 tens is the same as 40. So, the expanded form of 45 would be 40 plus 5. Let's write 40 and 5 next to the row on the right side.

(Write second factor in expanded form.)

Teacher Now that we have set up the problem, let's multiply. I like to multiply the second factor times the first factor but any order is okay – the commutative property helps us with this! Let's multiply 40 times 10. What's 40 times 10?

Students 400.

Teacher 40 times 10 equals 400. Let's write 400 in the part of the array in which the row and column meet.

(Write product.)

Teacher 400 is a partial product. Now, let's multiply the tens of the second factor times the ones of the first factor. What do we multiply?

Students 40 times 3.

Teacher What's 40 times 3?

Students 120.

Teacher Let's write 120 in the part of the array in which the row and column meet.

(Write product.)

Teacher Now, let's multiply the ones of the second factor times the tens of the first factor and ones of the first factor. Let's do that now. What are the ones of the second factor?

Students 5.

Teacher	**We have 5 ones in the second factor. Look at the first factor. What are the tens of the first factor?**
Students	1.
Teacher	**We have 1 ten in the first factor. 1 ten is the same as what?**
Students	10.
	So, let's multiply 5 times 10. What's 5 times 10?
Students	50.
Teacher	**5 times 10 equals 50. Let's write 50 in the part of the array in which the row and column meet.** (Write product.)
Teacher	**Finally, let's multiply the ones of the second factor times the ones of the first factor. What do we multiply?**
Students	5 times 3.
Teacher	**What's 5 times 3?**
Students	15.
Teacher	**Let's write 15 in the part of the array in which the row and column meet.** (Write product.)
Teacher	**To determine the final product, we add all the partial products together. I'll write all the partial products from greatest to least.** (Rewrite to 400 + 120 + 50 + 15.)
Teacher	**Let's add this in steps. What's 400 plus 120?**
Students	520.
Teacher	**What's 520 plus 50?**
Students	570.
Teacher	**What's 570 plus 15?**
Students	585.
Teacher	**585 is our final product. Let's write 585 under the equal line.**
Students	(Write product.)
Teacher	**That means 13 times 45 equals 585. Let's say that together.**
Students	13 times 45 equals 585.
Teacher	**Let's say it together again.**
Students	13 times 45 equals 585.
Teacher	**So, if you have 13 and multiply by 45, the product is 585. 13 times 45 equals 585. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to use an array?**
Students	We determined the expanded form for each factor. Then, we multiplied each factor for a partial product. Finally, we added the partial products to determine the final product.

D. Problems for Use During Instruction

See Module 11 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 11 Vocabulary Cards.

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Katherine A. Berry (kberry@austin.utexas.edu)

Module 11:
Multiplication of Whole Numbers

Problem Sets

A.

$$\begin{array}{r} 49 \\ \times \quad 3 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 76 \\ \times \quad 4 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 25 \\ \times \quad 2 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 33 \\ \times \quad 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 18 \\ \times \quad 7 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 54 \\ \times \quad 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 69 \\ \times \quad 3 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 42 \\ \times \quad 5 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 84 \\ \times \quad 2 \\ \hline \end{array}$$

A.

$$
\begin{array}{r}
75 \\
\times \quad 7 \\
\hline
\end{array}
$$

A.

$$\begin{array}{r} 56 \\ \times \quad 3 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 92 \\ \times \quad 6 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 41 \\ \times \quad 9 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 72 \\ \times \quad 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 67 \\ \times \quad 9 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 33 \\ \times \quad 7 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 48 \\ \times \quad 5 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 56 \\ \times \quad 6 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 30 \\ \times \quad 2 \\ \hline \end{array}$$

A.

$$
\begin{array}{r} 86 \\ \times \quad 5 \\ \hline \end{array}
$$

A.

$$\begin{array}{r} 34 \\ \times \quad 7 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 60 \\ \times \quad 5 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 53 \\ \times \quad 4 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 90 \\ \times \quad 8 \\ \hline \end{array}$$

A.

$$
\begin{array}{r} 83 \\ \times \quad 4 \\ \hline \end{array}
$$

A.

$$\begin{array}{r} 21 \\ \times \quad 9 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 10 \\ \times \quad 3 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 11 \\ \times \quad 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 27 \\ \times \quad 3 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 42 \\ \times \quad 8 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 44 \\ \times\ 46 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 72 \\ \times\ 19 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 48 \\ \times\ 49 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 13 \\ \times\ 90 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 46 \\ \times\ 16 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 61 \\ \times\ 10 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 25 \\ \times\ 55 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 41 \\ \times\ 63 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 97 \\ \times\ 42 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 36 \\ \times\ 56 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 48 \\ \times\ 15 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 77 \\ \times\ 88 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 84 \\ \times\, 84 \\ \hline \end{array}$$

B.

$$
\begin{array}{r}
59 \\
\times\ 18 \\
\hline
\end{array}
$$

B.

$$\begin{array}{r} 28 \\ \times\ 25 \\ \hline \end{array}$$

B.

$$
\begin{array}{r}
81 \\
\times\ 30 \\
\hline
\end{array}
$$

B.

$$\begin{array}{r} 14 \\ \times\ 57 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 68 \\ \times\ 41 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 34 \\ \times\ 45 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 99 \\ \times\ 92 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 78 \\ \times\ 43 \\ \hline \end{array}$$

B.

$$
\begin{array}{r}
94 \\
\times\ 12 \\
\hline
\end{array}
$$

B.

$$\begin{array}{r} 14 \\ \times\ 33 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 56 \\ \times\ 77 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 84 \\ \times\ 24 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 16 \\ \times\ 51 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 65 \\ \times\ 12 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 89 \\ \times\ 47 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 22 \\ \times\ 88 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 76 \\ \times\ 20 \\ \hline \end{array}$$

c.

$$
\begin{array}{r}
529 \\
\times \quad 65 \\
\hline
\end{array}
$$

c.

$$\begin{array}{r} 273 \\ \times \quad 86 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 300 \\ \times \quad 73 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 101 \\ \times \quad 67 \\ \hline \end{array}$$

c.

$$
\begin{array}{r} 904 \\ \times \quad 51 \\ \hline \end{array}
$$

c.

$$\begin{array}{r} 616 \\ \times \quad 41 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 504 \\ \times \quad 88 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 187 \\ \times \quad 59 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 720 \\ \times \quad 89 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 860 \\ \times \quad 22 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 749 \\ \times \quad 15 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 295 \\ \times \quad 35 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 257 \\ \times \quad 21 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 193 \\ \times \quad 57 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 236 \\ \times \quad 98 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 399 \\ \times \quad 43 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 244 \\ \times \quad 14 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 660 \\ \times \quad 63 \\ \hline \end{array}$$

c.

$$
\begin{array}{r}
879 \\
\times \quad 62 \\
\hline
\end{array}
$$

c.

$$\begin{array}{r} 523 \\ \times \quad 12 \\ \hline \end{array}$$

Module 11:
Multiplication of Whole Numbers

Vocabulary Cards

algorithm
area
array
computation
commutative property
equal groups
equal sign
factor
hundreds column
multiply/multiplication
multiplication sign
ones column
partial products
product
regroup/trade/exchange
tens column

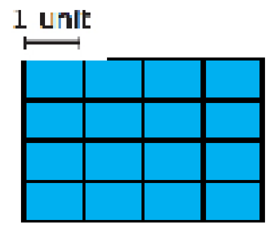
algorithm

A procedure or description of steps that can be used to solve a problem.

---

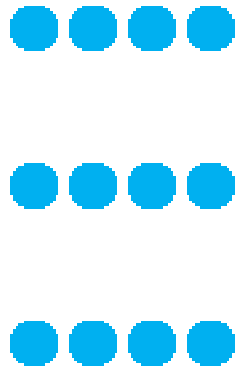
area

The number of square units that covers a closed figure.



array

A set of objects, pictures, (… **ed in columns and rows.**



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computation

The action used to solve a problem.

commutative property (of multiplication)

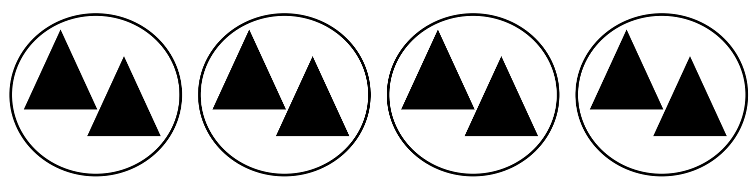
Two factors can be multiplied in any order.

$$2 \times 8 = 8 \times 2$$

---

equal groups

Groups with the same number of objects or items in each group.



equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$2 \times 8 = 16$$

= is the equal sign

---

factor

A number you multiply with another number to get the product.

$$2 \times 8 = 16$$

2 and 8 are the factors

hundreds column

The column with digits in the hundreds place.

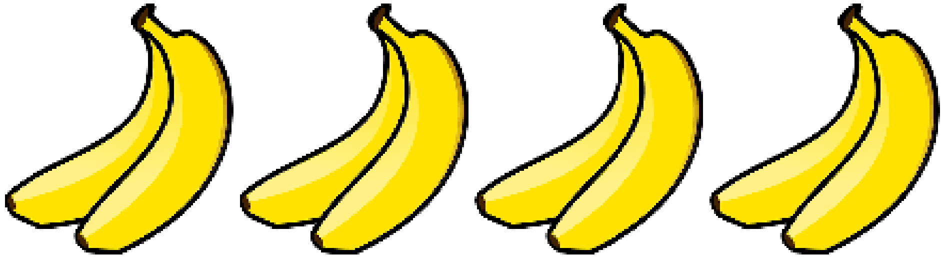
In the number 423, 4 is in the hundreds place.

---

multiply/multiplication

The process of adding a number to itself a number of times.

$$4 \times 2 = 8$$



multiplication sign

The symbol that tells you to multiply.

$$2 \times 8 = 16$$

× is the multiplication sign

---

ones column

The column with digits in the ones place.

In the number 423, 3 is in the ones place.

partial products

The product of parts of each factor.

$$
\begin{array}{rl}
13 & \\
\times\ 45 & \\
\hline
\mathbf{400} & (40 \times 10) \\
\mathbf{120} & (40 \times 3) \\
\mathbf{50} & (10 \times 5) \\
+\ \mathbf{15} & (5 \times 3) \\
\hline
585 &
\end{array}
$$

---

product

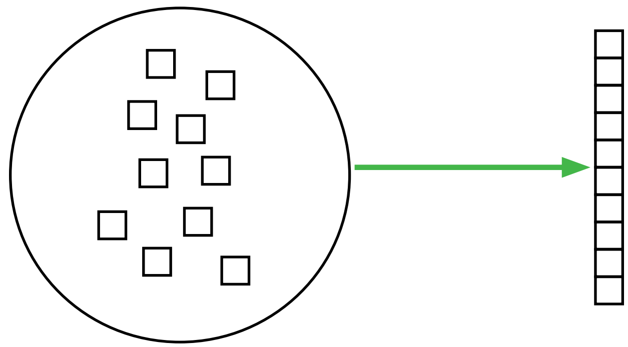
The result of multiplying two or more factors.

$$2 \times 8 = 16$$

16 is the product

regroup/trade/exchange

The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



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tens column

The column with digits in the tens place.

In the number 423, 2 is the in the tens column.

Instructional Routines for Mathematics Intervention

MODULE 12

Multiplication of Rational Numbers

Module 12: Multiplication of Rational Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A set of steps to solve a problem.
decimal	A number based on powers of ten.
denominator	The term in a fraction that tells the number of equal parts in a whole.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
factor	A number that you multiply with another number to get the product.
fraction	A number representing part of a whole or set.
hundredths	The digit in representing $\frac{1}{100}$.
improper fraction	Any fraction in which the numerator is greater than the denominator.
mixed number	A whole number and a fraction combined.
multiply/multiplication	The process of adding a number to itself a number of times.
multiplication sign	The symbol that tells you to multiply.
numerator	The term in a fraction that tells how many parts of a fraction.
ones	The digit representing 1.
partial products	The product of parts of each factor.
product	The result of multiplying two or more factors.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
tenths	The digit in representing $\frac{1}{10}$.

B. Background Information

Background Information:

In this module, we focus on multiplication with fractions and decimals. As you focus on computation of rational numbers, continue to emphasize multiplication as equal groups and multiplication as comparison because students will see these concepts within word problems.

For multiplication of fractions, we recommend using several models of fractions to help students understand concepts related to multiplication of fractions. We also recommend demonstrating several algorithms for multiplication of decimals. Every student should develop efficiency with strategies for multiplication of fractions and decimals. In the following sections, we provide examples of (1) multiplication of fractions, (2) multiplication of decimals with the traditional algorithm, and (3) multiplication of decimals with the partial products algorithm.

C. Routines and Examples

(1) Multiplication of Fractions*

*Most students know the *procedure* for multiply decimals but do not have *conceptual* understanding of multiplication of fractions. Here, we provide two conceptual **Routines** (one with manipulatives and one with drawings) as well as a procedural **Routine**. Our **Example** is conceptual and uses manipulatives. Consider reading the **Example** before reading the **Routines**.

Routine

Materials:

- Module 12 Problem Sets
- Module 12 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
    - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

(Only use manipulatives with simpler problems)

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.

Teacher	**Let's do this problem with fraction tiles.** (Move fraction tiles to workspace.)
Teacher	**With multiplication of fractions, we interpret this problem as __** (first fraction) **of __** (second fraction). **How do we interpret this problem?**
Students	__ of __.
Teacher	**We want to determine __** (first fraction) **of __** (second fraction)**. If you wanted to determine half of 8, you would show 8 and then find half of that amount. The same works with fractions. We'll show the second fraction (or factor) and then find the first fraction of the second fraction. Which fraction will we show?**
Students	Second fraction.
Teacher	**So, let's show the second fraction with the fraction tiles.** (Show second fraction with fraction tiles.)
Teacher	**Now, let's find __** (first fraction) **of __** (second fraction)**. There are several ways to do this, but an easy way is to find __** (first fraction) **of each one-__** (second fraction denominator) **part. Let's focus on one-__ part at a time. What should we focus on?**
Students	One-__ part.
Teacher	**Let's just think about this one-__ part** (second fraction denominator). **What's __** (first fraction) **of this part?**
Students	__.
Teacher	**If that's hard to answer, think about it this way. What's __** (first fraction) **times one-__** (second fraction denominator)**?**
Students	__.
Teacher	**__** (first fraction) **of this one-__ part** (second fraction denominator) **would be __. Let's place that/those fraction tiles on top of the one-__ part.** (Place fraction tiles.)
Teacher	**Now, I do that again for each one-__ part. I find __** (first fraction) **of each one-__ part.** (Place fraction tiles.)
Teacher	**We're multiplying by finding __** (first fraction) **of each of the one-__ parts. How are we multiplying?**
Students	Finding __ (first fraction) of each of the one-__ parts.
Teacher	**We've determined __** (first fraction) **of each of the one-__ parts with the fraction tiles, these are our partial products. What are these?**
Students	Partial products.
Teacher	**Let's add the partial products to determine the final product. What should we add?**
Students	The partial products.
Teacher	**We have __ plus __ plus … . That equals __. Say that with me.**
Students	__.
Teacher	**So, __** (first fraction) **of __** (second fraction) **equals __. What's the product?**

Students	__.
Teacher	**__ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**So, if you have a set of __** (second fraction) **and you determine __** (first fraction) **of the second fraction, __ times __ equals __. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**How could you explain multiplying of fractions to a friend?**
Students	We used fraction tiles to show the second fraction. Then, we found the first fraction of the second fraction to determine the product.

ROUTINE WITH DRAWING

(Only use drawings with simpler problems)

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem by drawing. What could we use to draw?**
Students	Pencil and graph paper.
Teacher	**I like to use a pencil and graph paper when I draw fractions. Good idea. Now, with multiplication of fractions, we interpret this problem as __** (first fraction) **of __** (second fraction). **How do we interpret this problem?**
Students	__ of __.
Teacher	**We want to determine __** (first fraction) **of __** (second fraction)**. For example, if you wanted to determine one-third of 12, you would show 12 cupcakes and then find one-third of the 12 cupcakes. The same works with fractions. We'll show the second fraction (or factor) and then find the first fraction of the second fraction. Which fraction will we show?**
Students	Second fraction.
Teacher	**So, let's draw the second fraction. Today, I'll draw a rectangle, divide that rectangle into __ equal parts** (denominator from second fraction) **and shade in __ parts** (numerator from the second fraction)**.** (Draw fraction and shade with pencil.)

Teacher	**Now, let's find __** (first fraction) **of __** (second fraction)**. There are several ways to do this, but an easy way is to find __** (first fraction) **of each one-__** (second fraction denominator) **part. Let's focus on one-__ part at a time. What should we focus on?**
Students	One-__ part at a time.
Teacher	**Let's just think about this one-__ part** (second fraction denominator)**. The first fraction has a denominator of __. What's the denominator?**
Students	__.
Teacher	**Let's divide this one-__ part** (second fraction denominator) **into __ equal parts** (first fraction denominator) **by drawing.** (By drawing, mark __ equal parts.)
Teacher	**Now, what's the numerator of the first fraction?**
Students	__.
Teacher	**__** (first fraction numerator) **of this one-__ part** (second fraction denominator) **would be __. Let's shade – with a highlighter or colored pencil – the __ parts of the first fraction.** (Highlight or color __ equal parts.)
Teacher	**Now, I do that again for each one-__ part. I draw and highlight or color __** (first fraction) **of each one-__ part** (second fraction denominator)**.** (Mark equal parts and highlight or color __ equal parts.)
Teacher	**Even though we only focused on determining __** (first fraction) **of __** (second fraction), **I want to divide any non-shaded parts of the second fraction into equal parts of the first fraction. This will help us learn of the denominator for the product.** (By drawing, mark __ equal parts.)
Teacher	**We're multiplying by finding __** (first fraction) **of each of the one-__ parts** (second fraction denominator)**. How are we multiplying?**
Students	Finding __ (first fraction) of each of the one-__ parts.
Teacher	**We've determined __** (first fraction) **of each of the one-__ parts by highlighting or coloring. Those are our partial products. Now, let's add the partial products to determine the final product. What should be add?**
Students	The partial products.
Teacher	**We have __ plus __ plus … . That equals __. Say that with me.**
Students	__.
Teacher	**So, __** (first fraction) **of __** (second fraction) **equals __. What's the product?**
Students	__.
Teacher	**__ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**So, if you have a set of __** (second fraction) **and you determine __** (first fraction) **of the second fraction, __ times __ equals __. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.

Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**How could you explain multiplying of fractions to a friend?**
Students	We drew the second fraction. Then, we highlighted the first fraction of each of the one-__ parts. Those were our partial products. We added the partial products to determine the product of __ and __.

ROUTINE WITHOUT MANIPULATIVES OR DRAWINGS

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**When we multiply, we multiply the numerators then we multiply the denominators. How do we multiply?**
Students	Multiply the numerators then multiply the denominators.
Teacher	**Let's focus on the numerators. What are the numerators in this problem?**
Students	__ and __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**__ times __ equals __, so let's write __ as the numerator of our product.** (Write numerator.)
Teacher	**Let's focus on the denominators. What are the denominators in this problem?**
Students	__ and __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**__ times __ equals __, so let's write __ as the denominator of our product.** (Write denominator.)
Teacher	**So, __** (first fraction) **of __** (second fraction) **equals __. What's the product?**
Students	__. (If product is not in simplest form, use greatest common factor to determine an equivalent fraction in simplest form.)
Teacher	**__ times __ equals __. Let's say that together.**
Students	__ times __ equals __.

Teacher — **So, if you have a set of __** (second fraction) **and you determine __** (first fraction) **of the second fraction, __ times __ equals __. Let's review. What's a factor?**

Students — The numbers multiplied in a multiplication problem.

Teacher — **What's a product?**

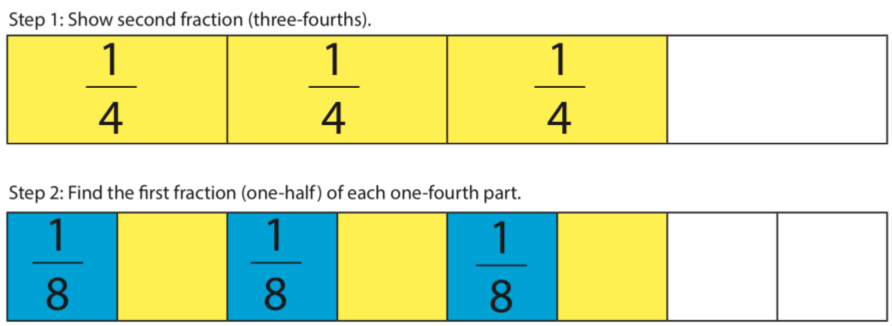
Students — The result of multiplying factors.

Teacher — **How could you explain multiplying of fractions to a friend?**

Students — We multiplied the numerators. Then, we multiplied the denominators. The product was __.

Example

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$



EXAMPLE WITH MANIPULATIVES

Teacher — **Let's work on multiplication. What does it mean to multiply?**

Students — To make equal groups or to compare.

Teacher — **Multiplication means to make equal groups or to compare. Look at this problem.**
(Show problem.)

Teacher — **First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**

Students — To multiply.

Teacher — **Let's do this problem with fraction tiles.**
(Move fraction tiles to workspace.)

Teacher — **With multiplication of fractions, we interpret this problem as $\frac{1}{2}$ of $\frac{3}{4}$. How do we interpret this problem?**

Students — $\frac{1}{2}$ of $\frac{3}{4}$.

Teacher — **Because we want to determine one-half of three-fourths, we show $\frac{3}{4}$. What fraction do we show?**
(Show 3 one-fourth parts compared to a whole.)

Students — $\frac{3}{4}$.

Teacher	**Now, let's find $\frac{1}{2}$ of $\frac{3}{4}$. I could do the multiplication by multiplying $\frac{1}{2}$ of $\frac{1}{4}$ to find $\frac{1}{2}$ of each $\frac{1}{4}$ part. Let's see. If I have $\frac{1}{4}$, what's $\frac{1}{2}$ of $\frac{1}{4}$?**
Students	$\frac{1}{8}$.
Teacher	**Yes, if I divide a $\frac{1}{4}$ part in half, that would be $\frac{1}{8}$. I'll place one $\frac{1}{8}$ piece on top of the $\frac{1}{4}$ part.** (Place one $\frac{1}{8}$ piece on a $\frac{1}{4}$ part.)
Teacher	**Now, I do that again for each $\frac{1}{4}$ part. I find $\frac{1}{2}$ of each $\frac{1}{4}$ part.** (Place one $\frac{1}{8}$ piece on each $\frac{1}{4}$ part.)
Teacher	**We're multiplying by finding $\frac{1}{2}$ of each of the three $\frac{1}{4}$ parts or $\frac{3}{4}$. How are we multiplying?**
Students	Finding $\frac{1}{2}$ of each of the three $\frac{1}{4}$ parts.
Teacher	**Now that we've determined $\frac{1}{2}$ of each $\frac{1}{4}$ part, let's add the $\frac{1}{8}$ pieces to determine the product. What should we add?**
Students	The $\frac{1}{8}$ pieces.
Teacher	**We have $\frac{1}{8}$ plus $\frac{1}{8}$ plus $\frac{1}{8}$. That's $\frac{3}{8}$. Say that with me.**
Students	$\frac{3}{8}$.
Teacher	**So, $\frac{1}{2}$ of $\frac{3}{4}$ equals $\frac{3}{8}$. What's the product?**
Students	$\frac{3}{8}$.
Teacher	**$\frac{1}{2}$ times $\frac{3}{4}$ equals $\frac{3}{8}$. Let's say that together.**
Students	$\frac{1}{2}$ times $\frac{3}{4}$ equals $\frac{3}{8}$.
Teacher	**So, if you have a set of $\frac{3}{4}$ and you find $\frac{1}{2}$ of the three-fourths, $\frac{1}{2}$ of $\frac{3}{4}$ equals $\frac{3}{8}$. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**How could you explain multiplying of fractions to a friend?**
Students	We used fraction tiles to show the second fraction. Then, we found the first fraction of the second fraction to determine the product.

(2) Multiplication of Decimals with Traditional Algorithm

Routine

Materials:

- Module 12 Problem Sets
- Module 12 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching

2-DIGIT × 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones and the tenths. Let's draw a vertical line between the ones column and the tenths column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by multiplying the tenths of the second factor. This means we'll write these products starting in the tenths column below the equal line. Where will we write the products?**
Students	Below the equal line in the tenths.
Teacher	**We first multiply the tenths of the second factor times the tenths of the first factor. What should we multiply first?**
Students	The tenths of the second factor times the tenths of the first factor.
Teacher	**Which tenths do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)
Students	__.
Teacher	**__ times __ equals __. Let's write __ below the equal line in the tenths column.** IF REGROUPING: **Our product is greater than 9, so we have to regroup. That means we write the ones in the tenths place and regroup the tens above the ones column.** (Write product.)

Teacher	**Now, we multiply the tenths of the second factor times the ones of the first factor. What do we multiply?**	
Students	The tenths of the second factor times the ones of the first factor.	
Teacher	**So, what do we multiply?**	
Students	__ times __.	
Teacher	**What's __ times __?**	
Students	__.	
Teacher	IF REGROUPING:	**Remember, we regrouped __ from when we multiplied the tenths of the second factor by the tenths of the first factor. Now, we add that regrouped amount to our product of __ times __. So, what's __ plus __?**
Students		__.
Teacher	**Let's write __ below the equal line in the ones column.** (Write product.)	
Teacher	**So, we multiplied the tenths of the second factor times the tenths of the first factor then the tenths of the second factor times the ones of the first factor. Who can describe what we multiplied so far?**	
Students	We multiplied the tenths of the second factor times the tenths of the first factor then the tenths of the second factor times the ones of the first factor.	
Teacher	**We've multiplied the tenths of the second factor. Now, it's time to multiply the ones of the second factor. What will we multiply now?**	
Students	The ones of the second factor.	
Teacher	**When writing the products of multiplying the ones of the second factor, we'll write them below this first line of products. Because we're now multiplying the ones, we will write our products starting in the ones column. I like to place an X or zero in the tenths column below the equal line to remember to start writing my products in the ones column.** (Write X or 0.)	
Teacher	**Now, let's multiply the ones of the second factor times the tenths of the first factor. What should we multiply?**	
Students	The ones of the second factor times the tenths of the first factor.	
Teacher	**What numbers do we multiply?**	
Students	__ times __.	
Teacher	**What's __ times __?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)	
Students	__.	
Teacher	**__ times __ equals __. Let's write __ below the equal line in the ones column.**	
	IF REGROUPING:	**Our product is greater than 9, so we have to regroup. That means we write the ones and regroup the tens above the problem.**
	(Write product.)	
Teacher	**Now, we multiply the ones of the second factor times the ones of the first factor. What do we multiply?**	

Students The ones of the second factor times the ones of the first factor.

Teacher So, what do we multiply?

Students __ times __.

Teacher What's __ times __?

Students __.

Teacher IF REGROUPING: **Remember, we regrouped __ from when we multiplied the ones of the second factor by the tenths of the first factor. Now, we add that regrouped amount to our product of __ times __. So, what's __ plus __?**

Students __.

Teacher Let's write __ below the equal line.
(Write product.)

Teacher So, we multiplied the ones of the second factor times the tenths of the first factor and then the ones of the second factor times the ones of the first factor. Who can describe what we multiplied?

Students We multiplied the ones of the second factor times the tenths of the first factor then the ones of the second factor times the ones of the first factor.

Teacher Now, we did all the multiplication but we are not finished! We call these numbers here (point to numbers under equal line) **our partial products. We have to add the partial products together to determine the final product. Let's draw another equal line and write in a plus sign. What should we draw?**

Students An equal line and plus sign.
(Write equal line and plus sign.)

Teacher So, let's add __ plus __. What's __ plus __? (If students need help with addition of whole numbers, see Module 5.)

Students __.

Teacher Yes. So, I write __ under the equal line.
(Write final product.)

Teacher Now, we seem finished but we're not. In this problem, we multiplied decimals. So, we have to place the decimal point in the product. What do we have to place in the product?

Students A decimal point.

Teacher To place the decimal point, we determine the number of decimal places in the two factors. Let's see. The first factor had __ decimal place. The second factor also had __ decimal place. What's __ plus __?

Students __.

Teacher So, in the product, we need to put in __ decimal places starting from the least place value (or the right). That means I'll place a decimal point between the __ and __.
(Write decimal point.)

Teacher So, __ times __ equals __. What's the product?

Students __.

Teacher Let's say it together again.

Students __ times __ equals __.

Teacher	**So, if you have __ and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	One of the numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to make equal groups?**
Students	To have groups with an equal number in each group.
Teacher	**How could you explain multiplication of double-digit numbers to a friend?**
Students	We multiplied the tenths of the second factor times the tenths and ones of the first factor. Then, we multiplied the ones of the second factor times the tenths and ones of the first factor. Finally, we added the partial products to determine the final product. We multiplied two decimal places so we added in a decimal point two decimal places from the right of the number.

Example

$$\begin{array}{r} 7.3 \\ \times \quad 6.1 \\ \hline 44.53 \end{array}$$

2-DIGIT × 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones and the tenths. Let's draw a vertical line between the ones column and the tenths column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by multiplying the tenths of the second factor. This means we'll write these products starting in the tenths column below the equal line. Where will we write the products?**
Students	Below the equal line of the tenths.
Teacher	**We first multiply the tenths of the second factor times the tenths of the first factor. What should we multiply first?**
Students	The tenths of the second factor times the tenths of the first factor.
Teacher	**Which tenths do we multiply?**
Students	1 times 3.

Teacher	**What's 1 times 3?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)
Students	3.
Teacher	**1 times 3 equals 3. Let's write 3 below the equal line in the tenths column.** (Write 3.)
Teacher	**Now, we multiply the tenths of the second factor times the ones of the first factor. What do we multiply?**
Students	The tenths of the second factor times the ones of the first factor.
Teacher	**So, what do we multiply?**
Students	1 times 7.
Teacher	**What's 1 times 7?**
Students	7.
Teacher	**Let's write 7 below the equal line in the ones column.** (Write product.)
Teacher	**So, we multiplied the tenths of the second factor times the tenths of the first factor then the tenths of the second factor times the ones of the first factor. Who can describe what we multiplied so far?**
Students	We multiplied the tenths of the second factor times the tenths of the first factor then we multiplied the tenths of the second factor times the ones of the first factor.
Teacher	**We've multiplied the tenths of the second factor. Now, it's time to multiply the ones of the second factor. What will we multiply now?**
Students	The ones of the second factor.
Teacher	**When writing the products of multiplying the ones of the second factor, we'll write them below this first line of products. Because we're now multiplying the ones, we will write our products starting in the ones column. I like to place an X or zero in the tenths column below the equal line to remember to start writing my products in the ones column.** (Write X or 0.)
Teacher	**Now, let's multiply the ones of the second factor times the tenths of the first factor. What should we multiply?**
Students	The ones of the second factor times the tenths of the first factor.
Teacher	**What numbers do we multiply?**
Students	6 times 3.
Teacher	**What's 6 times 3?** (If a student has difficulty with multiplication, use a multiplication table or other resource.)
Students	18.
Teacher	**6 times 3 equals 18. Let's write 8 below the equal line in the ones column and regroup the 1.** (Write 8 and regroup the 1.)
Teacher	**Now, we multiply the ones of the second factor times the ones of the first factor. What do we multiply?**

Students	The ones of the second factor times the ones of the first factor.
Teacher	**So, what do we multiply?**
Students	6 times 7.
Teacher	**What's 6 times 7?**
Students	42.
	Remember, we regrouped 1 from when we multiplied the ones of the second factor by the tenths of the first factor. Now, we add that regrouped amount to our product of 42. So, what's 42 plus 1?
Students	43.
Teacher	**Let's write 43 below the equal line.** (Write 43.)
Teacher	**So, we multiplied the ones of the second factor times the tenths of the first factor and then the ones of the second factor times the ones of the first factor. Who can describe what we multiplied?**
Students	We multiplied the ones of the second factor times the tenths of the first factor then we multiplied the ones of the second factor times the ones of the first factor.
Teacher	**Now, we did all the multiplication but we are not finished! We call these numbers here** (point to numbers under equal line) **our partial products. We have to add the partial products together to determine the final product. Let's draw another equal line and write in a plus sign. What should we draw?**
Students	An equal line and plus sign. (Write equal line and plus sign.)
Teacher	**So, let's add 73 plus 4380. What's 73 plus 4380?** (If students need help with addition of whole numbers, see Module 5.)
Students	4453.
Teacher	**Yes. So, I write 4453 under the equal line.** (Write 4453.)
Teacher	**Now, are we finished?**
Students	No!
Teacher	**We seem finished but we're not. In this problem, we multiplied decimals. So, we have to place the decimal point in the product. What do we have to place in the product?**
Students	A decimal point.
Teacher	**To place the decimal point, we determine the number of decimal places in the two factors. Let's see. The first factor had 1 decimal place. The second factor also had 1 decimal place. What's 1 plus 1?**
Students	2.
Teacher	**So, in the product, we need to put in 2 decimal places starting from the right of the number. That means I'll place a decimal point between the 4 and 5.** (Write decimal point.)
Teacher	**So, 7.3 times 6.1 equals 44.53. Let's say that together.**
Students	7.3 times 6.1 equals 44.53.
Teacher	**Let's say it together again.**

Students	7.3 times 6.1 equals 44.53.
Teacher	**So, if you have 7.3 and multiply by 6.1, the product is 44.53. Let's review. What's a factor?**
Students	One of the numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to make equal groups?**
Students	To have groups with an equal number in each group.
Teacher	**How could you explain multiplication of double-digit numbers to a friend?**
Students	We multiplied the tenths of the second factor times the tenths and ones of the first factor. Then, we multiplied the ones of the second factor times the tenths and ones of the first factor. Finally, we added the partial products to determine the final product. We placed in the decimal point because we multiplied by two decimals.

(3) Multiplication with Partial Products Algorithm*

*For clarity, read **Example** before using **Routines**.

Routine

Materials:

- Module 12 Problem Sets
- Module 12 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching

2-DIGIT × 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tenths. Let's draw a vertical line between the ones column and the tenths column.** (Draw vertical lines to separate place value columns.)
Teacher	**Let's use the partial products strategy. What strategy?**
Students	Partial products.
Teacher	**With the partial products strategy, we do the multiplication for each factor then we add the partial products together for a final product. With the partial products strategy, we work from the greatest place value to the least place value. How do we work?**
Students	From the greatest place value to the least place value.
Teacher	**First, we'll multiply the ones of the second factor times the ones of the first factor and tenths of the first factor. Let's do that now. What are the ones of the second factor?**
Students	__.
Teacher	**We have __ ones in the second factor. How many ones?**
Students	__.
Teacher	**Look at the first factor. What are the ones of the first factor?**
Students	__.
Teacher	**We have __ ones in the first factor. How many ones?**
Students	__.
	So, let's multiply __ times __. What's __ times __?
Students	__.

Teacher	**__ times __ equals __. Let's write __ below the equal line and make sure to line up by place value.** (Write product.)
Teacher	**__ is our first partial product. Now, let's multiply the ones of the second factor times the tenths of the first factor? What do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**Let's write __ below the equal line. We'll write this second partial product under the first partial product and make sure to line up by place value. That is – line up tens with tens, ones with ones, tenths with tenths, and hundredths with hundredths.** (Write product.)
Teacher	**Now, let's multiply the tenths of the second factor times the ones of the first factor and tenths of the first factor. Let's do that now. What are the tenths of the second factor?**
Students	__.
Teacher	**We have __ tenths in the second factor. Look at the first factor. What are the ones of the first factor?**
Students	__.
Teacher	**We have __ ones in the first factor. How many ones?**
Students	__.
	So, let's multiply __ times __. What's __ times __?
Students	__.
Teacher	**__ times __ equals __. Let's write __ below the equal line under our other partial products and make sure to line up by place value.** (Write product.)
Teacher	**Finally, let's multiply the tenths of the second factor times the tenths of the first factor. What do we multiply?**
Students	__ times __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**Let's write __ below the equal line under our other partial products and make sure to line up by place value.** (Write product.)
Teacher	**To determine the final product, we add all the partial products together. I'll write a plus sign and another equal line.** (Write plus sign and equal line.)
Teacher	**So, what's __ plus __ plus __ plus __?** (For assistance with the partial sums algorithm for addition, see Module 5.)
Students	__.
Teacher	**__ is our final product. Let's write __ under the equal line.**
Students	(Write product.)
Teacher	**That means __ times __ equals __. Let's say that together.**

Students	__ times __ equals __.
Teacher	**Let's say it together again.**
Students	__ times __ equals __.
Teacher	**So, if you have __ groups and multiply by __, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**How could you explain multiplying to a friend?**
Students	We multiplied the ones of the second factor times the ones and tenths of the first factor. Then, we multiplied the tenths of the second factor times the ones and tenths of the first factor. We added the partial products to determine the final product.

Example

```
    7.3
  × 6.1
   42
    1.8
    0.7
+   0.03
   44.53
```

2-DIGIT × 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tenths. Let's draw a vertical line between the ones column and the tenths column.** (Draw vertical lines to separate place value columns.)
Teacher	**Let's use the partial products strategy. What strategy?**
Students	Partial products.
Teacher	**With the partial products strategy, we do the multiplication for each factor then we add the partial products together for a final product. With the partial**

	products strategy, we work from the greatest place value to the least place value. How do we work?
Students	From the greatest place value to the least place value.
Teacher	**First, we'll multiply the ones of the second factor times the ones of the first factor and tenths of the first factor. Let's do that now. What are the ones of the second factor?**
Students	6.
Teacher	**We have 6 ones in the second factor. How many ones?**
Students	6.
Teacher	**Look at the first factor. What are the ones of the first factor?**
Students	7.
Teacher	**We have 7 ones in the first factor. How many ones?**
Students	7.
	So, let's multiply 6 times 7. What's 6 times 7?
Students	42.
Teacher	**6 times 7 equals 42. Let's write 42 below the equal line and make sure to place the 2 in the ones column and 4 in the tens column.** (Write 42.)
Teacher	**42 is our first partial product. Now, let's multiply the ones of the second factor times the tenths of the first factor. What do we multiply?**
Students	6 times 0.3.
Teacher	**What's 6 times 0.3?**
Students	1.8.
Teacher	**Let's write 1.8 below the equal line. We'll write this partial product under the first partial product. We'll write the 1 in the ones column and 0.8 in the tenths column.** (Write 1.8.)
Teacher	**Now, let's multiply the tenths of the second factor times the ones of the first factor and tenths of the first factor. Let's do that now. What are the tenths of the second factor?**
Students	0.1.
Teacher	**We have 0.1 in the second factor. Look at the first factor. What are the ones of the first factor?**
Students	7.
	So, let's multiply 0.1 times 7. What's 0.1 times 7?
Students	0.7.
Teacher	**0.1 times 7 equals 0.7. Let's write 0.7 below the equal line under our other partial products. Let's make sure to write the 7 in the tenths column.** (Write 0.7.)
Teacher	**Finally, let's multiply the tenths of the second factor times the tenths of the first factor. What do we multiply?**
Students	0.1 times 0.3.
Teacher	**What's 0.1 times 0.3?**
Students	0.03.

Teacher	**Let's write 0.03 below the equal line under our other partial products. Let's make sure to write the 3 in the hundredths column.** (Write 0.03.)
Teacher	**To determine the final product, we add all the partial products together. I'll write a plus sign and another equal line.** (Write plus sign and equal line.)
Teacher	**I like to add in steps. What's 42 plus 1.8?**
Students	43.8.
Teacher	**What's 43.8 plus 0.7?**
Students	44.5.
Teacher	**What's 44.5 plus 0.03?**
Students	44.53.
Teacher	**44.53 is our final product. Let's write 44.53 under the equal line.**
Students	(Write product.)
Teacher	**That means 7.3 times 6.1 equals 44.53. Let's say that together.**
Students	7.3 times 6.1 equals 44.53.
Teacher	**So, if you have 7.3 and multiply by 6.1, the product is 44.53. Let' review. What's a factor?**
Students	The numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The result of multiplying factors.
Teacher	**What does it mean to use the partial products strategy?**
Students	We multiplied each factor for a partial product. Then, we added the partial products to determine the final product.
Teacher	**How could you explain multiplying to a friend?**
Students	We multiplied 6 times 7 then 6 times 0.3. Then, we multiplied 0.1 times 7 then 0.1 times 0.03. We added the partial products for a final product of 44.53.

D. Problems for Use During Instruction

See Module 12 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 12 Vocabulary Cards.

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Module 12:
Multiplication of Rational Numbers

Problem Sets

A. Proper fractions (30)
B. Improper fractions (20)
C. Mixed numbers (20)

D. Decimals with tenths (20)
E. Decimals with hundredths (20)
F. Decimals with tenths and hundredths (30)

A.

$$\frac{3}{4} \times \frac{2}{3} =$$

A.

$$\frac{2}{3} \times \frac{1}{2} =$$

A.

$$\frac{2}{5} \times \frac{2}{3} =$$

A.

$$\frac{1}{2} \times \frac{1}{10} =$$

A.

$$\frac{7}{10} \times \frac{2}{5} =$$

A.

$$\frac{5}{6} \times \frac{1}{2} =$$

A.

$$\frac{1}{8} \times \frac{1}{4} =$$

A.

$$\frac{5}{6} \times \frac{1}{3} =$$

A.

$$\frac{4}{5} \times \frac{2}{3} =$$

A.

$$\frac{3}{5} \times \frac{3}{12} =$$

A.

$$\frac{1}{2} \times \frac{9}{12} =$$

A.

$$\frac{4}{5} \times \frac{3}{8} =$$

A.

$$\frac{5}{8} \times \frac{2}{3} =$$

A. $\frac{3}{4} \times \frac{3}{4} =$

A.

$$\frac{1}{3} \times \frac{4}{5} =$$

A.

$$\frac{1}{2} \times \frac{1}{2} =$$

A.

$$\frac{2}{8} \times \frac{1}{4} =$$

A.

$$\frac{1}{3} \times \frac{9}{10} =$$

A.

$$\frac{2}{5} \times \frac{6}{8} =$$

A.

$$\frac{1}{4} \times \frac{2}{3} =$$

A.

$$\frac{3}{8} \times \frac{3}{4} =$$

A.

$$\frac{1}{5} \times \frac{3}{4} =$$

A.

$$\frac{1}{6} \times \frac{2}{3} =$$

A.

$$\frac{2}{4} \times \frac{1}{5} =$$

A. $\frac{2}{6} \times \frac{5}{10} =$

A.

$$\frac{4}{5} \times \frac{4}{5} =$$

A. $\frac{5}{6} \times \frac{3}{4} =$

A.

$$\frac{3}{8} \times \frac{5}{6} =$$

A.

$$\frac{1}{6} \times \frac{2}{5} =$$

A.

$$\frac{1}{3} \times \frac{4}{12} =$$

B. $\frac{1}{4} \times \frac{9}{6} =$

B.

$$\frac{1}{2} \times \frac{13}{2} =$$

B.

$$\frac{1}{3} \times \frac{5}{4} =$$

B.

$$\frac{2}{3} \times \frac{14}{12} =$$

B.

$$\frac{2}{8} \times \frac{4}{2} =$$

B.

$$\frac{3}{4} \times \frac{11}{6} =$$

B. $$\frac{1}{3} \times \frac{13}{7} =$$

B.

$$\frac{1}{6} \times \frac{7}{3} =$$

B.

$$\frac{4}{5} \times \frac{5}{3} =$$

B. $\frac{5}{6} \times \frac{5}{2} =$

B. $$\frac{1}{3} \times \frac{8}{6} =$$

B.

$$\frac{3}{4} \times \frac{5}{2} =$$

B.

$$\frac{2}{5} \times \frac{7}{6} =$$

B.

$$\frac{2}{4} \times \frac{9}{8} =$$

B. $$\frac{1}{8} \times \frac{7}{5} =$$

B.

$$\frac{2}{3} \times \frac{11}{2} =$$

B.

$$\frac{1}{3} \times \frac{8}{7} =$$

B.

$$\frac{1}{8} \times \frac{9}{4} =$$

B.

$$\frac{2}{6} \times \frac{10}{6} =$$

B.

$$\frac{1}{5} \times \frac{14}{5} =$$

c.

$$1\frac{1}{4} \times \frac{2}{5} =$$

c.

$$\frac{3}{5} \times 2\frac{4}{10} =$$

c.

$$2\frac{1}{8} \times \frac{1}{4} =$$

c.

$$\frac{2}{3} \times 1\frac{4}{5} =$$

c.

$$6\frac{3}{8} \times \frac{1}{3} =$$

c.

$$\frac{5}{6} \times 4\frac{1}{2} =$$

c.

$$4\frac{5}{8} \times \frac{1}{4} =$$

c.

$$\frac{3}{4} \times 6\frac{1}{4} =$$

c.

$$8\frac{3}{6} \times \frac{1}{8} =$$

c.

$$\frac{2}{6} \times 3\frac{1}{3} =$$

c.

$$9\frac{1}{5} \times \frac{3}{5} =$$

c.

$$\frac{3}{4} \times 7\frac{2}{4} =$$

c.

$$1\frac{1}{12} \times \frac{1}{2} =$$

c. $\frac{1}{8} \times 4\frac{1}{3} =$

c.

$$5\frac{2}{3} \times \frac{1}{12} =$$

c. $\frac{1}{10} \times 6\frac{5}{8} =$

c.

$$7\frac{4}{9} \times \frac{1}{3} =$$

c.

$$\frac{3}{9} \times 4\frac{6}{9} =$$

c.

$$4\frac{7}{8} \times \frac{1}{12} =$$

c.

$$\frac{3}{4} \times 7\frac{2}{3} =$$

D.

$$\begin{array}{r} 7.3 \\ \times \quad 6.1 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 0.2 \\ \times\ 1.8 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 4.4 \\ \times \quad 7.5 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 9.2 \\ \times\ 9.3 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 2.5 \\ \times \quad 0.4 \\ \hline \end{array}$$

D.

$$
\begin{array}{r} 8.9 \\ \times \quad 2.0 \\ \hline \end{array}
$$

D.

$$\begin{array}{r} 1.8 \\ \times\ 3.0 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 6.4 \\ \times \quad 8.8 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 0.3 \\ \times \quad 6.5 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 1.1 \\ \times \; 8.4 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 1.3 \\ \times \quad 4.3 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 6.4 \\ \times \quad 3.5 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 5.7 \\ \times \; 3.5 \\ \hline \end{array}$$

D.

$$
\begin{array}{r} 4.4 \\ \times \;\; 0.8 \\ \hline \end{array}
$$

D.

$$\begin{array}{r} 6.3 \\ \times \quad 8.2 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 1.2 \\ \times \quad 1.2 \\ \hline \end{array}$$

D.

$$
\begin{array}{r}
2.3 \\
\times \quad 4.8 \\
\hline
\end{array}
$$

D.

$$\begin{array}{r} 1.9 \\ \times \; 3.5 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 8.1 \\ \times\ 8.2 \\ \hline \end{array}$$

D.

$$
\begin{array}{r}
2.4 \\
\times \quad 7.0 \\
\hline
\end{array}
$$

E.

$$\begin{array}{r} 0.89 \\ \times \quad 0.93 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 0.54 \\ \times \quad 1.62 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 1.35 \\ \times \quad 2.71 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 4.43 \\ \times \quad 3.87 \\ \hline \end{array}$$

E.

$$
\begin{array}{r}
3.85 \\
\times \quad 0.88 \\
\hline
\end{array}
$$

E.

$$\begin{array}{r} 0.35 \\ \times \quad 0.77 \\ \hline \end{array}$$

E.

$$
\begin{array}{r}
1.93 \\
\times \quad 0.13 \\
\hline
\end{array}
$$

E.

$$\begin{array}{r} 5.63 \\ \times \quad 0.61 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 1.30 \\ \times \quad 3.57 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 0.12 \\ \times \quad 0.27 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 0.73 \\ \times \quad 0.49 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 1.92 \\ \times \quad 4.58 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 5.38 \\ \times \quad 2.24 \\ \hline \end{array}$$

E.

$$
\begin{array}{r}
6.89 \\
\times \quad 1.92 \\
\hline
\end{array}
$$

E.

$$
\begin{array}{r}
14.21 \\
\times \quad 0.53 \\
\hline
\end{array}
$$

E.

$$\begin{array}{r} 6.46 \\ \times \quad 4.11 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 0.33 \\ \times \quad 2.12 \\ \hline \end{array}$$

E.

$$
\begin{array}{r}
3.17 \\
\times \quad 1.34 \\
\hline
\end{array}
$$

E.

$$\begin{array}{r} 0.45 \\ \times \quad 0.54 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 11.27 \\ \times \quad 0.68 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 8.61 \\ \times \quad 7.9 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 6.95 \\ \times \quad 2.8 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 9.07 \\ \times \quad 6.6 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 2.25 \\ \times \quad 1.5 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 3.89 \\ \times \quad 4.3 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 5.61 \\ \times \quad 2.4 \\ \hline \end{array}$$

F.

$$
\begin{array}{r}
1.39 \\
\times \quad 6.7 \\
\hline
\end{array}
$$

F.

$$\begin{array}{r} 1.14 \\ \times \quad 2.0 \\ \hline \end{array}$$

F.

$$
\begin{array}{r}
2.78 \\
\times \quad 4.1 \\
\hline
\end{array}
$$

F.

$$\begin{array}{r} 17.98 \\ \times \quad 3.8 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 6.61 \\ \times \quad 8.2 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 9.62 \\ \times \quad 4.3 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 2.33 \\ \times \quad 5.3 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 6.98 \\ \times \quad 4.9 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 7.05 \\ \times \quad 8.8 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 2.30 \\ \times \quad 9.7 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 4.73 \\ \times \quad 8.6 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 11.03 \\ \times \quad 4.0 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 3.37 \\ \times \quad 1.4 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 2.88 \\ \times \quad 2.3 \\ \hline \end{array}$$

F.

$$
\begin{array}{r} 65.21 \\ \times \quad 8.4 \\ \hline \end{array}
$$

F.

$$\begin{array}{r} 7.91 \\ \times \quad 0.9 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 6.02 \\ \times \quad 5.1 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 13.15 \\ \times \quad 0.4 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 0.92 \\ \times \quad 0.5 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 0.38 \\ \times \quad 1.7 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 71.89 \\ \times \quad 0.2 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 1.35 \\ \times \quad 9.6 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 31.78 \\ \times \quad 0.9 \\ \hline \end{array}$$

F.

$$
\begin{array}{r}
9.16 \\
\times \quad 1.3 \\
\hline
\end{array}
$$

Module 12:
Multiplication of Rational Numbers

Vocabulary Cards

- algorithm
- decimal
- denominator
- equal groups
- equal sign
- equivalent
- factor
- fraction
- hundredths
- improper fraction
- least common multiple
- mixed number
- multiply/multiplication
- multiplication sign
- numerator
- ones
- partial products
- product
- regroup/trade/exchange
- tenths

algorithm

A set of steps to solve a problem.

---

decimal

A number based on powers of ten.

34.107

tens ones tenths hundredths thousandths

denominator

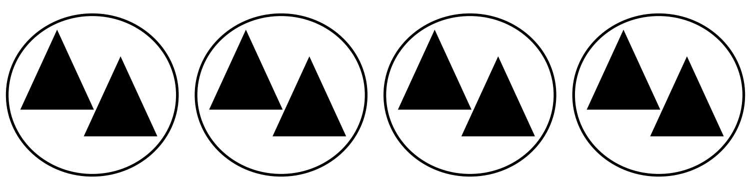
The term in a fraction that tells the number of equal parts in a whole.

$2 / 3$ $\frac{2}{3}$ **In these fractions, 3 is the denominator.**

---

equal groups

Groups with the same number of objects or items in each group.



equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$2 \times 8 = 16$$

= is the equal sign

---

equivalent

Two numbers that have the same value.

$$\frac{1}{4} = \frac{2}{8} \qquad \frac{2}{3} = \frac{8}{12}$$

factor

A number that you multiply with another number to get the product.

$$2 \times 8 = 16$$

2 and 8 are the factors

---

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \qquad \frac{10}{12} \qquad \frac{8}{3}$$

hundredths

The digit in representing $\frac{1}{100}$.

In the number 4.23, 3 is in the hundredths place.

---

improper fraction

Any fraction in which the numerator is greater than the denominator.

$$\frac{9}{4} \qquad \frac{17}{12} \qquad \frac{10}{3}$$

mixed number

A whole number and a fraction combined.

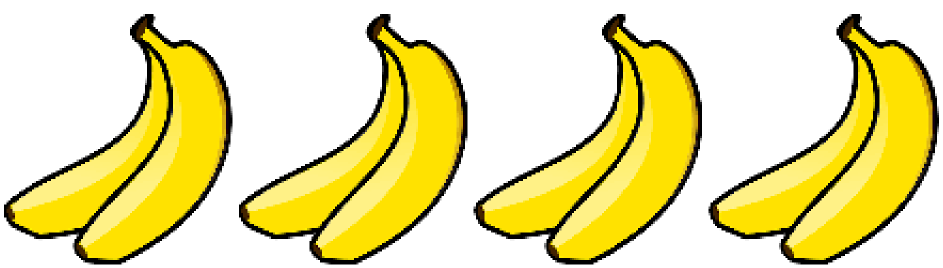
$$1\frac{1}{6} \qquad 4\frac{5}{12} \qquad 12\frac{4}{3}$$

---

multiply/multiplication

The process of adding a number to itself a number of times.

$$4 \times 2 = 8$$



multiplication sign

The symbol that tells you to multiply.

$$2 \times 8 = 16$$

× is the multiplication sign

---

numerator

The term in a fraction that tells how many parts of a fraction.

$2 / 3$ $\frac{2}{3}$ **In these fractions, 2 is the numerator.**

ones

The digit representing 1.

In the number 4.23, 4 is in the ones place.

---

partial products

The product of parts of each factor.

$$\begin{array}{r l} 13 & \\ \times\ 45 & \\ \hline \mathbf{400} & (40 \times 10) \\ \mathbf{120} & (40 \times 3) \\ \mathbf{50} & (10 \times 5) \\ +\ \mathbf{15} & (5 \times 3) \\ \hline 585 & \end{array}$$

product

The result of multiplying two or more factors.

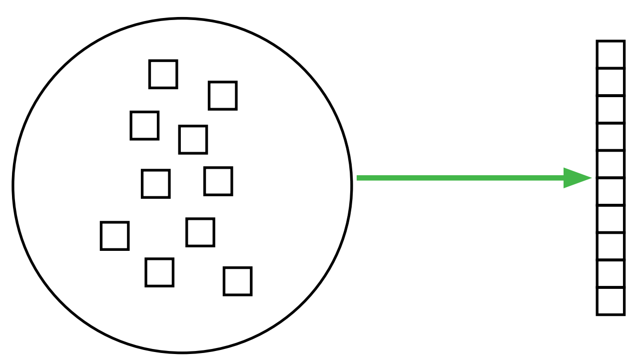
$$2 \times 8 = 16$$

16 is the product

---

regroup/trade/exchange

The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



tenths

The digit in representing $\frac{1}{10}$.

In the number 4.23, 2 is in the tenths place.

Instructional Routines for Mathematics Intervention

MODULE 13

Concepts of Division

Module 13: Concepts of Division

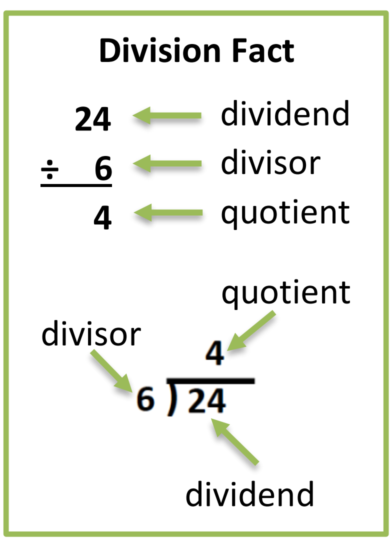
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
divide/division	To separate into equal groups or among groups.
dividend	The number to be divided.
division sign	The symbol that tells you to divide.
divisor	The number the dividend is divided by.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
partitive division	A way of dividing where you share items into a pre-determined number of groups.
quotative division	A way of dividing where you measure a pre-determined amount of items into an unknown number of groups.
quotient	The result when one number is divided by another number.

B. Background Information

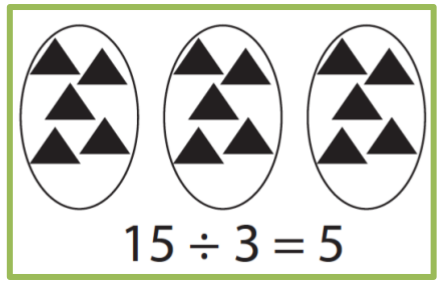
Students need to learn two concepts of division: (1) division as partitive and (2) division as measurement or quotative. Typically, students first learn about division as partitive. Then, students learn about division as measurement or quotative.



For learning the concepts of division, we recommend using *mathematics facts*. We define a division mathematics fact as a single- or double-digit dividend divided by a single-digit divisor for a single-digit quotient. You may present division facts vertically or horizontally.

C. Routines and Examples

(1) Division as Partitive



Routine

Materials:

- Module 13 Problems
- Module 13 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., cubes, clips) and any container (e.g., plates, cups)

Teacher	**Let's work on division. Today, let's think about division as partitioning or equal share. What does it mean to share equally?**
Students	Each person gets the same amount.
Teacher	**So, when you share equally, we'll give the same amount to each person or each group. To partition means the same thing as to share equally. What does partition mean?**
Students	To share equally.
Teacher	**Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign** (point). **The division sign tells us to divide. What does the division sign mean?**
Students	To divide.
Teacher	**We'll divide by partitioning or sharing equally. In a division problem, we'll use the dividend to tell us how many altogether we have to share. What will the dividend tell us?**
Students	The total number of objects to share.
Teacher	**And we'll use the divisor to tell us the number of groups we will make to then equally share the objects. What will the divisor tell us?**
Students	The number of groups we will make to then equally share the objects.
Teacher	**Great. Let's do this problem.** (Move clips to workspace.)
Teacher	**Our dividend is __. What's our dividend?**
Students	__.
Teacher	**Let's show this dividend by showing __ objects. We'll show the objects with the clips.** (Use clips to show dividend.)
Teacher	**How many clips?**
Students	__.

Teacher	**Our divisor is __. What's our divisor?**
Students	__.
Teacher	**Let's show the divisor by showing __ groups. We'll use plates to show each group.** (Show groups using plates.)
Teacher	**How many groups?**
Students	__.
Teacher	**So, we have __ clips to share equally among __ groups. Let's divide by sharing the __ clips equally among the __ groups. How will we divide?**
Students	Equally share the clips among the groups.
Teacher	**Let's put one object on each plate. 1 clips goes on this plate, 1 clip goes on this plate, 1 clip goes on this plate, …** (Equally share 1 clip on each plate.)
Teacher	**Now, do we have more clips to equally share?**
Students	Yes!
Teacher	**Let's keep sharing the clips among the groups. That means 1 clips goes on this plate, 1 clip goes on this plate, 1 clip goes on this plate, ….** (Equally share 1 clip on each plate.)
Teacher	**We keep sharing until we've shared all the clips equally. Now, to learn the quotient, let's count the number of clips in one group. We have __, __, __, …** (Count clips on 1 plate.)
Teacher	**How many clips in one group?**
Students	__.
Teacher	**Yes! There are __ clips. So, __ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**Let's say it together again.**
Students	__ divided by __ equals __.
Teacher	**So, if you have __ clips and share the clips equally among __ groups, the quotient is __. __ divided by __ equals __. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make to then equally share objects.
Teacher	**What's a quotient?**
Students	The result in each group after you equally share.
Teacher	**What does it mean to partition?**
Students	To equally share objects among groups.
Teacher	**How could you explain dividing to a friend?**
Students	We started a total number of clips. We equally shared the clips among groups. The quotient was the number of clips in each group.

Example

15 ÷ 3 = 5

Teacher	**Let's work on division. Today, let's think about division as partitioning or equal share. What does it mean to share equally?**
Students	Each person gets the same amount.
Teacher	**So, when you share equally, we'll give the same amount to each person or each group. To partition means the same thing as to share equally. What does partition mean?**
Students	To share equally.
Teacher	**Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign** (point). **The division sign tells us to divide. What does the division sign mean?**
Students	To divide.
Teacher	**We'll divide by partitioning or sharing equally. In a division problem, we'll use the dividend to tell us how many altogether we have to share. What will the dividend tell us?**
Students	The total number of objects to share.
Teacher	**And we'll use the divisor to tell us how many groups we make to then equally share the objects. What will the divisor tell us?**
Students	The number of groups we will make to then equally share the objects.
Teacher	**Great. Let's do this problem.** (Move cubes to workspace.)
Teacher	**Our dividend is 15. What's our dividend?**
Students	15.
Teacher	**Let's show this dividend by showing 15 cubes. We'll show the objects with the cubes.** (Show 15 cubes.)
Teacher	**How many cubes?**
Students	15.
Teacher	**Our divisor is 3. What's our divisor?**
Students	3.
Teacher	**Let's show the divisor by showing 3 groups. We'll use plates to show each group.** (Show 3 plates.)
Teacher	**How many groups?**
Students	3.
Teacher	**So, we have 15 cubes to share equally among 3 groups. Let's divide by sharing the 15 cubes equally among the 3 groups. How will we divide?**
Students	Equally share the cubes among the groups.
Teacher	**Let's put one object on each plate. 1 cube goes on this plate, 1 cube goes on this plate, 1 cube goes on this plate.**

(Equally share 1 cube on each plate.)

Teacher **Now, do we have more cubes to equally share?**

Students Yes!

Teacher **Let's keep sharing the cubes among the groups. That means 1 cube goes on this plate, 1 cube goes on this plate, 1 cube goes on this plate.**

(Equally share 1 cube on each plate.)

Teacher **We keep sharing until we've shared all the cubes equally. That means, 1 cube goes on this plate, 1 cube goes on this plate, 1 cube goes on this plate. Then, 1 cube goes on this plate, 1 cube goes on this plate, 1 cube goes on this plate. Finally, 1 cube goes on this plate, 1 cube goes on this plate, 1 cube goes on this plate. Do we have any more cubes to share?**

Students No!

Now, to learn the quotient, let's count the number of cubes in one group. We have 1, 2, 3, 4, 5 cubes in one group. How many cubes in one group?

Students 5.

Teacher **Yes! There are 5 cubes in one group. So, 15 divided by 3 equals 5. Let's say that together.**

Students 15 divided by 3 equals 5.

Teacher **Let's say it together again.**

Students 15 divided by 3 equals 5.

Teacher **So, if you have 15 cubes and share the cubes equally among 3 groups, the quotient is 5. 15 divided by 3 equals 5. Let's review. What's a dividend?**

Students The total number that will be divided.

Teacher **What's a divisor?**

Students The number of groups we make to equally share the objects.

Teacher **What's a quotient?**

Students The result in each group after you equally share.

Teacher **What does it mean to partition?**

Students To equally share objects among groups.

Teacher **How could you explain dividing to a friend?**

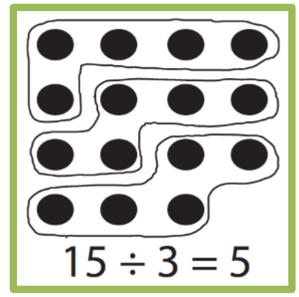
Students We started a total number of cubes. We equally shared the cubes among groups. The quotient was the number of cubes in each group.

(2) Division as Quotative or Measurement

Routine

Materials:

- [Module 13 Problems]
- [Module 13 Vocabulary Cards]
    - If necessary, review Vocabulary Cards before teaching
- Number line



Teacher	**Let's work on division. Today, let's think about division as quotative. That's a new word. Let's say it together.**
Students	Quotative.
Teacher	**Quotative division means we'll measure objects into groups. We can also call quotative division measurement division. What does quotative or measurement division mean?**
Students	We'll measure objects into groups.
Teacher	**When we use quotative or measurement division, we start with a set. Imagine you have a set of 12 pencils. You want to give your friends 4 pencils each. Quotative division helps you determine how many friends could get a set of 4 pencils. Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign** (point). **The division sign tells us to divide. What does the division sign mean?**
Students	To divide.
Teacher	**Today we'll divide using quotative or measurement division, but there are other ways to divide – like partitive division or equal shares. Let's start by getting our cubes.** (Move cubes to workspace.)
Teacher	**Our dividend is __. What's our dividend?**
Students	__.
Teacher	**Let's show this dividend by showing __ objects. We'll show the objects with the cubes.** (Use cubes to show dividend.)
Teacher	**How many cubes?**
Students	__.
Teacher	**Our divisor is __. What's our divisor?**
Students	__.
Teacher	**Let's show the divisor by measuring groups of __. The divisor tells us how many objects will be in each group. How many will be in each group?**
Students	__.

Teacher	**So, we have __ cubes to measure into groups of __. Let's divide by measuring the objects into groups of __. How will we divide?**
Students	Measure the objects into groups of __.
Teacher	**So, let's make a group of __. I'll place __, __, __, … objects into this group.** (Place objects into a group.)
Teacher	**Now, do we have more cubes to make another group?**
Students	Yes!
Teacher	**Let's keep measuring the objects into groups. That means, I'll place __, __, __, … objects into this group.** (Place objects into a group.)
Teacher	**We keep measuring groups until we've placed all the cubes into groups.** (Place objects into a group.)
Teacher	**Now, to learn the quotient, let's count the number of groups we created. We have __, __, __, … groups.** (Count groups.)
Teacher	**How many groups?**
Students	__.
Teacher	**Yes! There are __ groups. So, __ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**Let's say it together again.**
Students	__ divided by __ equals __.
Teacher	**So, if you have __ cubes and measure the cubes into groups of __, the quotient is __. __ divided by __ equals __. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number we place into each group.
Teacher	**What's a quotient?**
Students	The number of groups we made by measuring the cubes into groups.
Teacher	**What does it mean to use quotative or measurement division?**
Students	To place objects into groups.
Teacher	**How could you explain dividing to a friend?**
Students	We started a total number of cubes. We placed the cubes into groups. The quotient was the number of groups we created.

Example

15 ÷ 3 = 5

Teacher	**Let's work on division. Today, let's think about division as quotative. That's a new word. Let's say it together.**
Students	Quotative.

Teacher	**Quotative or measurement division means we'll measure objects into groups. What does quotative or measurement division mean?**
Students	We'll measure objects into groups.
	(Show problem.)
Teacher	**First, I see a division sign** (point). **The division sign tells us to divide. What does the division sign mean?**
Students	To divide.
Teacher	**Today we'll divide using quotative or measurement division, but there are other ways to divide – like partitive division or equal shares. Let's start by getting our beans.**
	(Move beans to workspace.)
Teacher	**Our dividend is 15. What's our dividend?**
Students	15.
Teacher	**Let's show this dividend by showing 15 beans.**
	(Use beans to show dividend.)
Teacher	**How many beans?**
Students	15.
Teacher	**Our divisor is 3. What's our divisor?**
Students	3.
Teacher	**Let's show the divisor by measuring groups of 3. The divisor tells us how many objects will be in each group. How many will be in each group?**
Students	3.
Teacher	**So, we have 15 beans to measure into groups of 3. Let's divide by measuring the objects into groups of 3. How will we divide?**
Students	Measure the objects into groups of 3.
Teacher	**So, let's make a group of 3. I'll place 1, 2, 3 beans into this group.**
	(Place objects into a group.)
Teacher	**Now, do we have more beans to make another group?**
Students	Yes!
Teacher	**Let's keep measuring the objects into groups. That means, I'll place 1, 2, 3 beans into this group.**
	(Place objects into a group.)
Teacher	**We keep measuring groups until we've placed all the beans into groups.**
	(Place objects into a group.)
Teacher	**Now, to learn the quotient, let's count the number of groups we created. We have 1, 2, 3, 4, 5 groups.**
	(Count groups.)
Teacher	**How many groups?**
Students	5.
Teacher	**Yes! There are 5 groups. So, 15 divided by 3 equals 5. Let's say that together.**
Students	15 divided by 3 equals 5.
Teacher	**Let's say it together again.**
Students	15 divided by 3 equals 5.

Teacher	**So, if you have 15 beans and measure the beans into groups of 3, the quotient is 5. 15 divided by 3 equals 5. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number we place into each group.
Teacher	**What's a quotient?**
Students	The number of groups we made by measuring the cubes into groups.
Teacher	**What does it mean to use quotative or measurement division?**
Students	To place objects into groups.
Teacher	**How could you explain dividing to a friend?**
Students	We started a total number of beans. We placed the beans into groups. The quotient was the number of groups we created.

D. Problems for Use During Instruction

See Module 13 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 13 Vocabulary Cards.

Developed by:
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Katherine A. Berry (kberry@austin.utexas.edu)

Module 13:

Concepts of Division

Problem Sets

$$
\begin{array}{r}
1 \\
\div \quad 1 \\
\hline
\end{array}
$$

$$\begin{array}{r} 30 \\ \div \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ \div \quad 4 \\ \hline \end{array}$$

$$
\begin{array}{r}
18 \\
\div \quad 3 \\
\hline
\end{array}
$$

$$\begin{array}{r} 12 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \div \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 81 \\ \div \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ \div \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \div \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 15 \\ \div \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \div \quad 2 \\ \hline \end{array}$$

$$
\begin{array}{r}
18 \\
\div \quad 9 \\
\hline
\end{array}
$$

$$
\begin{array}{r} 24 \\ \div \quad 6 \\ \hline \end{array}
$$

$$
\begin{array}{r}
64 \\
\div \quad 8 \\
\hline
\end{array}
$$

$$\begin{array}{r} 20 \\ \div \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \div \quad 3 \\ \hline \end{array}$$

$$
\begin{array}{r}
32 \\
\div \quad 4 \\
\hline
\end{array}
$$

$$\begin{array}{r} 25 \\ \div \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \div \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \div \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ \div \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ \div \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \div \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \div \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 18 \\ \div \quad 2 \\ \hline \end{array}$$

$$
\begin{array}{r} 9 \\ \div \quad 9 \\ \hline \end{array}
$$

$$
\begin{array}{r}
36 \\
\div \quad 6 \\
\hline
\end{array}
$$

$$\begin{array}{r} 4 \\ \div \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 16 \\ \div \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \div \quad 3 \\ \hline \end{array}$$

$$
\begin{array}{r} 16 \\ \div \quad 4 \\ \hline \end{array}
$$

$$
\begin{array}{r}
36 \\
\div \quad 9 \\
\hline
\end{array}
$$

$$\begin{array}{r} 49 \\ \div \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \div \quad 5 \\ \hline \end{array}$$

$$
\begin{array}{r}
12 \\
\div \quad 6 \\
\hline
\end{array}
$$

$$\begin{array}{r} 8 \\ \div \quad 2 \\ \hline \end{array}$$

$$
\begin{array}{r}
40 \\
\div \quad 5 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
56 \\
\div \quad 8 \\
\hline
\end{array}
$$

$$\begin{array}{r} 6 \\ \div \quad 6 \\ \hline \end{array}$$

$$
\begin{array}{r}
16 \\
\div \quad 2 \\
\hline
\end{array}
$$

$$\begin{array}{r} 20 \\ \div \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 72 \\ \div \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 63 \\ \div \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \div \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \div \quad 2 \\ \hline \end{array}$$

$$
\begin{array}{r}
7 \\
\div \quad 7 \\
\hline
\end{array}
$$

$$\begin{array}{r} 42 \\ \div \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 45 \\ \div \quad 9 \\ \hline \end{array}$$

$$
\begin{array}{r}
54 \\
\div \quad 6 \\
\hline
\end{array}
$$

$$\begin{array}{r} 10 \\ \div \quad 5 \\ \hline \end{array}$$

$$
\begin{array}{r} 36 \\ \div \quad 4 \\ \hline \end{array}
$$

$$\begin{array}{r} 35 \\ \div \quad 7 \\ \hline \end{array}$$

$$
\begin{array}{r} 21 \\ \div \quad 3 \\ \hline \end{array}
$$

$$
\begin{array}{r} 14 \\ \div \quad 2 \\ \hline \end{array}
$$

$$\begin{array}{r} 42 \\ \div \quad 6 \\ \hline \end{array}$$

$$
\begin{array}{r}
54 \\
\div \quad 9 \\
\hline
\end{array}
$$

$$\begin{array}{r} 15 \\ \div \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \div \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 48 \\ \div \quad 8 \\ \hline \end{array}$$

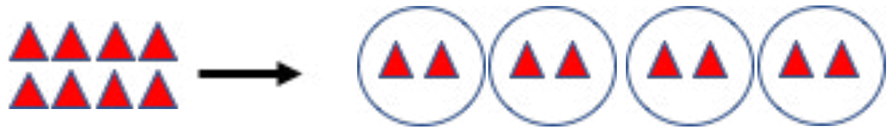
Module 13:
Concepts of Division

Vocabulary Cards

divide/division
dividend
division sign
divisor
equal groups
equal sign
partitive division
quotative division
quotient

divide/division

To separate into equal groups or among groups.



8 ÷ 4 = 2

---

dividend

The number to be divided.

16 ÷ 8 = 2

16 is the dividend

division sign

The symbol that tells you to divide.

$$16 \div 8 = 2$$

÷ is the division sign

---

divisor

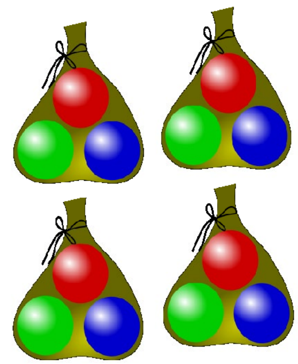
The number the dividend is divided by.

$$16 \div 8 = 2$$

8 is the divisor

equal groups

Groups with the same number of objects or items in each group.



equal sign

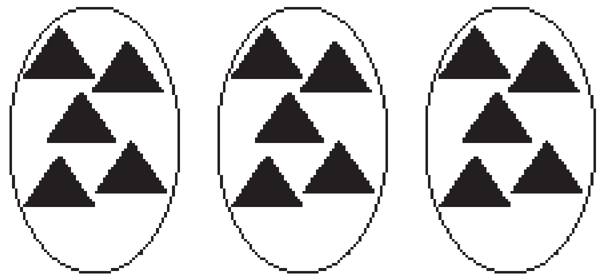
The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$16 \div 8 = 2$$

= is the equal sign

partitive division

A way of dividing where you share items into a pre-determined number of groups.



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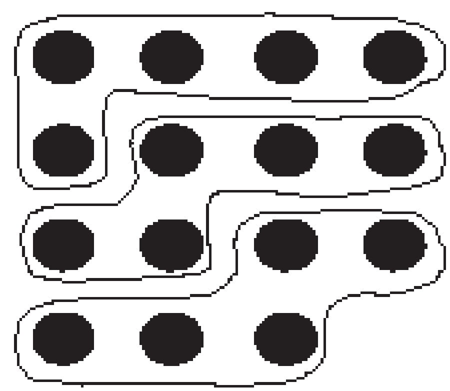
quotative division

A way of dividing where you measure a pre-determined amount of items into an unknown number of groups



quotient

The result when one number is divided by another number.

$$16 \div 8 = 2$$

2 is the quotient

Instructional Routines for Mathematics Intervention

MODULE 14

Division of Whole Numbers

Module 14: Division of Whole Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A procedure or description of steps that can be used to solve a problem.
computation	The action used to solve a problem.
divide/division	To separate into equal groups.
dividend	The number that is to be divided in a division problem.
division sign	The symbol that tells you to divide.
divisor	The number that the dividend is divided by.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
hundreds column	The column with digits in the hundreds place.
ones column	The column with digits in the ones place.
quotient	The number that results when one number is divided by another number.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
remainder	The amount left over in a division problem.
tens column	The column with digits in the tens place.

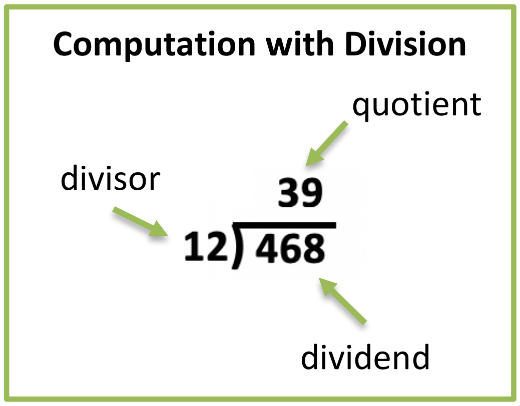
B. Background Information

Background Information:

If your focus is on the conceptual understanding of division, see *Module 13: Concepts of Division.* This module, *Module 14*, focuses on computation with division of whole numbers. As you focus on computation, continue to emphasize division as partitive (i.e., equal shares) and division as quotative (i.e., measurement) because students will see these concepts within word problems.

For learning computation with division, we recommend presenting problems with a division bracket. Some students may require explicit instruction on translating a horizontal problem (e.g., 245 ÷ 15) to the presentation with a division bracket (see below). Depending upon the algorithm, leave enough space above or below the problem for students to complete their written work.

Every student should develop efficiency with a division computation strategy. In the following sections, we provide examples of (1) division with traditional algorithm and (2) division with partial quotients algorithm. Teachers should help students develop competency with at least one algorithm.



C. Routines and Examples

(1) Division with Traditional Algorithm

Routine

Materials:

- Module 14 Problem Sets
- Module 14 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
    - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT ÷ 1-DIGIT: ROUTINE WITH MANIPULATIVES
(Only use manipulatives with simpler problems)

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**With our Base-10 blocks, each cube represents one thousand. What do the cubes represent?**
Students	Thousands.

Teacher	**The flats represent hundreds. What do the flats represent?**
Students	Hundreds.
Teacher	**The rods represent tens. What do the rods represent?**
Students	Tens.
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**
Students	Ones.
Teacher	**Our dividend is __. What's our dividend?**
Students	__.
Teacher	**Let's show this dividend by showing __ with the Base-10 blocks.** (Use Base-10 blocks to show dividend.)
Teacher	**How many blocks?**
Students	__.
Teacher	**Our divisor is __. What's our divisor?**
Students	__.
Teacher	**Let's solve this problem using division as partitive or equal shares. What does equal shares mean?**
Students	We divide the blocks equally among groups.
Teacher	**If we want to equally share the blocks, our divisor tells us the number of groups. What's the divisor tell us?**
Students	The number of groups.
Teacher	**Let's show the groups by showing __ plates.** (Show plates.)
Teacher	**Now, let's use the Base-10 blocks and divide the blocks equally among the __ groups. Let's start dividing with the greatest place value. In this example, that means I'll divide the tens then I'll divide the ones. How will we divide?**
Students	Tens then ones.
Teacher	**So, let's equally share the tens among the groups. I place 1 ten on each plate until I either have no remaining tens or I don't have enough tens to share equally.** (Equally share the tens blocks on each plate.)
Teacher	**Do we have any remaining tens?**
Students	*OPTION 1:* No. *OPTION 2:* Yes.
Teacher	***OPTION 2:*** **We have tens remaining that cannot be equally shared unless we regroup the tens into ones. Let's regroup the tens into ones.** (Regroup remaining tens into ones.)
Teacher	**So, how many ones do we have to equally share?**
Students	__.
Teacher	**Let's equally share the ones among the groups. I place 1 one on each plate until I have no remaining ones.** (Equally share the ones blocks on each plate.)
Teacher	**Now, to learn the quotient, let's count the number of blocks in one group. We have __, __, __, …**

(Count blocks on 1 plate.)

Teacher **How many blocks in one group?**

Students ___.

Teacher **Yes! There are ___ blocks. So, ___ divided by ___ equals ___. Let's say that together.**

Students ___ divided by ___ equals ___.

Teacher **Let's say it together again.**

Students ___ divided by ___ equals ___.

Teacher **So, if you have ___ blocks and share the blocks equally among ___ groups, the quotient is ___. ___ divided by ___ equals ___. Let's review. What's a dividend?**

Students The total number that will be divided.

Teacher **What's a divisor?**

Students The number of groups we will make to then equally share objects.

Teacher **What's a quotient?**

Students The result in each group after you equally share.

Teacher **What does it mean to partition?**

Students To equally share objects among groups.

Teacher **How could you explain dividing to a friend?**

Students We started with a total number of blocks. We equally shared the blocks among groups. The quotient was the number of blocks in each group.

3-DIGIT ÷ 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on division. What does it mean to divide?**

Students To share equally or measure into groups.

Teacher **Division means to share equally or to measure into groups. Look at this problem.**

(Show problem.)

Teacher **First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**

Students To divide.

Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the hundreds from the tens and the tens from the ones. Let's draw a vertical line between each of the columns in the dividend.**

(Draw vertical lines to separate place value columns.)

Teacher **When we divide using this method, for each place value in the dividend, the first thing we do is divide. If we can divide, then we multiply, subtract, and then bring in the next place value. So, the pattern is: divide, multiply, subtract, bring in. Say that with me.**

Students Divide, multiply, subtract, bring in.

Teacher **And we keep repeating that pattern until we have solved the problem. Let's see how it works. Are you ready?**

Students Yes!

Teacher **Now, we start by dividing the dividend by the divisor. What's our dividend?**

Students __.

Teacher **And we'll divide the dividend by the divisor. What's the divisor?**

Students __.

Teacher **Okay, how many groups of __** (divisor) **can we make if we have _** (digit in hundreds column of dividend)**?**

Students We can't make any groups of __.

Teacher **We can't make a group of __. So, now I think how many groups of __** (divisor) **can we make if we have __** (digits in the hundreds and tens columns of dividend)**?**

Students *OPTION 1:* __ groups.

OPTION 2: We can't make any groups of __.

Teacher ***OPTION 1:*** **We can make __ groups. So, let's write __ above the division bracket in the tens column.**

(Write.)

Teacher **So, __** (divisor) **times __** (digit above tens column) **equals __.**

What's __ (divisor) **times __** (digit above tens column)**?**

Students __.

Teacher **Let's write that product below the __** (digits in the hundreds and tens columns of dividend).

(Write.)

Teacher **Now, let's write a minus sign and an equal line. What sign?**

Students Minus sign.

Teacher **And let's subtract __** (digits in the hundreds and tens column of dividend) **minus __** (product). **What do we subtract?**

Students __ minus __.

Teacher **What's __ minus __?**

Students __.

Teacher **Let's write the difference here under the equal line.**

(Write.)

Teacher **Now, we bring in the one to our difference. __** (difference) **becomes __. This is our new dividend. What's our new dividend?**

Students __.

(Draw arrow and write.)

Teacher ***OPTION 2:*** **We can't make any groups of __. So, now I think about how many groups of __** (divisor) **can we make if we have __** (digits in the hundreds, tens, and ones columns of dividend)**?**

Teacher **How many groups of __** (divisor) **can we make with our new dividend?**

Students __ groups.

Teacher **We can make __ groups. So, let's write __ above the division bracket in the ones column.**

(Write.)

Teacher	**So,** __ (divisor) **times** __ (digit above ones column) **equals** __**. What's** __ (divisor) **times** __ (digit above ones column)**?**
Students	__.
Teacher	**Let's write that product below the** __ (digits in the new dividend). (Write.)
Teacher	**Now, let's write a minus sign and an equal line. What sign?**
Students	Minus sign.
Teacher	**And let's subtract** __ (digits in the new dividend) **minus** __ (product). **What do we subtract?**
Students	__ minus __.
Teacher	**What's** __ **minus** __**?**
Students	__.
Teacher	**Let's write the difference here under the equal line.** (Write.)
Teacher	**Now, do we have any remaining?**
Students	*OPTION 1:* No. *OPTION 2:* Yes.
Teacher	***OPTION 1:*** **We have nothing remaining in our difference. It's time to determine the quotient.**
Teacher	***OPTION 2:*** **We have some remaining in our difference. This will be our remainder. Say that with me.**
Students	Remainder.
Teacher	**I'll show the remainder like this. First, up by the quotient, I'll write a letter R for remainder.** (Write R.)
Teacher	**What does the R stand for?**
Students	Remainder.
Teacher	**Then, I write the remainder amount next to the R. What's the remainder amount?**
Students	__.
Teacher	**Let's write** __ **next to R.** (Write.)
Teacher	**It's time to determine the quotient.**
Teacher	**Our quotient is the number above the division bracket. Where can you find the quotient?**
Students	Above the division bracket.
Teacher	**What's the quotient?**
Students	__.
Teacher	**So,** __ **divided by** __ **equals** __**. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**Let's say it together again.**
Students	__ divided by __ equals __.
Teacher	**So, if you have** __ (dividend) **and divide by** __ (divisor), **the quotient is** __**. Let's review. What's a dividend?**

Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you equally share or measure groups.
Teacher	**How could you explain dividing to a friend?**
Students	We asked ourselves about how many groups we can make with the divisor from the dividend. The number of groups is the quotient.

Example

$$\begin{array}{r} 39 \\ 12\overline{)468} \end{array}$$

3-DIGIT ÷ 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the hundreds from the tens and the tens from the ones. Let's draw a vertical line between each of the columns in the dividend.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by dividing the dividend by the divisor. What's our dividend?**
Students	468.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	12.
Teacher	**When we divide using this method, for each place value in the dividend, the first thing we do is divide. If we can divide, then we multiply, subtract, and then bring in the next place value. So, the pattern is: divide, multiply, subtract, bring in. Say that with me.**
Students	Divide, multiply, subtract, bring in.
Teacher	**And we keep repeating that pattern until we have solved the problem. Let's see how it works. Are you ready?**
Students	Yes!
Teacher	**Okay, so we start with the greatest place value of the dividend. Where do we start?**
Students	Greatest place value of the dividend.
Teacher	**In this problem, the greatest place value of the dividend is 4. What number?**

Students	4.
Teacher	**How many groups of 12 can we make with 4?**
Students	We can't make any groups of 4 if we have 12.
Teacher	**We can't make a group of 12. So, now we bring in the 6 to make 46. I think how many groups of 46 can we make if we have 12?**
Students	We can make 3 groups of 12.
	We can make 3 groups. So, let's write 3 above the division bracket in the tens column. (Write 3.)
Teacher	**So, now let's multiply 12 times 3. What's 12 times 3?**
Students	36.
Teacher	**Let's write that product of 36 below the 46 in the dividend.** (Write 36.)
Teacher	**Now, let's write a minus sign and an equal line to help us subtract 36 from 46. What sign?**
Students	Minus sign.
Teacher	**What do we subtract?**
Students	46 minus 36.
Teacher	**What's 46 minus 36?**
Students	10.
Teacher	**Let's write the difference here under the equal line.** (Write 10.)
Teacher	**Now, we bring in the one to our difference. I like to show this by drawing an arrow from the 8 and rewriting the 8 next to 10.** (Draw arrow and write 8.)
Teacher	**When I bring in the 8, 10 now becomes 108. This is our new dividend. What's our new dividend?**
Students	108.
Teacher	**So, we followed the steps of division: divide, multiply, subtract, bring in. Say that with me.**
Students	Divide, multiply, subtract, bring in.
Teacher	**But the problem isn't finished. Let's follow the steps again: divide, multiply, subtract, bring in. What do we do?**
Students	Divide, multiply, subtract, bring in.
Teacher	**How many groups of 12 can we make with our new dividend of 108?**
Students	9 groups.
Teacher	**We can make 9 groups. So, let's write 9 above the division bracket in the ones column.** (Write 9.)
Teacher	**So, let's multiply. What's 12 times 9?**
Students	108.
Teacher	**Let's write 108 below the 108**. (Write 108.)
Teacher	**Now, let's write a minus sign and an equal line. What sign?**

Students Minus sign.

Teacher And let's subtract 108 minus 108. What do we subtract?

Students 108 minus 108.

Teacher What's 108 minus 108?

Students 0.

Teacher Let's write the difference here under the equal line.
(Write 0.)

Teacher Now, do we have any remaining?

Students No.

Teacher We have nothing remaining in our difference. It's time to determine the quotient. Our quotient is the number above the division bracket. Where can you find the quotient?

Students Above the division bracket.

Teacher What's the quotient?

Students 39

Teacher So, 468 divided by 12 equals 39. Let's say that together.

Students 468 divided by 12 equals 39.

Teacher Let's say it together again.

Students 468 divided by 12 equals 39.

Teacher So, if you have 468 and divide by 12, the quotient is 39. Let's review. What's a dividend?

Students The total number that will be divided.

Teacher What's a divisor?

Students The number of groups we will make.

Teacher What's a quotient?

Students The result in each group after you equally share or measure groups.

Teacher How could you explain dividing to a friend?

Students We asked ourselves about how many groups we can make with the divisor from the dividend. The number of groups is the quotient.

(2) Division with Partial Quotients Algorithm*

*For clarity, read **Example** before using **Routines**.

Routine

Materials:

- Module 14 Problem Sets
- Module 14 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching

3-DIGIT ÷ 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with our pencil, and let's use the partial quotients strategy. If I want to use the partial quotients strategy, I first draw a vertical line down from the end of the division bracket.** (Draw vertical line from end of division bracket.)
Teacher	**Now, we start by dividing the dividend by the divisor. What's our dividend?**
Students	__.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	__.
Teacher	**I don't know exactly how many groups of __** (divisor) **I can make with __** (dividend)**, so the partial quotients strategy can be used with computation that I do know. Which strategy are we using?**
Students	Partial quotients.
Teacher	**With the partial quotients strategy, we divide the dividend a few times. Each time we create a partial quotient. At the end, we add the partial quotients to determine the final quotient. Which strategy are we using again?**
Students	Partial quotients.
Teacher	**How many groups of __** (divisor) **can we make with __** (dividend)**?**
Students	I'm not sure.
Teacher	**I don't know the exact answer either, so I'll use a partial quotient to start solving this problem. I know that __** (friendly number) **groups of __** (divisor) **would be __** (product of friendly number times divisor)**, so I'll write __** (product) **under the __** (original dividend)**. I'll also write __** (partial quotient) **to the right of the vertical line. __** (partial quotient) **is one of my partial quotients.**

	(Write.)
Teacher	**Now, I'll subtract __** (product) **from the dividend of __ to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	__ (dividend) **minus __** (product) **equals what?**
Students	__.
Teacher	**Let's write the difference of __ below the equal line.** (Write.)
Teacher	**Now, how many groups of __** (divisor) **can we make with __** (new dividend)**?**
Students	I'm not sure.
Teacher	**Again, I don't know the exact answer either, so I'll use a partial quotient. I know that __** (friendly number) **groups of __** (divisor) **would be __** (product of friendly number times divisor)**. I'm using computation that's easier for me to do. So, __** (friendly number) **groups of __** (divisor) **equals __** (product)**. I'll write __** (product) **under the __** (new dividend)**. I'll also write __** (partial quotient) **to the right of the vertical line. __** (partial quotient) **is one of my partial quotients.** (Write.)
Teacher	**Now, I'll subtract __** (product) **from the dividend of __ to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	__ (dividend) **minus __** (product) **equals what?**
Students	__.
Teacher	**Let's write the difference of __ below the equal line.** (Write.)
Teacher	**Now, how many groups of __** (divisor) **can we make with __** (new dividend)**?**
Students	I'm not sure.
Teacher	**Let's use a partial quotient. I know that __** (friendly number) **groups of __** (divisor) **would be __** (product of friendly number times divisor)**. I'll write __** (product) **under the __** (new dividend)**. I'll also write __** (partial quotient) **to the right of the vertical line. __** (partial quotient) **is one of my partial quotients.** (Write.)
Teacher	**Now, I'll subtract __** (product) **from the dividend of __ to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	__ (dividend) **minus __** (product) **equals what?**
Students	__.
Teacher	**Let's write the difference of __ below the equal line.** (Write.)
Teacher	**Now, how many groups of __** (divisor) **can we make with __** (new dividend)**?**
Students	__!
Teacher	**I know that __** (friendly number) **groups of __** (divisor) **would be __** (product of friendly number times divisor)**. I'll write __** (product) **under the __** (new

	dividend)**. I'll also write __** (partial quotient) **to the right of the vertical line. __** (partial quotient) **is one of my partial quotients.** (Write.)
Teacher	**Now, I'll subtract __** (product) **from the dividend of __ to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**__ minus __ equals what?**
Students	__.
Teacher	**Let's write the difference of __ below the equal line.** (Write.)
Teacher	**Can we make any more groups of __** (dividend)**?**
Students	No!
Teacher	**We can't make any more groups of __** (dividend)**, so let's determine our quotient. We do this by adding the partial quotients together. How do we determine the quotient?**
Students	Add the partial quotients together.
Teacher	**Let's write a plus sign and equal line.** (Write plus sign and equal line.)
Teacher	**What's __ plus __ plus …?** (partial quotients)
Students	__.
Teacher	**Let's write the sum of the partial quotients below the equal line.** (Write.)
Teacher	**We could also write the quotient above the division bracket.** (Write.)
Teacher	**What's the quotient?**
Students	__.
Teacher	**So, __ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**Let's say it together again.**
Students	__ divided by __ equals __.
Teacher	**Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you equally share or measure groups.
Teacher	**How could you explain dividing to a friend?**
Students	We kept asking how many groups we could make with the dividend. We didn't know the exact answer, so we used computation we did know as partial quotients. At the end, we added the partial quotients for the final quotient.

Example

	39	
12)	468	
−	240	20
	228	
−	120	10
	108	
−	72	6
	36	
−	36	+ 3
	0	39

3-DIGIT ÷ 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with our pencil, and let's use the partial quotients strategy. If I want to use the partial quotients strategy, I first draw a vertical line down from the end of the division bracket.** (Draw vertical line from end of division bracket.)
Teacher	**With the partial quotients strategy, we divide the dividend a few different times. Each time we create a partial quotient. At the end, we add the partial quotients to determine the final quotient. Which strategy are we using again?**
Students	Partial quotients.
Teacher	**Now, we start by dividing the dividend by the divisor. What's our dividend?**
Students	468.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	12.
Teacher	**I don't know exactly how many groups of 12 I can make with 468, so the partial quotients strategy can be used with computation that I do know. Which strategy are we using?**
Students	Partial quotients.
Teacher	**How many groups of 12 can we make with 468?**

Students	I'm not sure.
Teacher	**I don't know the exact answer either, so I'll use a partial quotient to start solving this problem. I know that 20 groups of 12 would be 240, so I'll write 240 under the 468. I'll also write 20 to the right of the vertical line. 20 is one of my partial quotients.** (Write 240 and 20.)
Teacher	**Now, I'll subtract 240 from the dividend of 468 to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**468 minus 240 equals what?**
Students	228.
Teacher	**Let's write the difference of 228 below the equal line.** (Write 228.)
Teacher	**Now, how many groups of 12 can we make with 228?**
Students	I'm not sure.
Teacher	**Again, I don't know the exact answer either, so I'll use a partial quotient. I know that 10 groups of 12 would be 120. I'm using computation that's easier for me to do – so I like to think about 20 groups of 12 or 10 groups of 12. What's another group of 12 that might be easy to remember or to figure out?**
Students	12 groups of 2 (or 4 or 5).
Teacher	**So, 10 groups of 12 equals 120. I'll write 120 under the 228. I'll also write 10 to the right of the vertical line. 10 is one of my partial quotients.** (Write 120 and 10.)
Teacher	**Now, I'll subtract 120 from the dividend of 228 to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**228 minus 120 equals what?**
Students	108.
Teacher	**Let's write the difference of 108 below the equal line.** (Write 108.)
Teacher	**Now, how many groups of 12 can we make with 108?**
Students	I'm not sure.
Teacher	**Let's use a partial quotient. I know that 6 groups of 12 would be 72. I'll write 72 under the 108. I'll also write 6 to the right of the vertical line. 6 is one of my partial quotients.** (Write 72 and 6.)
Teacher	**Now, I'll subtract 72 from the dividend of 108 to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**108 minus 72 equals what?**
Students	36.
Teacher	**Let's write the difference of 36 below the equal line.** (Write 36.)
Teacher	**Now, how many groups of 12 can we make with 36?**

Students	3!
Teacher	**Yes! I know that 3 groups of 12 would be 36. I'll write 36 under the 36. I'll also write 3 to the right of the vertical line. 3 is one of my partial quotients.** (Write 36 and 3.)
Teacher	**Now, I'll subtract 36 from the dividend of 36 to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**36 minus 36 equals what?**
Students	0.
Teacher	**Let's write the difference of 0 below the equal line.** (Write 0.)
Teacher	**This 0 is our new dividend. Can we make any more groups of 12?**
Students	No!
Teacher	**We can't make any more groups of 12, so let's determine our quotient. We do this by adding the partial quotients together. How do we determine the quotient?**
Students	Add the partial quotients together.
Teacher	**Let's write a plus sign and equal line.** (Write plus sign and equal line.)
Teacher	**What's 20 plus 10 plus 6 plus 3?**
Students	39.
Teacher	**Let's write the sum of the partial quotients below the equal line.** (Write 39.)
Teacher	**We could also write the quotient above the division bracket.** (Write 39.)
Teacher	**What's the quotient?**
Students	39.
Teacher	**So, 468 divided by 12 equals 39. Let's say that together.**
Students	468 divided by 12 equals 39.
Teacher	**Let's say it together again.**
Students	468 divided by 12 equals 39.
Teacher	**So, if you have 468 and divide by 12, the quotient is 39. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you equally share or measure groups.
Teacher	**How could you explain partial quotients to a friend?**
Students	We kept asking how many groups of 12 we could make with the dividend. We didn't know the exact answer, so we used computation we did know as partial quotients. At the end, we added the partial quotients for the final quotient.

D. Problems for Use During Instruction

See Module 14 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 14 Vocabulary Cards.

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Module 14:

Division of Whole Numbers

Problem Sets

A.

$$2\overline{)48}$$

A.

$$3\overline{)90}$$

A.

$$4\overline{)64}$$

A.

$$2\overline{)62}$$

A.

$$6\overline{)72}$$

A.

$$4\overline{)80}$$

A.

$$2\overline{)84}$$

A.

$$5\overline{)75}$$

A.

$$2\overline{)46}$$

A.

$$4\overline{)52}$$

B.

$$8 \overline{)29}$$

B.

$$9\overline{)37}$$

B.

$$7\overline{)25}$$

B.

$$6\overline{)21}$$

B.

$$5\overline{)47}$$

B.

$$4\overline{)59}$$

B.

$$7\overline{)46}$$

B.

$$9\overline{)28}$$

B.

$$3\overline{)37}$$

B.

$$8\overline{)51}$$

c.

$$4\overline{)236}$$

c.

$$5\overline{)165}$$

c.

$$7\overline{)518}$$

c.

$$2\overline{)720}$$

c.

$$8\overline{)448}$$

c.

$$6\overline{)516}$$

c.

$$8\overline{)304}$$

c.

$$9\overline{)774}$$

c.

$$3\overline{)162}$$

c.

$$5\overline{)285}$$

D.

$$5\overline{)785}$$

D.

$$9\overline{)296}$$

D.

$$5\overline{)158}$$

D.

$$3\overline{)538}$$

D.

$$6\overline{)719}$$

D.

$$2\overline{)971}$$

D.

$$5\overline{)497}$$

D.

$$9\overline{)917}$$

D.

$$7\overline{)192}$$

D.

$$8\overline{)966}$$

E.

$$2\overline{)5236}$$

E.

$$7 \overline{)\,1204}$$

E.

$$9\overline{)7227}$$

E.

$$5\overline{)8455}$$

E.

$$9 \overline{\smash{)}6840}$$

E.

$$8\overline{)7848}$$

E.

$$6\overline{)9552}$$

E.

$$2\overline{)9112}$$

E.

$$6\overline{)1410}$$

E.

$$5\overline{)2835}$$

F.

$$4\overline{)6743}$$

F.

$$2\overline{)7685}$$

F.

$$5\overline{)4817}$$

F.

$$4\overline{)5910}$$

F.

$$8\overline{)6029}$$

F.

$$5\overline{)7664}$$

F.

$$3\overline{)2009}$$

F.

$$9\overline{)1824}$$

F.

$$3\overline{)3622}$$

F.

$$7\overline{)5925}$$

G.

$$28\overline{)112}$$

G.

$$12\overline{)528}$$

G.

$$58\overline{)986}$$

G.

$$11\overline{)836}$$

G.

$$35\overline{)420}$$

G.

$$15\overline{)585}$$

G.

$$13\overline{)637}$$

G.

$$35\overline{)735}$$

G.

$$10\overline{)900}$$

G.

$$94\overline{)188}$$

H.

$$23 \overline{)202}$$

H.

$$16\overline{)124}$$

H.

$$11\overline{)104}$$

H.

$$28\overline{)114}$$

H.

$$17\overline{)122}$$

H.

$$24 \overline{)108}$$

H.

$$63\overline{)933}$$

H.

$$18\overline{)547}$$

H.

$$24\overline{)786}$$

H.

$$35\overline{)265}$$

Module 14:
Division of Whole Numbers

Vocabulary Cards

algorithm
computation
divide/division
dividend
division sign
divisor
equal groups
equal sign
hundreds column
ones column
quotient
regroup/trade/exchange
remainder
tens column

algorithm

A procedure or description of steps that can be used to solve a problem.

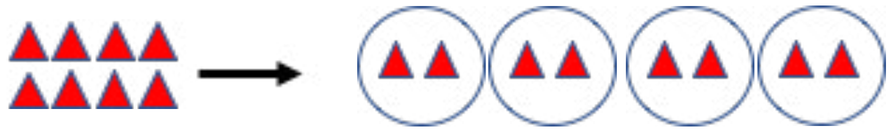
---

computation

The action used to solve a problem.

divide/division

To separate into equal groups.



8 ÷ 4 = 2

---

dividend

The number that is to be divided in a division problem.

16 ÷ 8 = 2

16 is the dividend

division sign

The symbol that tells you to divide.

$16 \div 8 = 2$

÷ is the division sign

---

divisor

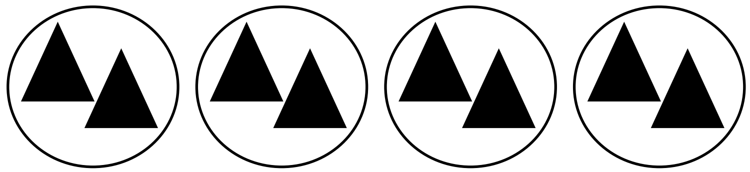
The number that the dividend is divided by.

$16 \div 8 = 2$

8 is the divisor

equal groups

Groups with the same number of objects or items in each group.



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equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$16 \div 8 = 2$$

= is the equal sign

hundreds column

The column with digits in the hundreds place.

In the number 423, 4 is in the hundreds place.

---

ones column

The column with digits in the ones place.

In the number 423, 3 is in the ones place.

quotient

The number that results when one number is divided by another number.

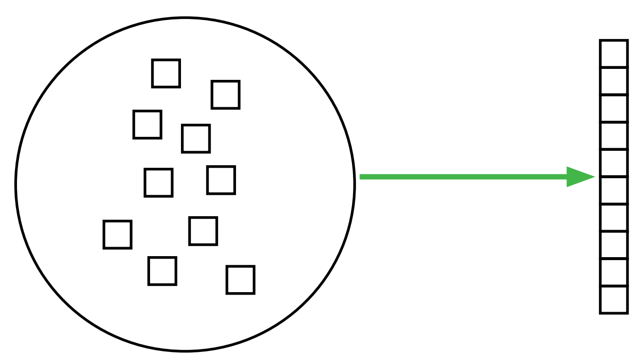
$$16 \div 8 = 2$$

2 is the quotient

---

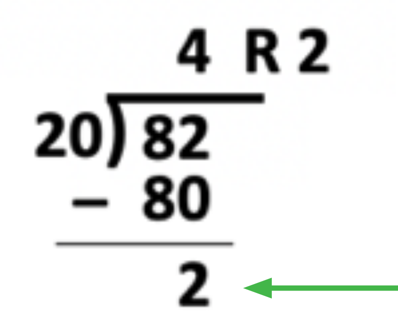
regroup/trade/exchange

The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



remainder

The amount remaining in a division problem.



2 is the remainder

---

tens column

The column with digits in the tens place.

In the number 423, 2 is the in the tens column.

Instructional Routines for Mathematics Intervention

MODULE 15

Division of Rational Numbers

Module 15: Division of Rational Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A procedure or description of steps that can be used to solve a problem.
computation	The action used to solve a problem.
decimal	A number based on powers of ten.
denominator	The term in a fraction that tells the number of equal parts in a whole.
divide/division	To separate into equal groups.
dividend	The number that is to be divided in a division problem.
division sign	The symbol that tells you to divide.
divisor	The number that the dividend is divided by.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
hundredths	The digit in representing $\frac{1}{100}$.
improper fraction	Any fraction in which the numerator is greater than the denominator.
least common multiple	The common multiple with the least value.
mixed number	A whole number and a fraction combined.
multiply/multiplication	The process of adding a number to itself a number of times.
multiplication sign	The symbol that tells you to multiply.
numerator	The term in a fraction that tells how many parts of a fraction.
ones	The digit representing 1.
quotient	The number that results when one number is divided by another number.
remainder	The amount left over in a division problem.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
tenths	The digit in representing $\frac{1}{10}$.

B. Background Information

Background Information:

In this module, we focus on division with fractions and decimals. As you focus on computation of rational numbers, continue to emphasize division as partitive (i.e., equal shares) and division as quotative (i.e., measurement) because students will see these concepts within word problems.

For division of fractions, we recommend using several models of fractions to help students understand concepts related to division of fractions. We also recommend demonstrating several algorithms for division of decimals. Every student should develop efficiency with strategies for division of fractions and decimals. In the following sections, we provide examples of (1) division of fractions, (2) division of decimals with the traditional algorithm, and (3) division of decimals with the partial quotients algorithm.

C. Routines and Examples

(1) Division of Fractions*

*Most students know the *procedure* for dividing decimals but do not have *conceptual understanding* of division of fractions. Here, we provide two conceptual **Routines** (one with manipulatives and one with drawings) as well as a procedural **Routine**. Our **Example** is conceptual and uses manipulatives. Consider reading the **Example** before reading the **Routines**.

Routine

Materials:

- Module 15 Problem Sets
- Module 15 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles or two-color counters
  - Note that drawings can be used alongside or instead of manipulatives

ROUTINE WITH MANIPULATIVES

(Only use manipulatives with simpler problems)

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign or bracket** (point). **The division sign or bracket tells us to divide. What does the division sign or bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with fraction tiles.**

	(Move fraction tiles to workspace.)
Teacher	**With division of fractions, we interpret this problem as __** (first fraction/dividend) **divided by __** (second fraction/divisor). **How do we interpret this problem?**
Students	__ divided by __.
Teacher	**When something is divided, we want to determine how many groups of the divisor we can make with the dividend. If the problem was 12 divided by 3, you would determine how many groups of 3 you could make if you had 12 of something. The same works with fractions. We'll show the dividend (or first fraction). Which fraction will we show?**
Students	The dividend or first fraction.
Teacher	**And then we'll determine how many groups of the divisor (or second fraction) we can make with the dividend. We'll determine how many groups of which fraction?**
Students	The divisor or second fraction.
Teacher	**So, let's show the dividend with the fraction tiles.** (Show dividend with fraction tiles.)
Teacher	**Now, let's find __** (divisor) **of __** (dividend)**. What's the divisor?**
Students	__.
Teacher	**Let's get out the divisor with our fraction tiles and think of it as one group.** (Show divisor with fraction tiles.)
Teacher	**Now, I want to learn how many groups of this divisor I can make with the dividend. What do I want to learn?**
Students	How many groups of the divisor we can make with the dividend.
Teacher	**To do this, I hold the divisor group under the dividend fraction tiles to see how many groups I can make. Let's see, I can make __ groups of __** (divisor). (Place divisor fraction tile group under dividend.)
Teacher	**Now, I do that again until I can't make any more groups of __** (divisor) **with the dividend.** (Place divisor fraction tile group under dividend.)
Teacher	**We're dividing by finding __** (divisor) **groups of the dividend. How are we dividing?**
Students	Finding __ (divisor) groups of the dividend.
Teacher	**We've determined __** (divisor) **groups of the dividend is __.**
Teacher	**So, __** (divisor) **groups of __** (dividend) **equals __. What's the quotient?**
Students	__.
Teacher	**__ divided by __ equals __. Let's say that together.**
Students	__ divided __ equals __.
Teacher	**So, if you have a group of __** (divisor) **and you determine you can make __ groups of __** (dividend)**, __ divided by __ equals __. Let's review. What's a dividend?**
Students	The total number that will be divided.

Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you make groups.
Teacher	**How could you explain dividing to a friend?**
Students	We showed the dividend with the fraction tiles. Then, we determined how many groups of the divisor we could make with the dividend. The quotient was the number of groups we could make.

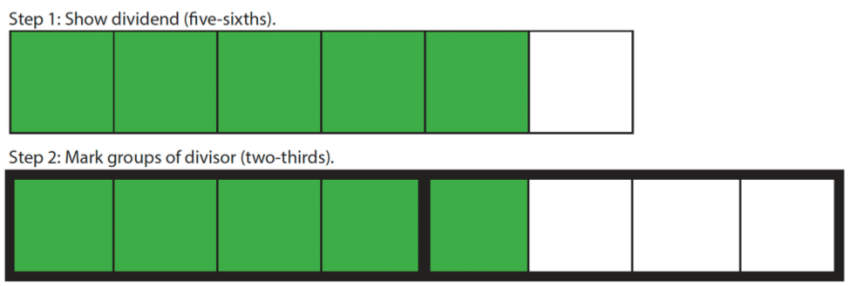
ROUTINE WITHOUT MANIPULATIVES OR DRAWINGS

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign or bracket** (point). **The division sign or bracket tells us to divide. What does the division sign or bracket mean?**
Students	To divide.
Teacher	**When we divide, we divide the numerators then we divide the denominators. How do we divide?**
Students	Divide the numerators then divide the denominators.
Teacher	**Sometimes that's easy to do, but sometimes dividing the numerators or denominators gives us another fraction. And that gets tricky. So, often we divide fractions by using the reciprocal of the divisor. Say reciprocal with me.**
Students	Reciprocal.
Teacher	**With a reciprocal of a fraction, the numerator becomes the denominator and the denominator becomes the numerator. What happens with a fraction reciprocal?**
Students	The numerator becomes the denominator and the denominator becomes the numerator.
Teacher	**What's the reciprocal of __** (divisor)?
Students	__.
Teacher	**The reciprocal of __** (divisor) **is __. So, instead of dividing by __** (divisor), **we multiply by the reciprocal of the divisor. What do we do?**
Students	Multiply by the reciprocal of the divisor.
Teacher	**So, we multiply __** (dividend) **times __** (reciprocal of divisor). **What do we multiply?**
Students	__ (dividend) times __ (reciprocal of divisor).
Teacher	**First, focus on the numerators. What are the numerators in this problem?**
Students	__ and __.
Teacher	**What's __ times __?**

Students	__.
Teacher	**__ times __ equals __, so let's write __ as the numerator of our quotient.** (Write numerator.)
Teacher	**Let's focus on the denominators. What are the denominators in this problem?**
Students	__ and __.
Teacher	**What's __ times __?**
Students	__.
Teacher	**__ times __ equals __, so let's write __ as the denominator of our quotient.** (Write denominator.)
Teacher	**So, __** (dividend) **divided by __** (divisor) **equals __. What's the quotient?**
Students	__. (If quotient is not in simplest form, use greatest common factor to determine an equivalent fraction in simplest form.)
Teacher	**__ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**So, if you have __** (dividend) **and you divide by __** (divisor)**, __ divided by __ equals __. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you make groups.
Teacher	**How could you explain dividing to a friend?**
Students	We used the reciprocal of the divisor and multiplied the dividend by the reciprocal.

Examples

$$\frac{5}{6} \div \frac{2}{3} = \frac{5}{4}$$



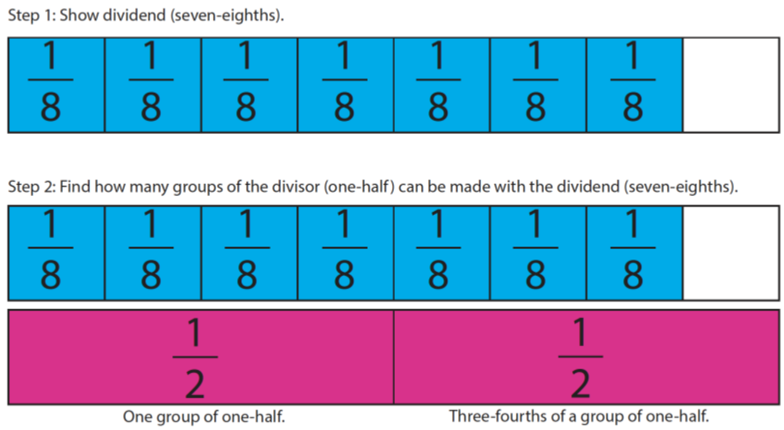
EXAMPLE WITH DRAWING

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign or bracket** (point). **The division sign or bracket tells us to divide. What does the division sign or bracket mean?**
Students	To divide.
Teacher	**Let's do this problem by drawing. How will we do this problem?**
Students	By drawing
Teacher	**With division of fractions, we interpret this problem as five-sixths divided by two-thirds. How do we interpret this problem?**
Students	Five-sixths divided by two-thirds.
Teacher	**When something is divided, we want to determine how many groups of the divisor we can make with the dividend. With our drawing, we'll show the dividend (or first fraction). Which fraction will we draw?**
Students	The dividend or first fraction.
Teacher	**And then we'll determine how many groups of the divisor (or second fraction) we can make with the dividend. We'll determine how many groups of which fraction?**
Students	The divisor or second fraction.
Teacher	**So, let's show the dividend. I'll draw a rectangle divided into six equal parts.** (Draw.)
Teacher	**I need to shade the numerator. How many equal parts should I shade?**
Students	5.
Teacher	**So, I'll shade 5 equal parts.** (Shade.)

Teacher	**Now, let's find two-thirds of five-sixths. What's the divisor?**
Students	Two-thirds.
Teacher	**I want to figure out how much one group of two-thirds would be when I have five-sixths. Let's see, two-thirds is equivalent to four-sixths. What's two-thirds equivalent to?**
Students	Four-sixths.
Teacher	**So, I'll draw a dark rectangle around one group of two-thirds (or four-sixths).** (Draw.)
Teacher	**I can make one full group of two-thirds. In that group of two-thirds, I see I have 1, 2, 3, 4 equal parts. So, 4 will be my new denominator. What's the new denominator?**
Students	4.
Teacher	**Let's see. I keep drawing dark rectangles around groups of two-thirds (or four-sixths) until I've used all of the five-sixth shaded parts. Let's draw another dark rectangle around the same size as the first.** (Draw.)
Teacher	**Now, with this group of two-thirds, is it a full group?**
Students	No!
Teacher	**It isn't a full group of two-thirds. How much of the group did we make? Remember, we determined 4 would be our new denominator. So, how much of this group of two-thirds is shaded?**
Students	One-fourth.
Teacher	**That's right. One-fourth of this group of two-thirds is shaded. So, when we divide five-sixths and make groups of two-thirds, we can make 1 full group of two-thirds and one-fourth of the next group of two-thirds. Our quotient is 1 and one-fourth or five-fourths. What's the quotient?**
Students	1 and one-fourth or five-fourths.
Teacher	**So, five-sixths divided by two-thirds equals five-fourths. Let's say that together.**
Students	Five-sixths divided by two-thirds equals five-fourths.
Teacher	**Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you make groups.
Teacher	**How could you explain dividing to a friend?**
Students	We drew the dividend. Then, we determined how many groups of the divisor we could make with the dividend. The quotient was the number of groups we could make.

Example

$$\frac{7}{8} \div \frac{1}{2} = \frac{7}{4}$$



EXAMPLE WITH MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division sign or bracket** (point). **The division sign or bracket tells us to divide. What does the division sign or bracket mean?**
Students	To divide.
Teacher	**Let's do this problem using the fraction tiles. How will we do this problem?**
Students	With the fraction tiles. (Show fraction tiles.)
Teacher	**With division of fractions, we interpret this problem as seven-eighths divided by one-half. How do we interpret this problem?**
Students	Seven-eighths divided by one-half.
Teacher	**When something is divided, we want to determine how many groups of the divisor we can make with the dividend. With our fraction tiles, we'll show the dividend (or first fraction). Which fraction will we show?**
Students	The dividend or first fraction.
Teacher	**And then we'll determine how many groups of the divisor (or second fraction) we can make with the dividend. We'll determine how many groups of which fraction?**
Students	The divisor or second fraction.
Teacher	**So, let's show the dividend. I'll show the whole and then show seven-eighths compared to the whole.** (Use fraction tiles.)
Teacher	**Now, let's find how many groups of one-half we can make with seven-eighths. What's the divisor?**

Students	One-half.
Teacher	**I want to figure out how many groups of one-half I can make if I have seven-eighths. So, I'll get out my one-half fraction tile to compare to the seven-eighths.** (Show one-half tile.)
Teacher	**Let's see how many groups of one-half I can make. I can make 1 group of one-half.** (Compare one-half tile to four one-eighth pieces.)
Teacher	**I can make one full group of one-half. How many groups?**
Students	1 full group of one-half.
Teacher	**Look closely. In this group of one-half, how many equal parts are represented?**
Students	4.
Teacher	**Yes. There are 4 equal parts in this group of one-half. That means 4 will be the denominator of our quotient. What will be the denominator?**
Students	4.
Teacher	**Let's keep going because I can make more than one group of one-half. Let's iterate (or copy) the one-half piece to see how much of the next group of one-half I can make.** (Move one-half fraction tile.)
Teacher	**Now, with this group of one-half, is it a full group?**
Students	No!
Teacher	**It isn't a full group of one-half. How much of the group did we make? Remember, we determined 4 would be our new denominator. So, how much of this group of one-half is covered by the one-half piece?**
Students	Three-fourths.
Teacher	**That's right. Three-fourths of this group of one-half is covered. So, when we divide seven-eighths and make groups of one-half, we can make 1 full group of one-half and three-fourths of the next group of one-half. Our quotient is 1 and three-fourth or seven-fourths. What's the quotient?**
Students	1 and three-fourths or seven-fourths.
Teacher	**So, seven-eighths divided by one-half equals seven-fourths. Let's say that together.**
Students	Seven-eighths divided by one-half equals seven-fourths.
Teacher	**Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you make groups.
Teacher	**How could you explain dividing to a friend?**
Students	We showed the dividend with the fraction tiles. Then, we determined how many groups of the divisor we could make with the dividend. The quotient was the number of groups we could make.

(2) Division of Decimals with Traditional Algorithm

Routine

Materials:

- Module 15 Problem Sets
- Module 15 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching

3-DIGIT ÷ 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tenths and the tenths from the hundredths. Let's draw a vertical line between each of the columns in the dividend.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by dividing the dividend by the divisor. What's our dividend?**
Students	__.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	__.
Teacher	**When we divide using this method, for each place value in the dividend, the first thing we do is divide. If we can divide, then we multiply, subtract, and then bring in the next place value. So, the pattern is: divide, multiply, subtract, bring in. Say that with me.**
Students	Divide, multiply, subtract, bring in.
Teacher	**And we keep repeating that pattern until we have solved the problem. Let's see how it works. Are you ready?**
Students	Yes!
Teacher	**Okay, so we start with the greatest place value of the dividend. Where do we start?**
Students	Greatest place value of the dividend.
Teacher	**In this problem, the greatest place value of the dividend is __. What number?**
Students	__.
Teacher	**How many groups of __** (divisor) **can we make with __?**

Students	We can't make any groups of __.
Teacher	**We can't make a group of __** (divisor)**. So, now we bring in the __** (next place value in dividend) **to make __. I think how many groups of __ can we make if we have __** (divisor)**?**
Students	We can make __ groups of __ (divisor).
Teacher	**We can make __ groups. So, let's write __ above the division bracket.** (Write.)
Teacher	**So, now let's multiply __ times __** (divisor)**. What's __ times __?**
Students	__.
Teacher	**Let's write that product of __ below the __ in the dividend.** (Write.)
Teacher	**Now, let's write a minus sign and an equal line to help us subtract __ from __. What sign?**
Students	Minus sign.
Teacher	**What do we subtract?**
Students	__ minus __.
Teacher	**What's __ minus __?**
Students	__.
Teacher	**Let's write the difference here under the equal line.** (Write.)
Teacher	**Now, we bring in the next digit of the dividend to our difference. I like to show this by drawing an arrow from the __ and rewriting the __ next to __.** (Draw arrow and write.)
Teacher	**When I bring in the __, __ now becomes __. This is our new dividend. What's our new dividend?**
Students	__.
Teacher	**So, we followed the steps of division: divide, multiply, subtract, bring in. Say that with me.**
Students	Divide, multiply, subtract, bring in.
Teacher	**But the problem isn't finished. Let's follow the steps again: divide, multiply, subtract, bring in. What do we do?**
Students	Divide, multiply, subtract, bring in.
Teacher	**How many groups of __** (divisor) **can we make with our new dividend of __?**
Students	__ groups.
Teacher	**We can make __ groups. So, let's write __ above the division bracket.** (Write.)
Teacher	**So, let's multiply. What's __ times __** (divisor)**?**
Students	__.
Teacher	**Let's write __ below the __.** (Write.)
Teacher	**Now, let's write a minus sign and an equal line. What sign?**
Students	Minus sign.
Teacher	**And let's subtract __ minus __. What do we subtract?**
Students	__ minus __.

Teacher	**What's __ minus __?**
Students	__.
Teacher	**Let's write the difference here under the equal line.** (Write.)
Teacher	**Now, do we have any remaining?**
Students	Yes!
Teacher	**Just like before, we have to bring in a number to keep dividing. I'll bring in the next digit from the dividend.** (Draw arrow and write.)
Teacher	**When I bring in the __, __ now becomes __. This is our new dividend. What's our new dividend?**
Students	__.
Teacher	**Let's follow the steps again: divide, multiply, subtract, bring in. What do we do?**
Students	Divide, multiply, subtract, bring in.
Teacher	**How many groups of __** (divisor) **can we make with our new dividend of __?**
Students	__ groups.
Teacher	**We can make __ groups. So, let's write __ above the division bracket.** (Write.)
Teacher	**So, let's multiply. What's __ times __** (divisor)**?**
Students	__.
Teacher	**Let's write __ below the __.** (Write.)
Teacher	**Now, let's write a minus sign and an equal line. What sign?**
Students	Minus sign.
Teacher	**And let's subtract __ minus __. What do we subtract?**
Students	__ minus __.
Teacher	**What's __ minus __?**
Students	__.
Teacher	**Let's write the difference here under the equal line.** (Write.)
Teacher	**Now, do we have any remaining?**
Students	No!
Teacher	**Now, we seem finished but we're not. In this problem, we divided decimals. So, we have to place the decimal point in the quotient. What do we have to place in the product?**
Students	A decimal point.
Teacher	**To place the decimal point, we determine the number of decimal places in the dividend and divisor. Let's see. The dividend had __ decimal places. The divisor also had __ decimal places. What's __ plus __?**
Students	_.
Teacher	**So, in the quotient, we need to put in __ decimal places starting from the least place value (or the right). That means I'll place a decimal point between the __ and __.**

Teacher **So, what's the quotient?**

Students __.

Teacher **The quotient is __. So, __** (dividend) **divided by __** (divisor) **equals __. Say that with me.**

Students __ divided by __ equals __.

Teacher **So, if you have __ and divide by __, the quotient is __. Let's review. What's a dividend?**

Students The total number that will be divided.

Teacher **What's a divisor?**

Students The number of groups we will make.

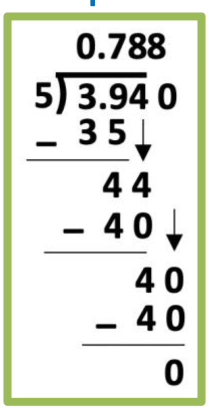
Teacher **What's a quotient?**

Students The result in each group after you equally share or measure groups.

Teacher **How could you explain dividing to a friend?**

Students We asked ourselves about how many groups we can make with the divisor from the dividend. The number of groups is the quotient.

Example



3-DIGIT ÷ 1-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher **Let's work on division. What does it mean to divide?**

Students To share equally or measure into groups.

Teacher **Division means to share equally or to measure into groups. Look at this problem.**
(Show problem.)

Teacher **First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**

Students To divide.

Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tenths and the tenths from the hundredths. Let's draw a vertical line between each of the columns in the dividend.**
(Draw vertical lines to separate place value columns.)

Teacher **Now, we start by dividing the dividend by the divisor. What's our dividend?**

Students	3.94.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	5.
Teacher	**When we divide using this method, for each place value in the dividend, the first thing we do is divide. If we can divide, then we multiply, subtract, and then bring in the next place value. So, the pattern is: divide, multiply, subtract, bring in. Say that with me.**
Students	Divide, multiply, subtract, bring in.
Teacher	**And we keep repeating that pattern until we have solved the problem. Let's see how it works. Are you ready?**
Students	Yes!
Teacher	**Okay, so we start with the greatest place value of the dividend. Where do we start?**
Students	Greatest place value of the dividend.
Teacher	**In this problem, the greatest place value of the dividend is 3. What number?**
Students	3.
Teacher	**How many groups of 5 can we make with 3?**
Students	We can't make any groups of 5 if we have 3.
Teacher	**We can't make a group of 5. So, now we bring in the 9 to make 39. I think how many groups of 39 can we make if we have 5?**
Students	We can make 7 groups of 5.
Teacher	**We can make 7 groups. So, let's write 7 above the division bracket in the tenths column.** (Write 7.)
Teacher	**So, now let's multiply 7 times 5. What's 7 times 5?**
Students	35.
Teacher	**Let's write that product of 35 below the 39 in the dividend.** (Write 35.)
Teacher	**Now, let's write a minus sign and an equal line to help us subtract 35 from 39. What sign?**
Students	Minus sign.
Teacher	**What do we subtract?**
Students	39 minus 35.
Teacher	**What's 39 minus 35?**
Students	4.
Teacher	**Let's write the difference here under the equal line.** (Write 4.)
Teacher	**Now, we bring in the hundredth to our difference. I like to show this by drawing an arrow from the 4 and rewriting the 4 next to 4.** (Draw arrow and write 4.)
Teacher	**When I bring in the 4, 4 now becomes 44. This is our new dividend. What's our new dividend?**
Students	44.

Teacher	**So, we followed the steps of division: divide, multiply, subtract, bring in. Say that with me.**
Students	Divide, multiply, subtract, bring in.
Teacher	**But the problem isn't finished. Let's follow the steps again: divide, multiply, subtract, bring in. What do we do?**
Students	Divide, multiply, subtract, bring in.
Teacher	**How many groups of 5 can we make with our new dividend of 44?**
Students	8 groups.
Teacher	**We can make 8 groups. So, let's write 8 above the division bracket in the hundredths column.** (Write 8.)
Teacher	**So, let's multiply. What's 8 times 5?**
Students	40.
Teacher	**Let's write 40 below the 44**. (Write 40.)
Teacher	**Now, let's write a minus sign and an equal line. What sign?**
Students	Minus sign.
Teacher	**And let's subtract 44 minus 40. What do we subtract?**
Students	44 minus 40.
Teacher	**What's 44 minus 40?**
Students	4.
Teacher	**Let's write the difference here under the equal line.** (Write 4.)
Teacher	**Now, do we have any remaining?**
Students	Yes!
Teacher	**Just like before, we have to bring in a number to keep dividing. This time, I'll bring in from the hundredths place. Is there a number written in the hundredths place?**
Students	No.
Teacher	**There is no number written there. But what number is in the thousandths place just holding place value?**
Students	Zero.
Teacher	**Yes, we assume a zero is in the thousandths place. So, I'll bring in a 0. I like to show this by drawing an arrow from the assumed 0 and writing the 0 next to 4.** (Draw arrow and write 0.)
Teacher	**When I bring in the 0, 4 now becomes 40. This is our new dividend. What's our new dividend?**
Students	40.
Teacher	**Let's follow the steps again: divide, multiply, subtract, bring in. What do we do?**
Students	Divide, multiply, subtract, bring in.
Teacher	**How many groups of 5 can we make with our new dividend of 40?**
Students	8 groups.

Teacher	**We can make 8 groups. So, let's write 8 above the division bracket in the thousandths column.** (Write 8.)
Teacher	**So, let's multiply. What's 8 times 5?**
Students	40.
Teacher	**Let's write 40 below the 40**. (Write 40.)
Teacher	**Now, let's write a minus sign and an equal line. What sign?**
Students	Minus sign.
Teacher	**And let's subtract 40 minus 40. What do we subtract?**
Students	40 minus 40.
Teacher	**What's 40 minus 40?**
Students	0.
Teacher	**Let's write the difference here under the equal line.** (Write 0.)
Teacher	**Now, do we have any remaining?**
Students	No!
Teacher	**Now, we seem finished but we're not. In this problem, we divided decimals. So, we have to place the decimal point in the quotient. What do we have to place in the product?**
Students	A decimal point.
Teacher	**To place the decimal point, we determine the number of decimal places in the dividend and divisor. Let's see. The dividend had 3 decimal places. The divisor had 0 decimal places. What's 3 plus 0?**
Students	3.
Teacher	**So, in the quotient, we need to put in 3 decimal places starting from the least place value (or the right). That means I'll place a decimal point between the 0 and 7.**
Teacher	**So, what's the quotient?**
Students	0.788.
Teacher	**The quotient is 0.788. So, 3.94 divided by 5 equals 0.788. Say that with me.**
Students	3.94 divided by 5 equals 0.788.
Teacher	**So, if you have 3.94 and divide by 5**, **the quotient is 0.788. Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you equally share or measure groups.
Teacher	**How could you explain dividing to a friend?**
Students	We asked ourselves about how many groups we can make with the divisor from the dividend. The number of groups is the quotient.

(3) Division with Partial Quotients Algorithm*

*For clarity, read **Example** before using **Routines**.

Routine

Materials:

- Module 15 Problem Sets
- Module 15 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching

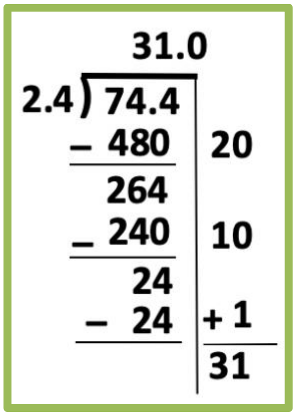
3-DIGIT ÷ 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with our pencil, and let's use the partial quotients strategy. If I want to use the partial quotients strategy, I first draw a vertical line down from the end of the division bracket.** (Draw vertical line from end of division bracket.)
Teacher	**With the partial quotients strategy, we divide the dividend a few different times. Each time we create a partial quotient. At the end, we add the partial quotients to determine the final quotient. Which strategy are we using again?**
Students	Partial quotients.
Teacher	**Now, we start by dividing the dividend by the divisor. What's our dividend?**
Students	__.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	__.
Teacher	**When we divide with decimals, let's ignore all the decimals for now. We'll interpret this as __divided by __. We'll bring back the decimals at the end. What will we ignore for now?**
Students	Decimals.
Teacher	**I don't know exactly how many groups of __** (divisor) **I can make with __** (dividend)**, so the partial quotients strategy can be used with computation that I do know. Which strategy are we using?**
Students	Partial quotients.
Teacher	**How many groups of __** (divisor) **can we make with __** (dividend)**?**
Students	I'm not sure.
Teacher	**I don't know the exact answer either, so I'll use a partial quotient to start solving this problem. I know that __** (friendly number) **groups of __** (divisor)

	would be __ (friendly number times divisor product)**, so I'll write __** (product) **under the __** (dividend)**. I'll also write __** (partial quotient) **to the right of the vertical line. __ is one of my partial quotients.** (Write.)
Teacher	**Now, I'll subtract __** (product) **from the dividend of __ to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**__** (dividend) **minus __** (product) **equals what?**
Students	__.
Teacher	**Let's write the difference of __ below the equal line.** (Write.)
Teacher	**Now, how many groups of __** (divisor) **can we make with __** (new dividend)**?**
Students	I'm not sure.
Teacher	**Again, I don't know the exact answer either, so I'll use a partial quotient. I know that __** (friendly number) **groups of __** (divisor) **would be __. I'll write __** (product) **under the __** (new dividend)**. I'll also write __** (partial quotient) **to the right of the vertical line. __ is one of my partial quotients.** (Write.)
Teacher	**Now, I'll subtract __** (product) **from the dividend of __** (new dividend) **to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**__** (new dividend) **minus __** (product) **equals what?**
Students	__.
Teacher	**Let's write the difference of __ below the equal line.** (Write.)
Teacher	**Now, how many groups of __** (divisor) **can we make with __?**
Students	__.
Teacher	**Yes! I know that __** (friendly number) **groups of __** (divisor) **would be __. I'll write __ under the __** (new dividend)**. I'll also write __** (partial quotient) **to the right of the vertical line. __ is one of my partial quotients.** (Write.)
Teacher	**Now, I'll subtract __ from the dividend of __ to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**__ minus __ equals what?**
Students	__.
Teacher	**Let's write the difference of __ below the equal line.** (Write.)
Teacher	**This is our new dividend. Can we make any more groups of __** (divisor)**?**
Students	No!
Teacher	**We can't make any more groups of __** (divisor)**, so let's determine our quotient. We do this by adding the partial quotients together. How do we determine the quotient?**
Students	Add the partial quotients together.

Teacher	**Let's write a plus sign and equal line.** (Write plus sign and equal line.)
Teacher	**What's __ plus __ plus …?**
Students	__.
Teacher	**Let's write the sum of the partial quotients below the equal line.** (Write.)
Teacher	**We also could write the quotient above the division bracket.** (Write.)
Teacher	**What's the quotient?**
Students	__.
Teacher	**Now, we seem finished but we're not. In this problem, we divided decimals. So, we have to place the decimal point in the quotient. What do we have to place in the quotient?**
Students	A decimal point.
Teacher	**To place the decimal point, we determine the number of decimal places in the dividend and divisor. Let's see. The dividend had __ decimal places. The divisor also had __ decimal places. What's __ plus __?**
Students	__.
Teacher	**So, in the quotient, we need to put in __ decimal places starting from the greatest place value of the quotient. That means I'll place a decimal point __.** (Write decimal point.)
Teacher	**So, what's the quotient?**
Students	__.
Teacher	**So, __ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**Let's say it together again.**
Students	__ divided by __ equals __.
Teacher	**Let's review. What's a dividend?**
Students	The total number that will be divided.
Teacher	**What's a divisor?**
Students	The number of groups we will make.
Teacher	**What's a quotient?**
Students	The result in each group after you equally share or measure groups.
Teacher	**How could you explain partial quotients to a friend?**
Students	We kept asking how many groups of the divisor we could make with the dividend. We didn't know the exact answer, so we used computation we did know as partial quotients. At the end, we added the partial quotients for the final quotient.

Example



3-DIGIT ÷ 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on division. What does it mean to divide?**
Students	To share equally or measure into groups.
Teacher	**Division means to share equally or to measure into groups. Look at this problem.** (Show problem.)
Teacher	**First, I see a division bracket** (point). **The division bracket tells us to divide. What does the division bracket mean?**
Students	To divide.
Teacher	**Let's do this problem with our pencil, and let's use the partial quotients strategy. If I want to use the partial quotients strategy, I first draw a vertical line down from the end of the division bracket.** (Draw vertical line from end of division bracket.)
Teacher	**With the partial quotients strategy, we divide the dividend a few different times. Each time we create a partial quotient. At the end, we add the partial quotients to determine the final quotient. Which strategy are we using again?**
Students	Partial quotients.
Teacher	**Now, we start by dividing the dividend by the divisor. What's our dividend?**
Students	74.4.
Teacher	**And we'll divide the dividend by the divisor. What's the divisor?**
Students	2.4.
Teacher	**When we divide with decimals, let's ignore all the decimals for now. We'll interpret this as 744 divided by 24. We'll bring back the decimals at the end. What will we ignore for now?**
Students	Decimals.
Teacher	**I don't know exactly how many groups of 24 I can make with 744, so the partial quotients strategy can be used with computation that I do know. Which strategy are we using?**
Students	Partial quotients.
Teacher	**How many groups of 24 can we make with 744?**
Students	I'm not sure.

Teacher	**I don't know the exact answer either, so I'll use a partial quotient to start solving this problem. I know that 20 groups of 24 would be 480, so I'll write 480 under the 744. I'll also write 20 to the right of the vertical line. 20 is one of my partial quotients.** (Write 480 and 20.)
Teacher	**Now, I'll subtract 480 from the dividend of 744 to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**744 minus 480 equals what?**
Students	264.
Teacher	**Let's write the difference of 264 below the equal line.** (Write 264.)
Teacher	**Now, how many groups of 24 can we make with 264?**
Students	I'm not sure.
Teacher	**Again, I don't know the exact answer either, so I'll use a partial quotient. I know that 10 groups of 24 would be 240. I'm using computation that's easier for me to do – so I like to think about 20 groups of 24 or 10 groups of 24. So, 10 groups of 24 equals 240. I'll write 240 under the 264. I'll also write 10 to the right of the vertical line. 10 is one of my partial quotients.** (Write 240 and 10.)
Teacher	**Now, I'll subtract 240 from the dividend of 264 to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**264 minus 240 equals what?**
Students	24.
Teacher	**Let's write the difference of 24 below the equal line.** (Write 24.)
Teacher	**Now, how many groups of 24 can we make with 24?**
Students	1!
Teacher	**Yes! I know that 1 group of 24 would be 24. I'll write 24 under the 24. I'll also write 1 to the right of the vertical line. 1 is one of my partial quotients.** (Write 24 and 1.)
Teacher	**Now, I'll subtract 24 from the dividend of 24 to determine a new dividend. I write a minus sign and an equal line.** (Write minus sign and equal line.)
Teacher	**24 minus 24 equals what?**
Students	0.
Teacher	**Let's write the difference of 0 below the equal line.** (Write 0.)
Teacher	**This 0 is our new dividend. Can we make any more groups of 24?**
Students	No!
Teacher	**We can't make any more groups of 24, so let's determine our quotient. We do this by adding the partial quotients together. How do we determine the quotient?**

Students Add the partial quotients together.
Teacher Let's write a plus sign and equal line.
(Write plus sign and equal line.)
Teacher What's 20 plus 10 plus 1?
Students 31.
Teacher Let's write the sum of the partial quotients below the equal line.
(Write 31.)
Teacher We also could write the quotient above the division bracket.
(Write 31.)
Teacher What's the quotient?
Students 31.
Teacher Now, we seem finished but we're not. In this problem, we divided decimals. So, we have to place the decimal point in the quotient. What do we have to place in the quotient?
Students A decimal point.
Teacher To place the decimal point, we determine the number of decimal places in the dividend and divisor. Let's see. The dividend had 1 decimal place. The divisor also had 1 decimal place. What's 1 plus 1?
Students 2.
Teacher So, in the quotient, we need to put in 2 decimal places starting from the greatest place value of the quotient. That means I'll place a decimal point after the 31. I would write 31.0 if I wanted to do so.
(Write decimal point.)
Teacher So, what's the quotient?
Students 31.0.
Teacher So, 74.4 divided by 2.4 equals 31.0. Let's say that together.
Students 74.4 divided by 2.4 equals 31.0.
Teacher Let's say it together again.
Students 74.4 divided by 2.4 equals 31.0.
Teacher Let's review. What's a dividend?
Students The total number that will be divided.
Teacher What's a divisor?
Students The number of groups we will make.
Teacher What's a quotient?
Students The result in each group after you equally share or measure groups.
Teacher How could you explain partial quotients to a friend?
Students We kept asking how many groups of 24 we could make with the dividend. We didn't know the exact answer, so we used computation we did know as partial quotients. At the end, we added the partial quotients for the final quotient.

D. Problems for Use During Instruction

See Module 15 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 15 Vocabulary Cards.

Developed by:
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Module 15:
Division of Rational Numbers

Problem Sets

A.

$$\frac{2}{3} \div \frac{1}{2} =$$

A. $\frac{5}{8} \div \frac{1}{2} =$

A. $$\frac{3}{4} \div \frac{1}{2} =$$

A.

$$\frac{3}{5} \div \frac{2}{3} =$$

A.

$$\frac{1}{6} \div \frac{2}{3} =$$

A.

$$\frac{8}{10} \div \frac{4}{5} =$$

A.

$$\frac{1}{4} \div \frac{1}{2} =$$

A.

$$\frac{1}{5} \div \frac{2}{5} =$$

A.

$$\frac{4}{8} \div \frac{1}{4} =$$

A.

$$\frac{1}{2} \div \frac{3}{4} =$$

A. $\frac{8}{12} \div \frac{1}{6} =$

A.

$$\frac{4}{5} \div \frac{1}{2} =$$

A.

$$\frac{1}{5} \div \frac{2}{3} =$$

A.

$$\frac{1}{3} \div \frac{3}{4} =$$

A.

$$\frac{5}{6} \div \frac{1}{8} =$$

A.

$$\frac{4}{5} \div \frac{3}{4} =$$

A.

$$\frac{4}{6} \div \frac{1}{2} =$$

A.

$$\frac{7}{10} \div \frac{1}{4} =$$

A.

$$\frac{8}{10} \div \frac{3}{4} =$$

A.

$$\frac{5}{6} \div \frac{1}{3} =$$

A.

$$\frac{4}{5} \div \frac{1}{6} =$$

A.

$$\frac{6}{8} \div \frac{1}{4} =$$

A.

$$\frac{5}{8} \div \frac{2}{3} =$$

A.

$$\frac{5}{10} \div \frac{1}{4} =$$

A.

$$\frac{8}{10} \div \frac{2}{5} =$$

A.

$$\frac{6}{9} \div \frac{1}{3} =$$

A. $\frac{1}{4} \div \frac{1}{2} =$

A.

$$\frac{9}{10} \div \frac{2}{5} =$$

A.

$$\frac{2}{5} \div \frac{1}{2} =$$

A.

$$\frac{5}{6} \div \frac{1}{2} =$$

B.

$$\frac{7}{3} \div \frac{1}{2} =$$

B.

$$\frac{6}{5} \div \frac{1}{3} =$$

B.

$$\frac{8}{3} \div \frac{1}{8} =$$

B. $\frac{12}{4} \div \frac{2}{3} =$

B. $\frac{6}{4} \div \frac{1}{8} =$

B.

$$\frac{9}{8} \div \frac{1}{5} =$$

B.

$$\frac{16}{3} \div \frac{1}{8} =$$

B.

$$\frac{6}{4} \div \frac{1}{6} =$$

B.

$$\frac{9}{3} \div \frac{4}{5} =$$

B.

$$\frac{8}{3} \div \frac{2}{3} =$$

B.

$$\frac{6}{5} \div \frac{2}{3} =$$

B. $\frac{4}{3} \div \frac{2}{4} =$

B.

$$\frac{7}{5} \div \frac{1}{2} =$$

B.

$$\frac{5}{2} \div \frac{2}{3} =$$

B. $$\frac{4}{3} \div \frac{3}{8} =$$

c.

$$4\frac{1}{4} \div \frac{1}{2} =$$

c.

$$3\frac{3}{4} \div \frac{1}{6} =$$

c.

$$6\frac{1}{2} \div \frac{1}{2} =$$

c.

$$2\frac{3}{5} \div \frac{2}{5} =$$

c.

$$1\frac{7}{8} \div \frac{3}{4} =$$

c.

$$4\frac{4}{5} \div \frac{4}{5} =$$

c.

$$5\frac{2}{3} \div \frac{1}{5} =$$

c.

$$6\frac{1}{2} \div \frac{1}{4} =$$

c.

$$4\frac{2}{5} \div \frac{1}{3} =$$

c.

$$3\frac{1}{6} \div \frac{2}{3} =$$

c.

$$2\frac{1}{6} \div \frac{1}{8} =$$

c.

$$2\frac{1}{5} \div \frac{1}{2} =$$

c.

$$8\frac{5}{6} \div \frac{5}{6} =$$

c.

$$2\frac{1}{4} \div \frac{1}{2} =$$

c.

$$9\frac{5}{6} \div \frac{1}{3} =$$

D.

$$0.3\overline{)2.4}$$

D.

$$0.5\overline{)4.5}$$

D.

$$0.7\overline{)9.1}$$

D.

$$5\overline{)7.0}$$

D.

$$0.1\overline{)0.8}$$

D.

$$0.6\overline{)4.2}$$

D.

$$0.8\overline{)7.2}$$

D.

$$0.1\overline{)3.6}$$

D.

$$0.2\overline{)2.8}$$

D.

$$0.9\overline{)5.4}$$

D.

$$0.4\overline{)9.6}$$

D.

$$3\overline{)3.6}$$

D.

$$0.7\overline{)7.7}$$

D.

$$0.6\overline{\smash{)}4.8}$$

D.

$$0.3\overline{)5.1}$$

D.

$$8\overline{)7.2}$$

D.

$$0.8\overline{)9.6}$$

D.

$$0.4\overline{)7.2}$$

D.

$$0.5\overline{)9.5}$$

D.

$$0.2\overline{)6.4}$$

E.

$$6\overline{)8.64}$$

E.

$$3\overline{)3.93}$$

E.

$$8\overline{)4.32}$$

E.

$$4\overline{)8.84}$$

E.

$$5\overline{)7.20}$$

E.

$$9\overline{)9.09}$$

E.

$$7\overline{)5.25}$$

E.

$$8\overline{)2.88}$$

E.

$$6\overline{)9.00}$$

E.

$$3\overline{)4.38}$$

E.

$$0.04\overline{)16}$$

E.

$$3\overline{).24}$$

E.

$$5\overline{)3.65}$$

E.

$$4\overline{)12.08}$$

E.

$$.08\overline{\big)\,.64}$$

E.

$$0.09\overline{)10.89}$$

E.

$$0.16\overline{)5.67}$$

E.

$$.45\overline{)14.85}$$

E.

$$6.2\overline{)23.25}$$

E.

$$3\overline{)96.3}$$

F.

$$3.6\overline{)27.68}$$

F.

$$4.18\overline{)41.80}$$

F.

$$0.8\overline{)1.68}$$

F.

$$4.2\overline{)14.28}$$

F.

$$3.3\overline{)20.46}$$

F.

$$2.3\overline{)19.32}$$

F.

$$1.8\overline{)15.48}$$

F.

$$4.8\overline{)5.28}$$

F.

$$1.9\overline{)93.1}$$

F.

$$5.1\overline{)19.38}$$

F.

$$0.9\overline{)9.54}$$

F.

$$7.9\overline{)51.35}$$

F.

$$2.1\overline{)31.5}$$

F.

$$6.8\overline{)88.4}$$

F.

$$6.1\overline{)12.2}$$

F.

$$4.5\overline{)\,17.1}$$

F.

$$7.8\overline{)31.98}$$

F.

$$3.1\overline{)24.18}$$

F.

$$1.7\overline{)54.23}$$

F.

$$3.7\overline{)92.5}$$

F.

$$3.8\overline{)83.6}$$

F.

$$0.3\overline{)79.5}$$

F.

$$3.8\overline{)47.5}$$

F.

$$5.1\overline{)75.99}$$

F.

$$2.1 \overline{)1.47}$$

F.

$$4.8\overline{)47.07}$$

F.

$$4.3\overline{)36.12}$$

F.

$$1.8\overline{)\,7.2\qquad}$$

F.

$$8.4\overline{)24.36}$$

F.

$$1.9\overline{)81.89}$$

G.

$$2\overline{\smash{)}1.09}$$

G.

$$6\overline{)7.18}$$

G.

$$9\overline{)2.69}$$

G.

$$4\overline{)9.83}$$

G.

$$3\overline{)8.41}$$

G.

$$9.13\overline{)9.92}$$

G.

$$5.99\overline{)41.9}$$

G.

$$6.04\overline{\smash{)}75.96}$$

G.

$$6.21\overline{)6.99}$$

G.

$$8.64\overline{)47.4}$$

Module 15:
Division of Rational Numbers

Vocabulary Cards

- algorithm
- computation
- decimal
- denominator
- divide/division
- dividend
- division sign
- divisor
- equal groups
- equal sign
- fractions
- hundredths
- improper fraction
- least common multiple
- mixed number
- numerator
- ones
- quotient
- reciprocal
- remainder
- regroup/trade/exchange
- tenths

algorithm

A procedure or description of steps that can be used to solve a problem.

---

computation

The action used to solve a problem.

decimal

A number based on powers of ten.

34.107

tens, ones, tenths, hundredths, thousandths

---

denominator

The term in a fraction that tells the number of equal parts in a whole.

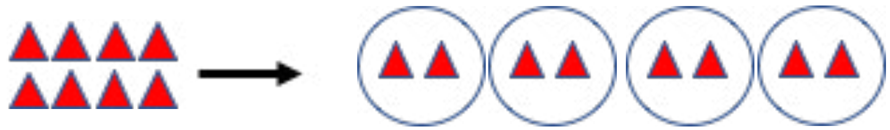
2 / 3

$$\frac{2}{3}$$

In these fractions, 3 is the denominator.

divide/division

To separate into equal groups.



8 ÷ 4 = 2

---

dividend

The number that is to be divided in a division problem.

16 ÷ 8 = 2

16 is the dividend

division sign

The symbol that tells you to divide.

16 ÷ 8 = 2

÷ is the division sign

---

divisor

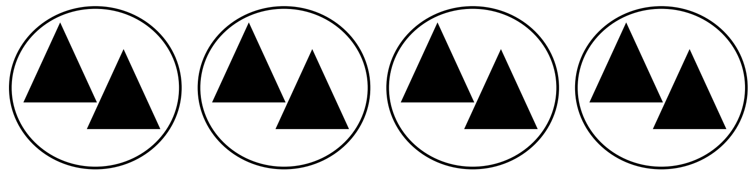
The number that the dividend is divided by.

16 ÷ 8 = 2

8 is the divisor

equal groups

Groups with the same number of objects or items in each group.



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equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$16 \div 8 = 2$$

= is the equal sign

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \qquad \frac{10}{12} \qquad \frac{8}{3}$$

---

hundredths

The digit in representing $\frac{1}{100}$.

In the number 4.23, 3 is in the hundredths place.

improper fraction

Any fraction in which the numerator is greater than the denominator.

$$\frac{9}{4} \qquad \frac{17}{12} \qquad \frac{10}{3}$$

---

least common multiple

The common multiple with the least value.

6: 6, 12, 18, 24, 30
8: 8, 16, 24, 32, 40

With multiples of 6 and 8, the least common multiple is 24.

mixed number

A whole number and a fraction combined.

$$1\frac{1}{6} \qquad 4\frac{5}{12} \qquad 12\frac{4}{3}$$

---

numerator

The term in a fraction that tells how many parts of a fraction.

$$2 / 3 \qquad \frac{2}{3}$$

In these fractions, 2 is the numerator.

ones

The digit representing 1.

In the number 4.23, 4 is in the ones place.

---

quotient

The number that results when one number is divided by another number.

$$16 \div 8 = 2$$

2 is the quotient

reciprocal

The reciprocal of a number is 1 divided by that number.

original number	reciprocal
$\frac{4}{9}$	$\frac{9}{4}$

---

remainder

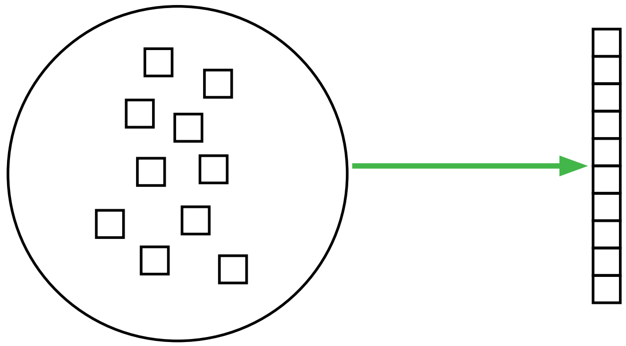
The amount left over in a division problem.

$$\begin{array}{r} 4\ R\,2 \\ 20\overline{)\,82} \\ -\ 80 \\ \hline 2 \end{array}$$

← **2 is the remainder**

regroup/trade/exchange

The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.



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tenths

The digit in representing $\frac{1}{10}$.

In the number 4.2 3, 2 is in the tenths place.

Instructional Routines for Mathematics Intervention

MODULE 16

Representing Decimals

Module 16: Representing Decimals
Mathematics Routines

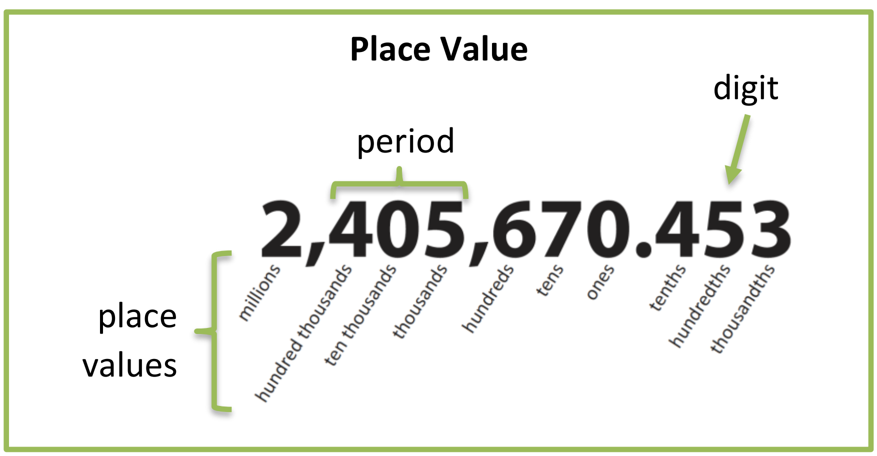
A. Important Vocabulary with Definitions

Term	Definition
decimal	A number based on powers of ten.
decimal point	A dot used to separate ones from tenths in a number or dollars from cents.
hundreds	The digit representing 100.
hundredths	The digit representing 1/100.
ones	The digit representing 1.
place value	The value of a digit depending on its place in a number.
tens	The digit representing 10.
tenths	The digit representing 1/10.
thousands	The digit representing 1,000.
thousandths	The digit representing 1/1,000.

B. Background Information

In this module, we focus on representing decimals. We use two models: (1) Proportional and (2) Non-Proportional.

When referring to decimals, be sure to emphasize place value.



C. Routines and Examples

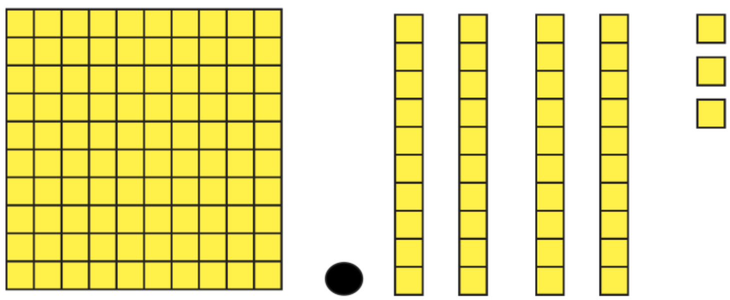
(1) Proportional Models

Routine

Materials:

- Module 16 Problem Sets
- Module 16 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks

ROUTINE WITH BASE-10 BLOCKS



Teacher	**Let's show different decimals. What's a decimal?**
Students	A number with tenths, hundredths, thousandths, etc.
Teacher	**A decimal is a number – just like 2 is a number or $\frac{6}{8}$ is a number. Except with a decimal, we have digits after the decimal point in the tenths, hundredths, thousandths, and so on. What does a decimal have?**
Students	A decimal point and tenths, hundredths, thousandths, etc.
Teacher	**So, let's show different decimals. We'll use these Base-10 blocks.** (Show manipulatives.)
Teacher	**When we show decimals with the Base-10 blocks, we can use them in a different way than we used with thousands, hundreds, tens, and ones. Today, with Base-10 blocks, one cube represents tens. What does a cube represent?**
Students	Tens.
Teacher	**The flat represents ones. What does the flat represent?**
Students	Ones.
Teacher	**The rod represents tenths. What does the rod represent?**
Students	Tenths.
Teacher	**And the unit represents hundredths. What does the unit represent?**
Students	Hundredths.
Teacher	**Let's show this decimal.** (Show decimal.)
Teacher	**What number?**
Students	___.

Teacher **When we read decimals, make sure to only say "and" at the decimal point. So, __** (read number and emphasize "and")**. Let's say that together.**

Students __.

Teacher **Let's show the decimal from the greatest place value to the least place value. For this number, what's the greatest place value?**

Students __.

Teacher **So, what digit is in the __** (place value)**?**

Students __.

Teacher **That means we need to show __** (digit) **__** (place value)**. How many __** (place value)**?**

Students __.

Teacher **How could we use the Base-10 blocks to show __** (digit) **__** (place value)**?**

Students Show __.
(Show using Base-10 blocks.)

Teacher **Did we show the entire number?**

Students No.

Teacher **That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**

Students __.

Teacher **So, what digit is in the __** (place value)**?**

Students __.

Teacher **That means we need to show __** (digit) **__** (place value)**. How many __** (place value)**?**

Students __.

Teacher **How could we use the Base-10 blocks to show __** (digit) **__** (place value)**?**

Students Show __.
(Show using Base-10 blocks.)

Teacher **Did we show the entire number?**

Students No.

Teacher **That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**

Students __.

Teacher **So, what digit is in the __** (place value)**?**

Students __.

Teacher **That means we need to show __** (digit) **__** (place value)**. How many __** (place value)**?**

Students __.

Teacher **How could we use the Base-10 blocks to show __** (digit) **__** (place value)**?**

Students Show __.
(Show using Base-10 blocks.)

Teacher **Did we show the entire number?**

Students Yes!

Teacher **What decimal did we show?**

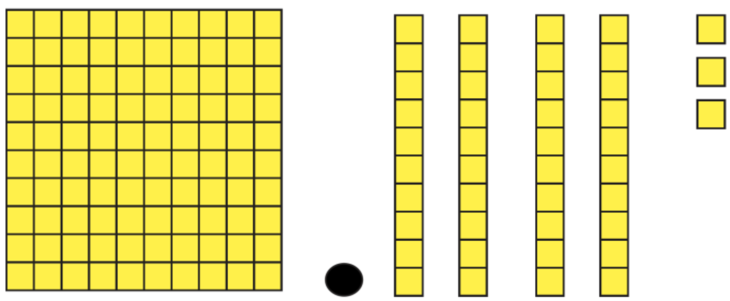
Students __.

Teacher	**Let's count the Base-10 blocks to make sure we showed __ (decimal). Ready?**
Students	___, ___, ___, ...
Teacher	**Great work! Using these Base-10 blocks helps you understand the value of different decimals. How can you use the Base-10 blocks to show a decimal?**
Students	You look at each digit starting with the greatest place value, and show that number of cubes, flats, rods, and units.

Example

1.43

EXAMPLE WITH BASE-10 BLOCKS



Teacher	**Let's show different decimals. What's a decimal?**
Students	A number with tenths, hundredths, thousandths, etc.
Teacher	**A decimal is a number – just like 17 is a number or $\frac{1}{9}$ is a number. Except with a decimal, we have digits after the decimal point in the tenths, hundredths, thousandths, and so on. What does a decimal have?**
Students	A decimal point and tenths, hundredths, thousandths, etc.
Teacher	**So, let's show different decimals. We'll use these Base-10 blocks.** (Show manipulatives.)
Teacher	**When we show decimals with the Base-10 blocks, we can use them in a different way than we used with thousands, hundreds, tens, and ones. Today, with Base-10 blocks, one cube represents tens. What does a cube represent?**
Students	Tens.
Teacher	**The flat represents ones. What does the flat represent?**
Students	Ones.
Teacher	**The rod represents tenths. What does the rod represent?**
Students	Tenths.
Teacher	**And the unit represents hundredths. What does the unit represent?**
Students	Hundredths.
Teacher	**Let's show this decimal.** (Show decimal.)
Teacher	**What number?**
Students	1.43.

Teacher	**When we read decimals, make sure to only say "and" at the decimal point. So, one and forty-three hundredths. Let's say that together.**
Students	One and forty-three hundredths.
Teacher	**Let's show the decimal from the greatest place value to the least place value. For this number, what's the greatest place value?**
Students	Ones place.
Teacher	**So, what digit is in the one place?**
Students	1.
Teacher	**That means we need to show 1 one. How many ones?**
Students	1.
Teacher	**How could we use the Base-10 blocks to show 1 one?**
Students	Show 1 flat. (Show using Base-10 blocks.)
Teacher	**Did we show the entire number?**
Students	No.
Teacher	**That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**
Students	Tenths place.
Teacher	**So, what digit is in the tenths place?**
Students	4.
Teacher	**That means we need to show 4 tenths. How many tenths?**
Students	4.
Teacher	**How could we use the Base-10 blocks to show 4 tenths?**
Students	Show 4 rods. (Show using Base-10 blocks.)
Teacher	**Did we show the entire number?**
Students	No.
Teacher	**That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**
Students	Hundredths place.
Teacher	**So, what digit is in the hundredths place?**
Students	3.
Teacher	**That means we need to show 3 hundredths. How many hundredths?**
Students	3.
Teacher	**How could we use the Base-10 blocks to show 3 hundredths?**
Students	Show 3 units. (Show using Base-10 blocks.)
Teacher	**Did we show the entire number?**
Students	Yes!
Teacher	**What decimal did we show?**
Students	1.43.
Teacher	**Let's count the Base-10 blocks to make sure we showed 1.43. Ready?**
Students	One: 1 tenth, 2 tenths, 3 tenths, 4 tenths, 41 hundredths, 42 hundredths, 43 hundredths. One and forty-three hundredths.

Teacher	**Great work! Using these Base-10 blocks helps you understand the value of different decimals. How can you use the Base-10 blocks to show a decimal?**
Students	You look at each digit starting with the greatest place value, and show the number of cubes, flats, rods, and units.

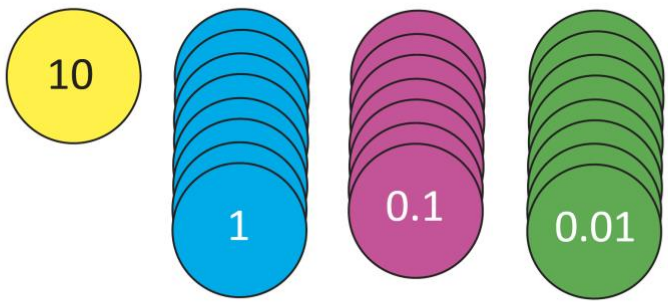
(2) Non-Proportional Models

Routine

Materials:

- Module 16 Problem Sets
- Module 16 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like money or place value disks

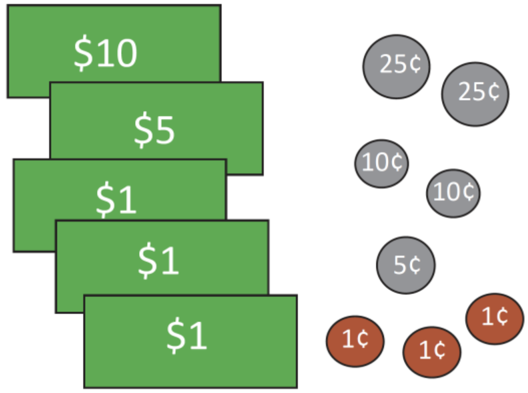
ROUTINE WITH PLACE VALUE DISKS



Teacher	**Let's show different decimals. What's a decimal?**
Students	A number with tenths, hundredths, thousandths, etc.
Teacher	**A decimal is a number – just like 17 is a number or $\frac{5}{8}$ is a number. Except with a decimal, we have digits after the decimal point in the tenths, hundredths, thousandths, and so on. What does a decimal have?**
Students	A decimal point and tenths, hundredths, thousandths, etc.
Teacher	**So, let's show different decimals. We'll use these place value disks.** (Show manipulatives.)
Teacher	**When we show decimals with the place value disks, we look at each disk to read the place value of the disk. I'll show you a few disks, and you tell me the place value of the disk.** (Show different colored disks and ask for the place value.)
Teacher	**Let's show this decimal.** (Show decimal.)
Teacher	**What number?**
Students	__.
Teacher	**When we read decimals, make sure to only say "and" at the decimal point. So, __** (read number and emphasize "and")**. Let's say that together.**
Students	__.
Teacher	**Let's show the decimal from the greatest place value to the least place value. For this number, what's the greatest place value?**
Students	__.
Teacher	**So, what digit is in the __** (place value)**?**
Students	__.

Teacher	**That means we need to show** __ (digit) __ (place value)**. How many** __ (place value)**?**
Students	__.
Teacher	**How could we use the disks to show** __ (digit) __ (place value)**?**
Students	Show __. (Show using place value disks.)
Teacher	**Did we show the entire number?**
Students	No.
Teacher	**That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**
Students	__.
Teacher	**So, what digit is in the** __ (place value)**?**
Students	__.
Teacher	**That means we need to show** __ (digit) __ (place value)**. How many** __ (place value)**?**
Students	__.
Teacher	**How could we use the disks to show** __ (digit) __ (place value)**?**
Students	Show __. (Show using place value disks.)
Teacher	**Did we show the entire number?**
Students	No.
Teacher	**That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**
Students	__.
Teacher	**So, what digit is in the** __ (place value)**?**
Students	__.
Teacher	**That means we need to show** __ (digit) __ (place value)**. How many** __ (place value)**?**
Students	__.
Teacher	**How could we use the disks to show** __ (digit) __ (place value)**?**
Students	Show __. (Show using place value disks.)
Teacher	**Did we show the entire number?**
Students	Yes!
Teacher	**What decimal did we show?**
Students	__.
Teacher	**Let's count the disks to make sure we showed** __ (decimal)**. Ready?**
Students	__, __, __, …
Teacher	**Great work! Using these place value disks helps you understand the value of different decimals. How can you use the disks to show a decimal?**
Students	You look at each digit, starting with the greatest place value, and show each place value using the place value disks.

ROUTINE WITH MONEY



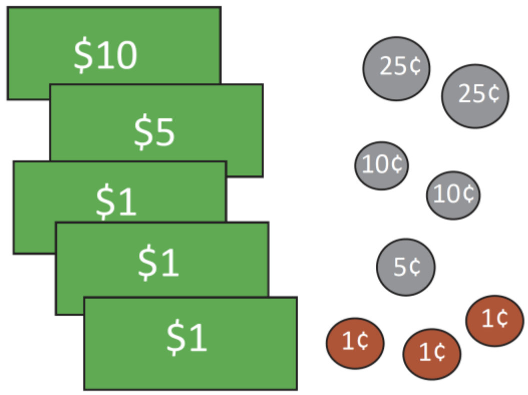
Teacher	**Let's show different decimals. What's a decimal?**
Students	A number with tenths, hundredths, thousandths, etc.
Teacher	**A decimal is a number – just like 43 is a number or $\frac{1}{2}$ is a number. Except with a decimal, we have digits after the decimal point in the tenths, hundredths, thousandths, and so on. What does a decimal have?**
Students	A decimal point and tenths, hundredths, thousandths, etc.
Teacher	**So, let's show different decimals. We'll use this money.** (Show manipulatives.)
Teacher	**When we show decimals with money, we use bills and coins to show the value of a number. Let's show this decimal.** (Show decimal.)
Teacher	**What number?**
Students	__.
Teacher	**When we read decimals, make sure to only say "and" at the decimal point. So, __** (read number and emphasize "and")**. Let's say that together.**
Students	__.
Teacher	**Let's show the decimal from the greatest place value to the least place value. For this number, what's the greatest place value?**
Students	__.
Teacher	**So, what digit is in the __** (place value)**?**
Students	__.
Teacher	**That means we need to show __** (digit) **__** (place value)**. How many __** (place value)**?**
Students	__.
Teacher	**How could we use the money to show __** (digit) **__** (place value)**?**
Students	Show __. (Show using money.)
Teacher	**Did we show the entire number?**
Students	No.
Teacher	**That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**
Students	__.

Teacher	**So, what digit is in the** __ (place value)**?**
Students	__.
Teacher	**That means we need to show** __ (digit) __ (place value)**. How many** __ (place value)**?**
Students	__.
Teacher	**How could we use the money to show** __ (digit) __ (place value)**?**
Students	Show __. (Show using money.)
Teacher	**Did we show the entire number?**
Students	No.
Teacher	**That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**
Students	__.
Teacher	**So, what digit is in the** __ (place value)**?**
Students	__.
Teacher	**That means we need to show** __ (digit) __ (place value)**. How many** __ (place value)**?**
Students	__.
Teacher	**How could we use the money to show** __ (digit) __ (place value)**?**
Students	Show __. (Show using money.)
Teacher	**Did we show the entire number?**
Students	Yes!
Teacher	**What decimal did we show?**
Students	__.
Teacher	**Let's count the money to make sure we showed** __ (decimal)**. Ready?**
Students	__, __, __, …
Teacher	**Great work! Using money helps you understand the value of different decimals. How can you use money to show a decimal?**
Students	You look at each digit, starting with the greatest place value, and use money to show each place value.

Example

$18.78

EXAMPLE WITH MONEY



Teacher	**Let's show different decimals. What's a decimal?**
Students	A number with tenths, hundredths, thousandths, etc.
Teacher	**A decimal is a number – just like 9 is a number or $\frac{1}{9}$ is a number. Except with a decimal, we have digits after the decimal point in the tenths, hundredths, thousandths, and so on. What does a decimal have?**
Students	A decimal point and tenths, hundredths, thousandths, etc.
Teacher	**So, let's show different decimals. We'll use this money.** (Show manipulatives.)
Teacher	**When we show decimals with money, we use bills and coins to show the value of a number. Let's show this decimal.** (Show decimal.)
Teacher	**What number?**
Students	18.78.
Teacher	**When we read decimals, make sure to only say "and" at the decimal point. So, eighteen and seventy-eight hundredths. Let's read that together.**
Students	Eighteen and seventy-eight hundredths.
Teacher	**Let's show the decimal from the greatest place value to the least place value. For this number, what's the greatest place value?**
Students	Tens place.
Teacher	**So, what digit is in the tens place?**
Students	1.
Teacher	**That means we need to show 1 ten. How many tens?**
Students	1.
Teacher	**How could we use the money to show 1 ten?**
Students	Show 1 $10 bill. (Show using money.)
Teacher	**Did we show the entire number?**
Students	No.

Teacher	**That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**
Students	Ones place.
Teacher	**So, what digit is in the ones place?**
Students	8.
Teacher	**That means we need to show 8 ones. How many ones?**
Students	8.
Teacher	**How could we use the money to show 8 ones?**
Students	Show 8 $1 bills or 1 $5 bill and 3 $1 bill.
Teacher	**There are different ways to show 8 ones. Let's use the way that requires the fewest bills: 1 $5 bill and 3 $1 bills.** (Show using money.)
Teacher	**Did we show the entire number?**
Students	No.
Teacher	**That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**
Students	Tenths place.
Teacher	**So, what digit is in the tenths?**
Students	7.
Teacher	**That means we need to show 7 tenths. How many tenths?**
Students	7.
Teacher	**How could we use the money to show 7 tenths?**
Students	Show 7 dimes or 2 quarters and 2 dimes.
Teacher	**There are different ways to show 7 tenths. Let's use the way that requires the fewest coins: 2 quarters and 2 dimes.** (Show using money.)
Teacher	**Did we show the entire number?**
Students	No.
Teacher	**That means we need to look at the next greatest place value. For this number, what's the next greatest place value?**
Students	Hundredths place.
Teacher	**So, what digit is in the hundredths?**
Students	8.
Teacher	**That means we need to show 8 hundredths. How many hundredths?**
Students	8.
Teacher	**How could we use the money to show 8 hundredths?**
Students	Show 8 pennies or 1 nickel and 3 pennies.
Teacher	**There are different ways to show 8 hundredths. Let's use the way that requires the fewest coins: 1 nickel and 3 pennies.** (Show using money.)
Teacher	**Did we show the entire number?**
Students	Yes!
Teacher	**What decimal did we show?**
Students	18.78.

Teacher	**Let's count the money to make sure we showed 18.78. Ready?**
Students	10, 15, 16, 17, 18 dollars. And 25, 50, 60, 70, 75, 76, 77, 78 cents.
Teacher	**Great work! Using money helps you understand the value of different decimals. How can you use money to show a decimal?**
Students	You look at each digit, starting with the greatest place value, and use money to show each place value.

D. Problems for Use During Instruction

See Module 16 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 16 Vocabulary Cards.

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Module 16:
Representing Decimals

Problem Sets

A.

1.4

A.

3.1

A.

16.4

A.

1.8

A.

4.7

A.

8.9

A.

0.2

A.

48.7

A.

79.9

A.

8.2

A.

0.5

A.

93.7

A.

35.2

A.

81.8

A.

0.3

A.

87.2

A.

13.1

A.

36.4

A.

0.4

A.

27.5

B.

0.34

B.

1.52

B.

38.81

B.

2.28

B.

670.04

B.

9.18

B.

0.84

B.

6.19

B.

10.33

B.

89.51

B.

337.69

B.

2.48

B.

0.97

B.

67.23

B.

100.37

B.

0.62

B.

26.75

B.

4.24

B.

51.07

B.

206.36

c.

0.999

c.

2.319

c.

7.751

c.

86.621

c.

9.688

c.

0.617

c.

9.363

c.

33.546

c.

8.514

c.

1.438

c.

2.903

c.

411.523

c.

132.109

c.

0.003

c.

28.052

c.

8.181

c.

674.903

c.

6.813

c.

1.049

c.

234.615

Module 16:
Representing Decimals

Vocabulary Cards

decimal
decimal point
hundreds
hundredths
ones
place value
tens
tenths
thousands
thousandths

decimal

A number based on powers of ten.

34.107

tens ones tenths hundredths thousandths

---

decimal point

A dot used to separate ones from tenths in a number or dollars from cents.

4.2

. is the decimal point

hundreds

The digit representing 100.

---

hundredths

The digit in representing $\frac{1}{100}$.

In the number 4.23, 3 is in the hundredths place.

ones

The digit representing 1.

---

place value

The value of a digit depending on its place in a number.

thousands	hundreds	tens	ones	.	tenths	hundredths	thousandths
8	7	6	5	.	4	3	2

tens

The digit representing 10.

---

tenths

The digit in representing $\frac{1}{10}$.

In the number 4.23, 2 is in the tenths place.

thousands

The digit representing 1,000.

---

thousandths

The digit in representing $\frac{1}{1{,}000}$.

In the number 4.238, 8 is in the thousandths place.

Instructional Routines for Mathematics Intervention

MODULE 17

Integers

Module 17: Integers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
absolute value	The distance of a number from 0 on a number line.
integer	A positive or negative whole number.
negative number	Any number less than 0.
number line	A straight line with numbers placed at equal intervals along its length.
opposites	Two numbers that are equal distance from 0 on a number line.
positive number	Any number greater than 0.
zero pair	A pair of numbers with a sum of 0.

B. Background Information

In this module, we focus on integers. An integer is a positive or negative whole number. We use the following different models to help students understand integers: (1) Number Line, (2) Two-Color Counters, and (3) Positive and Negative Mat with Cubes.

When referring to integers, be sure to emphasize that numbers without a negative symbol (-) are assumed positive. So:

 7 is "positive seven" or "seven."

-7 is "negative seven."

Be sure to use the negative symbol (-), instead of a minus sign (–), for representing negative numbers.

Emphasize *zero pairs* when teaching integers. A zero pair is a pair of numbers with a sum of 0. So, -7 + 7 = 0.

C. Routines and Examples

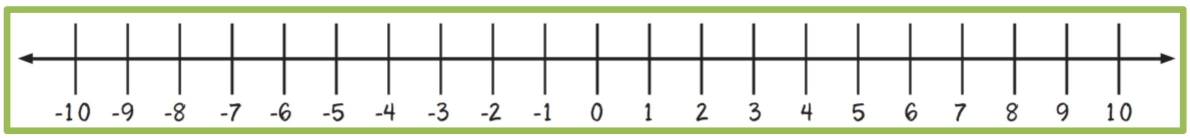
(1) Integers with a Number Line

Routine

Materials:

- Module 17 Problem Sets
- Module 17 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like a number line

ROUTINE WITH NUMBER LINE



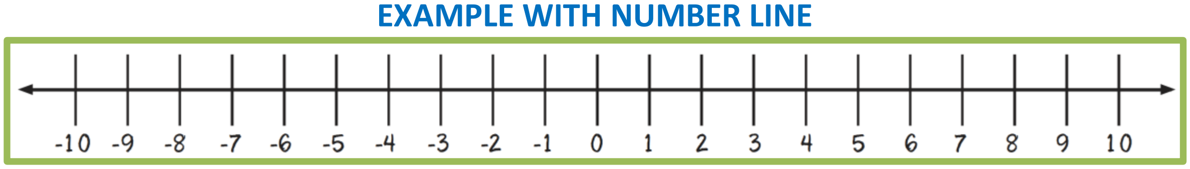
Teacher	**Let's show different integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have any sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**So, let's read a few different numbers. What's this number?** (Write 6.)
Students	Six or positive six.
Teacher	**This is six or positive six. What's this number?** (Write -2.)
Students	Negative two.
Teacher	**Is this number "two?"**
Students	No!
Teacher	**What's this number?**
Students	Negative two.

Teacher	**Yes. This is "negative two." What's this number?** (Write -14.)
Students	Negative fourteen.
Teacher	**This number is negative fourteen.** (Show number line.)
Teacher	**Today, let's show different integers on a number line. What's this number?**
Students	__.
Teacher	**If the number is positive, we will start at zero and move forward or right on the number line. What do we do if a number is positive?**
Students	Start at zero and move forward on the number line.
Teacher	**If the number is negative, we will start at zero and move backward or left on the number line. What do we do if a number is negative?**
Students	Start at zero and move backward on the number line.
Teacher	**Let's show __ on the number line. First, is __ a positive number or negative number?**
Students	__.
Teacher	**__ is a positive/negative number. So, let's place our finger on zero. Where?**
Students	Zero.
Teacher	**Because this number is positive/negative, we move forward/backward __ spaces on the number line. Ready? Count with me.**
Students	__, __, __, …
Teacher	**So, our finger shows where __ falls on the number line. What number did we show?**
Students	__.
Teacher	**Great work! Using this number line helps you understand the value of positive and negative integers. How can you use the number line to show integers?**
Students	Start at zero. If the number is positive, move forward on the number line. If the number is negative, move backward on the number line.

Example

-6

EXAMPLE WITH NUMBER LINE



Teacher	**Let's show different integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**

Students	It has a positive sign or it doesn't have any sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.) (Show number line.)
Teacher	**Today, let's show different integers on a number line. What's this number?**
Students	-6.
Teacher	**If the number is positive, we will start at zero and move forward or right on the number line. What do we do if a number is positive?**
Students	Start at zero and move forward on the number line.
Teacher	**If the number is negative, we will start at zero and move backward or left on the number line. What do we do if a number is negative?**
Students	Start at zero and move backward on the number line.
Teacher	**Let's show -6 on the number line. First, is -6 a positive number or negative number?**
Students	Negative.
Teacher	**-6 is a negative number. So, let's place our finger on zero. Where?**
Students	Zero.
Teacher	**Because this number is negative, we move backward 6 spaces on the number line. Ready? Count with me.**
Students	1, 2, 3, 4, 5, 6.
Teacher	**So, our finger shows where -6 falls on the number line. What number did we show?**
Students	-6.
Teacher	**Great work! Using this number line helps you understand the value of positive and negative integers. How can you use the number line to show integers?**
Students	Start at zero. If the number is positive, move forward on the number line. If the number is negative, move backward on the number line.

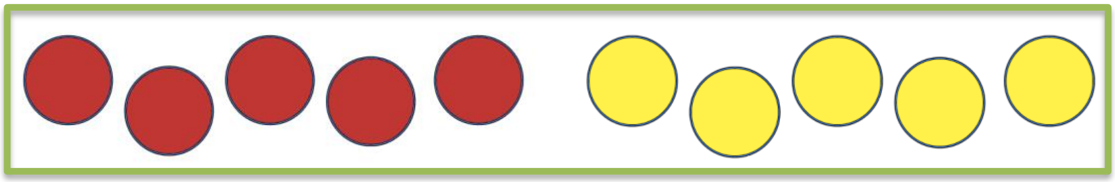
(2) Integers with Two-Color Counters

Routine

Materials:

- [Module 17 Problem Sets]
- [Module 17 Vocabulary Cards]
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like two-color counters or multi-colored cubes

ROUTINE WITH TWO-COLOR COUNTERS



Teacher	**Let's show different integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have any sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**So, let's read a few different numbers. What's this number?** (Write 3.)
Students	Three or positive three.
Teacher	**This is three or positive three. What's this number?** (Write -9.)
Students	Negative nine.
Teacher	**Is this number "nine?"**
Students	No!
Teacher	**What's this number?**
Students	Negative nine.
Teacher	**Yes. This is "negative nine." What's this number?** (Write -13.)

Students Negative thirteen.

Teacher **This number is negative thirteen.**

(Show counters.)

Teacher **Today, let's show different integers with two-color counters. With the two-color counters, we'll use the yellow side to show positive integers. What will the yellow side represent?**

Students Positive integers.

Teacher **We'll use the red side to show negative integers. What will the red side represent?**

Students Negative integers.

Teacher **Let's show a number. What's this number?**

Students __.

Teacher **Let's show __ with the two-color counters. First, is __ a positive number or negative number?**

Students __.

Teacher **__ is a positive/negative number. So, which color will we use?**

Students Yellow/red.

Teacher **Because this number is positive/negative, we'll use the yellow/red side. We need to show __, so let's show __ yellow/red counters. Count with me.**

Students __, __, __, …

Teacher **So, we showed __. What number did we show?**

Students __.

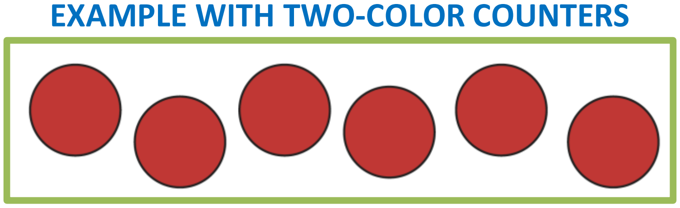
Teacher **Great work! Using the two-color counters helps you show positive and negative integers. How can you use the two-color counters to show integers?**

Students The yellow side represents positive integers. The red side represents negative integers. To show a positive integer, show the yellow counters. To show a negative integer, show the red counters.

Example

-6

EXAMPLE WITH TWO-COLOR COUNTERS



Teacher **Let's show different integers. An integer is a positive or negative whole number. What's an integer?**

Students A positive or negative whole number.

Teacher **Let's think about a positive number. How do you know a number is positive?**

Students It has a positive sign or it doesn't have any sign in front of the number.

Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.) (Show counters.)
Teacher	**Today, let's show different integers with two-color counters. With the two-color counters, we'll use the yellow side to show positive integers. What will the yellow side represent?**
Students	Positive integers.
Teacher	**We'll use the red side to show negative integers. What will the red side represent?**
Students	Negative integers.
Teacher	**Let's show a number. What's this number?**
Students	-6.
Teacher	**Let's show -6 with the two-color counters. First, is -6 a positive number or negative number?**
Students	Negative.
Teacher	**-6 is a negative number. So, which color will we use?**
Students	Red.
Teacher	**Because this number is negative, we'll use the red side. We need to show -6, so let's show 6 red counters. Count with me.**
Students	1, 2, 3, 4, 5, 6.
Teacher	**So, we showed -6. What number did we show?**
Students	-6.
Teacher	**Great work! Using the two-color counters helps you show positive and negative integers. How can you use the two-color counters to show integers?**
Students	The yellow side represents positive integers. The red side represents negative integers. To show a positive integer, show the yellow counters. To show a negative integer, show the red counters.

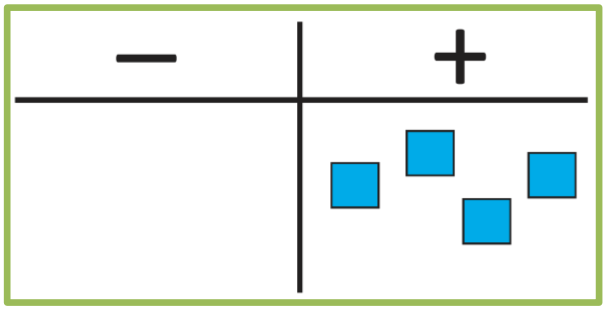
(3) Integers with Positive and Negative Mat

Routine

Materials:

- Module 17 Problem Sets
- Module 17 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes or paperclips

ROUTINE WITH POSITIVE AND NEGATIVE MAT



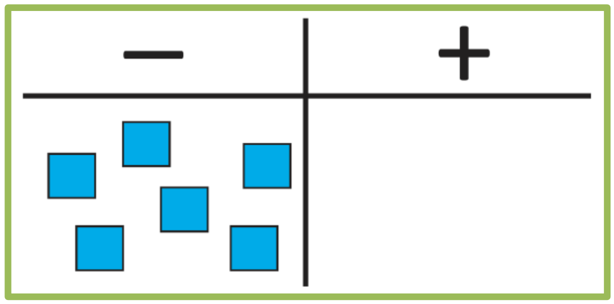
Teacher	**Let's show different integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have any sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**So, let's read a few different numbers. What's this number?** (Write 7.)
Students	Seven or positive seven.
Teacher	**This is seven or positive seven. What's this number?** (Write -1.)
Students	Negative one.
Teacher	**Is this number "one?"**
Students	No!

Teacher	**What's this number?**
Students	Negative one.
Teacher	**Yes. This is "negative one." What's this number?** (Write -24.)
Students	Negative twenty-four.
Teacher	**This number is negative twenty-four.** (Show mat and cubes.)
Teacher	**Today, let's show different integers with this positive and negative mat and these cubes. With the mat, we'll place positive integers on this positive side** (point)**. Where will we place positive integers?**
Students	Positive side of mat.
Teacher	**We'll place negative integers on this negative side** (point)**. Where will we place negative integers?**
Students	Negative side of mat.
Teacher	**Let's show a number. What's this number?**
Students	__.
Teacher	**Let's show __ with the cubes. First, is __ a positive number or negative number?**
Students	__.
Teacher	**__ is a positive/negative number. So, where will we place the cubes? On the positive side or negative side?**
Students	Positive/negative.
Teacher	**Because this number is positive/negative, we'll place the cubes on the positive/negative side. We need to show __, so let's show __ cubes on the positive/negative side of the mat. Count with me.**
Students	__, __, __, …
Teacher	**So, we showed __. What number did we show?**
Students	__.
Teacher	**Great work! Using the positive and negative mat helps you show positive and negative integers. How can you use the mat to show integers?**
Students	You use the cubes and place positive integers on the positive side of the mat. You use the cubes and place negative integers on the negative side of the mat.

Example

-6

EXAMPLE WITH POSITIVE AND NEGATIVE MAT



Teacher	**Let's show different integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have any sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.) (Show mat and cubes.)
Teacher	**Today, let's show different integers with this positive and negative mat and these cubes. With the mat, we'll place positive integers on this positive side** (point)**. Where will we place positive integers?**
Students	Positive side of mat.
Teacher	**We'll place negative integers on this negative side** (point)**. Where will we place negative integers?**
Students	Negative side of mat.
Teacher	**Let's show a number. What's this number?**
Students	-6.
Teacher	**Let's show -6 with the cubes. First, is -6 a positive number or negative number?**
Students	Negative.
Teacher	**-6 is a negative number. So, where will we place the cubes? On the positive side or negative side?**

Students	Negative.
Teacher	**Because this number is negative, we'll place the cubes on the negative side. We need to show -6, so let's show 6 cubes on the negative side of the mat. Count with me.**
Students	1, 2, 3, 4, 5, 6.
Teacher	**So, we showed -6. What number did we show?**
Students	-6_.
Teacher	**Excellent! Using the positive and negative mat helps you show positive and negative integers. How can you use the mat to show integers?**
Students	You use the cubes and place positive integers on the positive side of the mat. You use the cubes and place negative integers on the negative side of the mat.

D. Problems for Use During Instruction

See Module 17 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 17 Vocabulary Cards.

Developed by:
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Katherine A. Berry (kberry@austin.utexas.edu)

Module 17:

Integers

Problem Sets

A.

3

A.

26

A.

10

A.

4

A.

14

A.

24

A.

9

A.

15

A.

2

A.

13

A.

17

A.

5

A.

19

A.

16

A.

12

A.

29

A.

20

A.

1

A.

18

A.

27

A.

25

A.

6

A.

11

A.

22

A.

28

A.

23

A.

8

A.

0

A.

21

A.

7

B.

-5

B.

-8

B.

-25

B.

-14

B.

-11

B.

-19

B.

-16

B.

-21

B.

-6

B.

-2

B.

-13

B.

-23

B.

-7

B.

-1

B.

-20

B.

-9

B.

-26

B.

-17

B.

-27

B.

-15

B.

-30

B.

-10

B.

-28

B.

-3

B.

-29

B.

-24

B.

-12

B.

-22

B.

-18

B.

-4

Module 17:

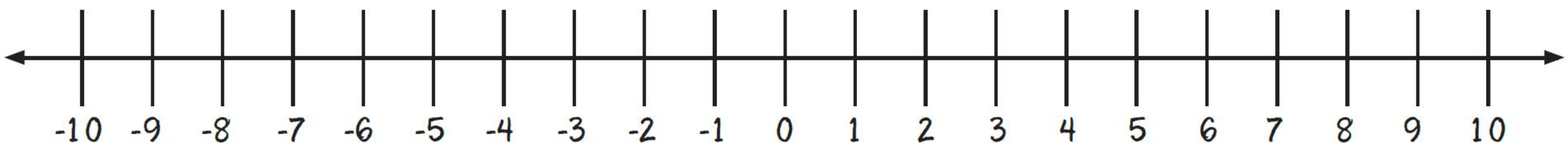
Integers

Vocabulary Cards

absolute value
integer
negative number
number line
opposites
positive number
zero pair

absolute value

The distance of a number from 0 on a number line.



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integer

A positive or negative whole number.

-3 -2 -1 1 2 3

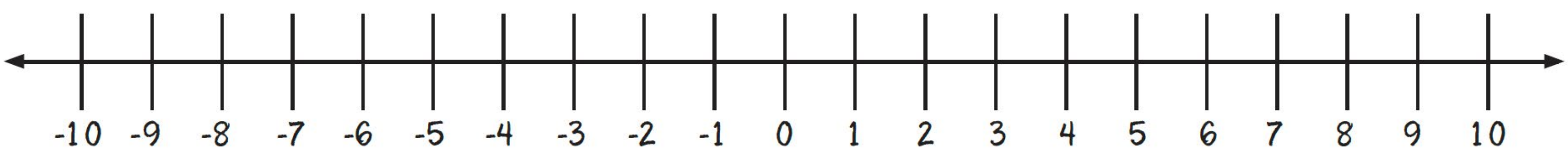
negative number

Any number less than 0.

-3 -2 -1

number line

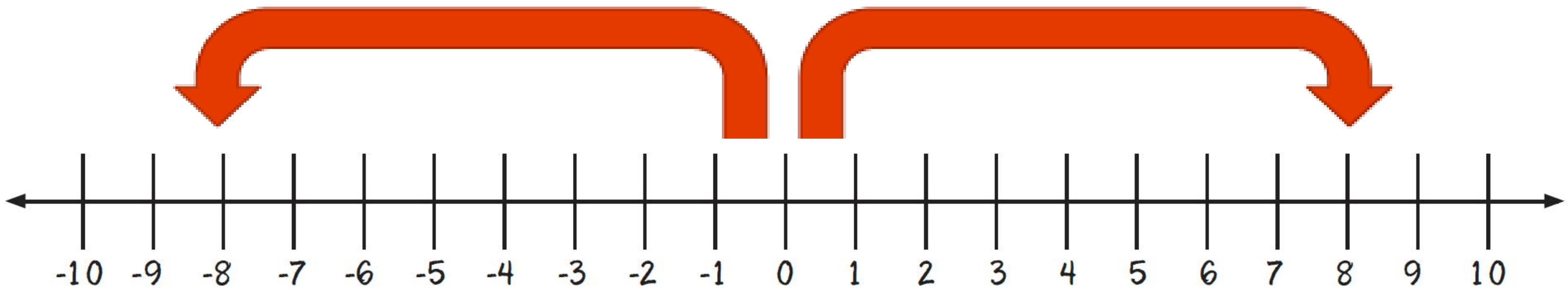
A straight line with numbers placed at equal intervals along its length.



opposites

Two numbers that are equal distance from 0 on a number line.

-8 and 8 are opposites



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positive number

Any number greater than 0.

1 2 3

zero pair

A pair of numbers with a sum of 0.

$$-7 + 7 = 0$$

Instructional Routines for Mathematics Intervention

MODULE 18

Addition and Subtraction of Integers

Module 18: Addition and Subtraction of Integers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
absolute value	The distance of a number from 0 on a number line.
addend	Any numbers added together.
difference	The result of subtracting one number from another number.
integer	A positive or negative whole number.
minuend	The number from which another number is subtracted.
negative number	Any number less than 0.
number line	A straight line with numbers placed at equal intervals along its length.
opposites	Two numbers that are equal distance from 0 on a number line.
positive number	Any number greater than 0.
subtrahend	The number to be subtracted.
sum	The result of adding two or more numbers or the total number when you combine sets.
zero pair	A pair of numbers with a sum of 0.

B. Background Information

In this module, we focus on addition and subtraction of integers. An integer is a positive or negative whole number. We use the following different models to help students understand addition and subtraction of integers:

(1) Addition with a Number Line
(2) Subtraction with a Number Line
(3) Addition with Two-Color Counters
(4) Subtraction with Two-Color Counters
(5) Addition with a Positive and Negative Mat with Cubes
(6) Subtraction with a Positive and Negative Mat with Cubes

When referring to integers, be sure to emphasize that numbers without a negative symbol (-) are assumed positive. So:

7 is “positive seven” or “seven.”
-7 is “negative seven.”

Be sure to use the negative symbol (-), instead of a minus sign (–), for representing negative numbers.

Emphasize *zero pairs* when teaching integers. A zero pair is a pair of numbers with a sum of 0. So, -7 + 7 = 0.

C. Routines and Examples

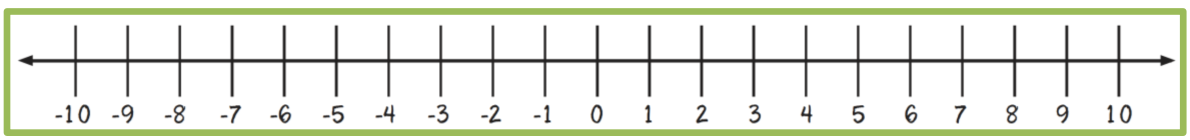
(1) Addition with a Number Line

Routine

Materials:

- Module 18 Problem Sets
- Module 18 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A number line and a manipulative with a face (e.g., duck or dinosaur)

ROUTINE WITH NUMBER LINE



Teacher	**Let's add integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on adding with this number line.** (Show number line.) (Show problem.)

Teacher **What numbers are we adding?**

Students ___ plus ___.

Teacher **So, let's start at the first addend. What's the first addend?**

Students ___.

Teacher **Let's place the duck on the number line at the first addend. When adding, we'll place duck so it is facing the increasing numbers on the number line.** (Place duck on first addend. Make sure duck is facing increasing number on number line.)

Teacher **Now, let's add. What number do we add?**

Students ___.

Teacher **If the second addend is positive, we move forward on the number line. What do we do if the second addend is positive?**

Students Move forward on the number line.

Teacher **If the second addend is negative, we move backward on the number line. What do we do if the second addend is negative?**

Students Move backward on the number line.

Teacher **So, which direction should we move?**

Students Forward/backward.

Teacher **Because the second addend is positive/negative, we move forward/backward ___ spaces. Let's do that together. Count with me.**

Students ___, ___, ___, …

Teacher **So, our duck shows the sum. What's ___ plus ___?**

Students ___.

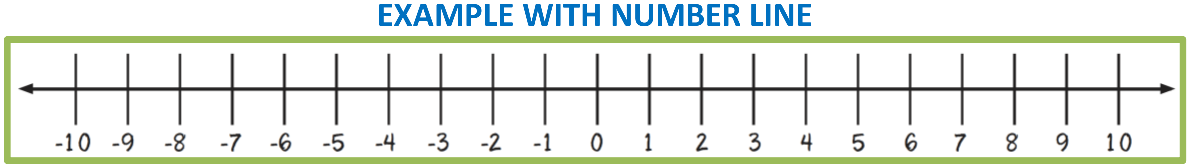
Teacher **Yes. ___ plus ___ equals ___. Using this number line helps you understand what it means to add integers. How can you use the number line to add integers?**

Students Start at the first addend. If the second addend is positive, move forward on the number line. If the second addend is negative, move backward on the number line.

Example

-3 + 5

EXAMPLE WITH NUMBER LINE



Teacher **Let's add integers. An integer is a positive or negative whole number. What's an integer?**

Students A positive or negative whole number.

Teacher **Let's think about a positive number. How do you know a number is positive?**

Students It has a positive sign or it doesn't have a sign in front of the number.

Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on adding with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we adding?**
Students	-3 plus 5.
Teacher	**So, let's start at the first addend. What's the first addend?**
Students	-3.
Teacher	**Let's place the duck on the number line at the first addend. When adding, we'll place the duck so it is facing the increasing numbers on the number line.** (Place duck on -3.)
Teacher	**Now, let's add. What number do we add?**
Students	5.
Teacher	**If the second addend is positive, we move forward on the number line. What do we do if the second addend is positive?**
Students	Move forward on the number line.
Teacher	**If the second addend is negative, we move backward on the number line. What do we do if the second addend is negative?**
Students	Move backward on the number line.
Teacher	**So, which direction should we move?**
Students	Forward.
Teacher	**Because the second addend is positive, we move forward 5 spaces. Let's do that together. Count with me.**
Students	1, 2, 3, 4, 5.
Teacher	**So, our duck shows the sum. What's -3 plus 5?**
Students	2.
Teacher	**Yes. -3 plus 5 equals 2. Using this number line helps you understand what it means to add integers. How can you use the number line to add integers?**
Students	Start at the first addend. If the second addend is positive, move forward on the number line. If the second addend is negative, move backward on the number line.

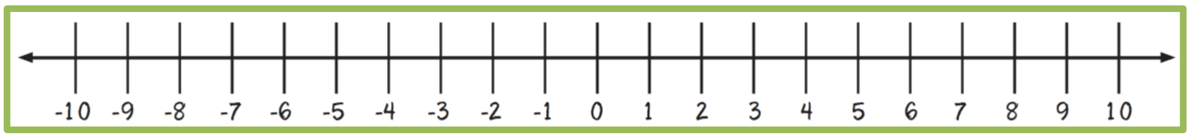
(2) Subtraction with a Number Line

Routine

Materials:

- Module 18 Problem Sets
- Module 18 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A number line and a manipulative with a face (e.g., duck or dinosaur)

ROUTINE WITH NUMBER LINE



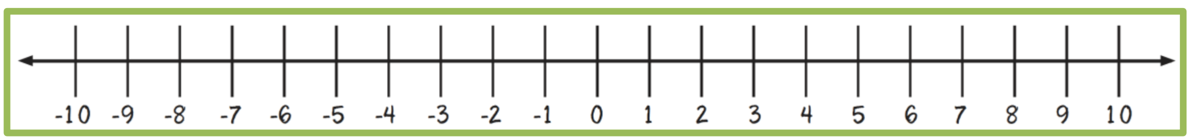
Teacher	**Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on subtracting with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we subtracting?**
Students	__ minus __.
Teacher	**So, let's start at the minuend. What's the minuend?**
Students	__.
Teacher	**Let's place the duck on the number line at the minuend. When subtracting, we'll place the duck so it is facing the decreasing numbers on the number line.** (Place duck on minuend. Place duck facing the decreasing numbers on the number line.)
Teacher	**Now, let's subtract. What number do we subtract? What's the subtrahend?**
Students	__.

Teacher	**If the subtrahend is positive, we move the duck forward on the number line from where the duck is facing. What do we do if the subtrahend is positive?**
Students	Move forward on the number line from where the duck is facing.
Teacher	**If the subtrahend is negative, we move backward on the number line from where the duck is facing. What do we do if the subtrahend is negative?**
Students	Move backward on the number line from where the duck is facing.
Teacher	**So, which direction should we move?**
Students	Forward/backward.
Teacher	**Because the subtrahend is positive/negative, we move forward/backward __ spaces. Let's do that together. Count with me.**
Students	___, ___, ___, …
Teacher	**So, our duck shows the difference. What's __ minus __?**
Students	___.
Teacher	**Yes. __ minus __ equals __. Using this number line helps you understand what it means to subtract integers. How can you use the number line to subtract integers?**
Students	Start at the minuend with the duck facing the decreasing numbers on the number line. If the subtrahend is positive, move the duck forward from its position. If the subtrahend is negative, move the duck backward from its position.

Example

4 – (-3)

EXAMPLE WITH NUMBER LINE



Teacher	**Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.

Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on subtracting with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we subtracting?**
Students	4 minus (-3).
Teacher	**So, let's start at the minuend. What's the minuend?**
Students	4.
Teacher	**Let's place the duck on the number line at the minuend. When subtracting, we'll place the duck so it is facing the decreasing numbers on the number line.** (Place duck on minuend. Place duck facing the decreasing numbers on the number line.)
Teacher	**Now, let's subtract. What number do we subtract? What's the subtrahend?**
Students	-3.
Teacher	**If the subtrahend is positive, we move the duck forward on the number line from where the duck is facing. What do we do if the subtrahend is positive?**
Students	Move forward on the number line from where the duck is facing.
Teacher	**If the subtrahend is negative, we move backward on the number line from where the duck is facing. What do we do if the subtrahend is negative?**
Students	Move backward on the number line from where the duck is facing.
Teacher	**So, which direction should we move?**
Students	Backward.
Teacher	**Because the subtrahend is negative, we move backward 3 spaces. Let's do that together. Count with me.**
Students	1, 2, 3.
Teacher	**So, our duck shows the difference. What's 4 minus -3?**
Students	7.
Teacher	**Yes. 4 minus -3 equals 7. Using this number line helps you understand what it means to subtract integers. How can you use the number line to subtract integers?**
Students	Start at the minuend with the duck facing the decreasing numbers on the number line. If the subtrahend is positive, move the duck forward from its position. If the subtrahend is negative, move the duck backward from its position.

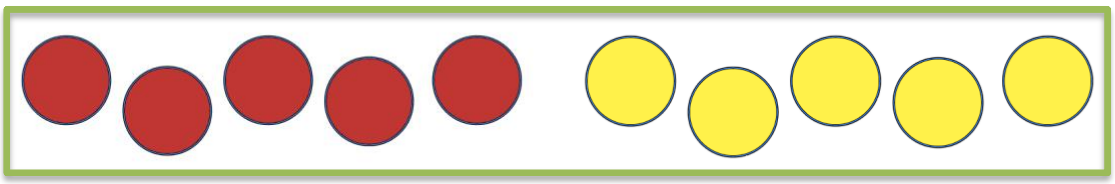
(3) Addition with Two-Color Counters

Routine

Materials:

- [Module 18 Problem Sets]
- [Module 18 Vocabulary Cards]
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like two-color counters or multi-colored cubes

ROUTINE WITH TWO-COLOR COUNTERS



Teacher	**Let's add integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on adding with these two-color counters.** (Show counters.)
Teacher	**With the two-color counters, the yellow side will represent positive integers. What does the yellow side represent?**
Students	Positive.
Teacher	**And the red side will represent negative integers. What does the red side represent?**
Students	Negative. (Show problem.)
Teacher	**What numbers are we adding?**
Students	__ plus __.
Teacher	**So, let's start at the first addend. What's the first addend?**

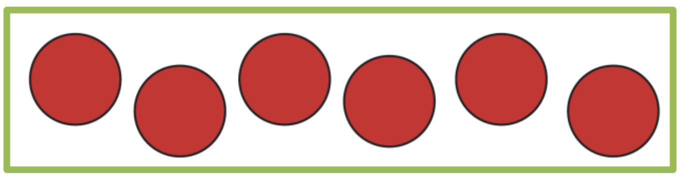
Students	__.	
Teacher	**Let's show the first addend with the two-color counters. How do we show __** (first addend)**?**	
Students	Show __ yellow/red counters.	
Teacher	**Yes, we'll show __ yellow/red counters.** (Show counters.)	
Teacher	**Now, let's add. What number do we add?**	
Students	__.	
Teacher	**Let's add the second addend to the first. How do we show __** (second addend)**?**	
Students	Show __ yellow/red counters.	
Teacher	**Yes, we'll show __ yellow/red counters.** (Show counters.)	
Teacher	**Let's add the counters of the second addend to the counters of the first addend.** (Add all counters together.)	
Teacher	**Are the counters all the same color?**	
Students	*OPTION 1:*	Yes!
	OPTION 2:	No!
Teacher	***OPTION 2:***	**When we have a mix of yellow and red counters, we'll make zero pairs until we only have all positive counters or all negative counters. What we will make?**
Students		Zero pairs.
Teacher		**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students		When you add a positive number and its opposite. The sum is zero.
Teacher		**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students		0.
Teacher		**Let's make zero pairs. I see a positive and a negative** (place counters side-by-side)**. What did we create?**
Students		A zero pair.
Teacher		**We made a zero pair. We can remove this pair from our workspace.** (Remove zero pair.)
Teacher		**Can we make another zero pair?** (Continue removing zero pairs until there are no more zero pairs.)
Teacher		**Are the counters all the same color?**
Students		Yes!
Teacher	**All of the counters are the same color. How many counters?**	
Students	__.	
Teacher	**So, what's __ plus __?**	
Students	__.	

Teacher **__ plus __ equals __. Let's say that together.**
Students __ plus __ equals __.
Teacher **Nice job! Using the two-color counters helps you add integers. How can you use the two-color counters to add integers?**
Students You show the first addend. Then, you add the second addend. You create zero pairs until all the counters are positive or negative. The sum is the remaining counters.

Example

2 + -8

EXAMPLE WITH TWO-COLOR COUNTERS



Teacher **Let's add integers. An integer is a positive or negative whole number. What's an integer?**
Students A positive or negative whole number.
Teacher **Let's think about a positive number. How do you know a number is positive?**
Students It has a positive sign or it doesn't have a sign in front of the number.
Teacher **We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.**
(Draw +.)
Teacher **We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students When there is not a negative sign directly in front of the number.
Teacher **How do you know a number is negative?**
Students It has a negative sign.
Teacher **We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.**
(Draw -.)
Teacher **Let's work on adding with these two-color counters.**
(Show counters.)
Teacher **With the two-color counters, the yellow side will represent positive integers. What does the yellow side represent?**
Students Positive.
Teacher **And the red side will represent negative integers. What does the red side represent?**
Students Negative.
(Show problem.)

Teacher	**What numbers are we adding?**
Students	2 plus -8.
Teacher	**So, let's start at the first addend. What's the first addend?**
Students	2
Teacher	**Let's show the first addend with the two-color counters. How do we show 2?**
Students	Show 2 yellow counters.
Teacher	**Yes, we'll show 2 yellow counters.** (Show counters.)
Teacher	**Now, let's add. What number do we add?**
Students	-8.
Teacher	**How do we show -8?**
Students	Show 8 red counters.
Teacher	**Yes, we'll show 8 red counters.** (Show counters.)
Teacher	**Let's add the counters of the second addend to the counters of the first addend.** (Add all counters together.)
Teacher	**Are the counters all the same color?**
Students	No!
Teacher	**When we have a mix of yellow and red counters, we'll make zero pairs until we only have all positive counters or all negative counters. What we will make?**
Students	Zero pairs.
Teacher	**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students	When you add a positive number and its opposite. The sum is zero.
Teacher	**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students	0.
Teacher	**Let's make zero pairs. I see a positive and a negative** (place counters side-by-side)**. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. We can remove this pair from our workspace.** (Remove zero pair.)
Teacher	**Can we make another zero pair?**
Students	Yes!
Teacher	**We can make another zero pair. We can remove this pair from our workspace.** (Remove zero pair.)
Teacher	**Can we make another zero pair?**
Students	No.
Teacher	**All of the counters are the same color. How many counters?**
Students	-6.
Teacher	**So, what's 2 plus -8?**
Students	-6.

Teacher	**2 plus -8 equals -6. Let's say that together.**
Students	2 plus -8 equals -6.
Teacher	**Nice job! Using the two-color counters helps you add integers. How can you use the two-color counters to add integers?**
Students	You show the first addend. Then, you add the second addend. You create zero pairs until all the counters are positive or negative. The sum is the remaining counters.

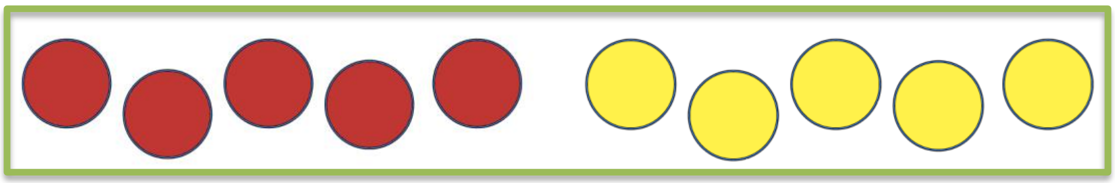
(4) Subtraction with Two-Color Counters

Routine

Materials:

- [Module 18 Problem Sets]
- [Module 18 Vocabulary Cards]
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like two-color counters or multi-colored cubes

ROUTINE WITH TWO-COLOR COUNTERS



Teacher	**Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on subtracting with these two-color counters.** (Show counters.)
Teacher	**With the two-color counters, the yellow side will represent positive integers. What does the yellow side represent?**
Students	Positive.
Teacher	**And the red side will represent negative integers. What does the red side represent?**
Students	Negative. (Show problem.)
Teacher	**What numbers are we subtracting?**
Students	__ minus __.
Teacher	**So, let's start with the minuend. What's the minuend?**

Students	__.	
Teacher	**Let's show the minuend with the two-color counters. How do we show __** (minuend)**?**	
Students	Show __ yellow/red counters.	
Teacher	**Yes, we'll show __ yellow/red counters.** (Show counters.)	
Teacher	**Now, let's subtract. What's the subtrahend?**	
Students	__.	
Teacher	**We need to subtract __** (subtrahend). **Look at the minuend. Do we have __ positive/negative counters to subtract or take away __** (subtrahend) **counters?**	
Students	*OPTION 1:*	Yes!
	OPTION 2:	No!
Teacher	***OPTION 2:***	**We don't have enough positive/negative counters to subtract the minuend. We can make zero pairs until we have enough counters to subtract the __** (subtrahend) **counters. What we will make?**
Students		Zero pairs.
Teacher		**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students		When you add a positive number and its opposite. The sum is zero.
Teacher		**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students		0.
Teacher		**Let's make zero pairs for our workspace. Let's add one zero pair. What did we create?**
Students		A zero pair.
Teacher		**We made a zero pair. Let's bring that zero pair to the workspace.** (Add zero pair.)
Teacher		**Look at the counters. Do we have enough __** (positive/negative) **counters to subtract __** (subtrahend) **counters?**
Students		Yes/no.
Teacher		**We keep making zero pairs until we have enough positive/negative counters to subtract __** (subtrahend) **counters.** (Continue making zero pairs until there are enough positive/negative counters to subtract.)
Teacher	**Now, let's subtract the subtrahend. That means we'll subtract or take away __ counters.** (Subtract.)	
Teacher	**How many counters?**	
Students	__.	

Teacher **So, what's __ minus __?**

Students __.

Teacher **__ minus __ equals __. Let's say that together.**

Students __ minus__ equals __.

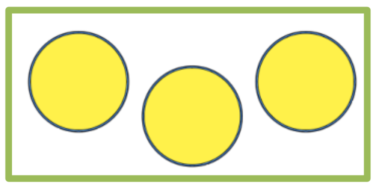
Teacher **Nice job! Using the two-color counters helps you subtract integers. How can you use the two-color counters to subtract integers?**

Students You show the minuend. Then, you subtract the subtrahend. If you don't have enough minuend counters to subtract, you can bring in zero pairs. Then, you subtract. The difference is the remaining counters.

Example

-1 – (-4)

EXAMPLE WITH TWO-COLOR COUNTERS



Teacher **Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**

Students A positive or negative whole number.

Teacher **Let's think about a positive number. How do you know a number is positive?**

Students It has a positive sign or it doesn't have a sign in front of the number.

Teacher **We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.**
(Draw +.)

Teacher **We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**

Students When there is not a negative sign directly in front of the number.

Teacher **How do you know a number is negative?**

Students It has a negative sign.

Teacher **We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.**
(Draw -.)

Teacher **Let's work on subtracting with these two-color counters.**
(Show counters.)

Teacher **With the two-color counters, the yellow side will represent positive integers. What does the yellow side represent?**

Students Positive.

Teacher **And the red side will represent negative integers. What does the red side represent?**

Students	Negative.
	(Show problem.)
Teacher	**What numbers are we subtracting?**
Students	-1 minus -4.
Teacher	**So, let's start with the minuend. What's the minuend?**
Students	-1.
Teacher	**Let's show the minuend with the two-color counters. How do we show -1?**
Students	Show 1 red counter.
Teacher	**Yes, we'll show 1 red counter.**
	(Show counter.)
Teacher	**Now, let's subtract. What's the subtrahend?**
Students	-4.
Teacher	**We need to subtract -4 or 4 red counters. Look at the minuend. Do we have enough negative counters to subtract or take away 4 counters?**
Students	No!
Teacher	**We don't have enough negative counters to subtract the minuend. We can make zero pairs until we have enough counters to subtract the 4 counters. What we will make?**
Students	Zero pairs.
Teacher	**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students	When you add a positive number and its opposite. The sum is zero.
Teacher	**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students	0.
Teacher	**Let's make zero pairs for our workspace. Let's add one zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**
	(Add zero pair.)
Teacher	**Look at the counters. Now we have 2 negative counters. Do we have enough negative counters to subtract 4 counters?**
	No.
Teacher	**Let's add another zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**
	(Add zero pair.)
Teacher	**Look at the counters. Now we have 3 negative counters. Do we have enough negative counters to subtract 4 counters?**
	No.
Teacher	**Let's add another zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**
	(Add zero pair.)

Teacher	**Look at the counters. Now we have 4 negative counters. Do we have enough negative counters to subtract 4 counters?**
Students	Yes!
Teacher	**Now, let's subtract the subtrahend. That means we'll subtract or take away 4 red counters.** (Subtract.)
Teacher	**How many counters do we have now?**
Students	3.
Teacher	**So, what's -1 minus -4?**
Students	3.
Teacher	**-1 minus -4 equals 3. Let's say that together.**
Students	-1 minus -4 equals 3.
Teacher	**Nice job! Using the two-color counters helps you subtract integers. How can you use the two-color counters to subtract integers?**
Students	You show the minuend. Then, you subtract the subtrahend. If you don't have enough minuend counters to subtract, you can bring in zero pairs. Then, you subtract. The difference is the remaining counters.

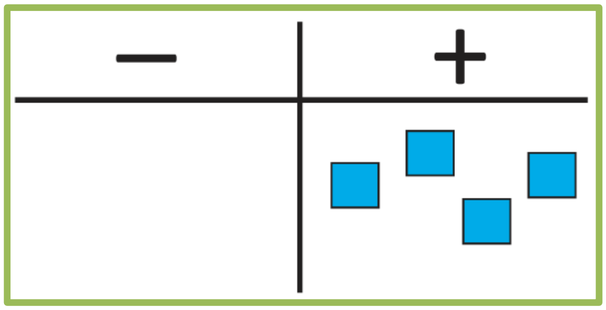
(5) Addition with Positive and Negative Mat with Cubes

Routine

Materials:

- Module 18 Problem Sets
- Module 18 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes or paperclips

ROUTINE WITH POSITIVE AND NEGATIVE MAT



Teacher	**Let's add integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on adding with this positive and negative mat and these cubes.** (Show mat and cubes.)
Teacher	**With the mat, we'll place positive integers on the positive side. Where will we place positive integers?**
Students	Positive side.
Teacher	**And we'll place negative integers on the negative side. Where will we place negative integers?**
Students	Negative side.

(Show problem.)

Teacher **What numbers are we adding?**

Students __ plus __.

Teacher **So, let's start at the first addend. What's the first addend?**

Students __.

Teacher **Let's show the first addend with the cubes. How do we show __** (first addend)**?**

Students Show __ cubes on the positive/negative side.

Teacher **Yes, we'll show __ cubes on the positive/negative side of the mat.**
(Show cubes.)

Teacher **Now, let's add. What number do we add?**

Students __.

Teacher **Let's add the second addend to the first. How do we show __** (second addend)**?**

Students Show __ cubes on the positive/negative side.

Teacher **Yes, we'll show __ cubes on the positive/negative side of the mat.**
(Show cubes.)

Teacher **Are the cubes on the same side of the mat?**

Students *OPTION 1:* Yes!
OPTION 2: No!

Teacher ***OPTION 2:*** **When we have cubes on the positive side and negative side, we'll make zero pairs until we only have all positive cubes or all negative cubes. What we will make?**

Students Zero pairs.

Teacher **A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**

Students When you add a positive number and its opposite. The sum is zero.

Teacher **So, positive 1 and negative 1 equals 0. What's +1 and -1?**

Students 0.

Teacher **Let's make zero pairs. I see a positive and a negative** (place cubes side-by-side)**. What did we create?**

Students A zero pair.

Teacher **We made a zero pair. We can remove this pair from our workspace.**
(Remove zero pair.)

Teacher **Can we make another zero pair?**
(Continue removing zero pairs until there are no more zero pairs.)

Teacher **All of the cubes are on the same side of the mat. How many cubes?**

Students __.

Teacher **So, what's __ plus __?**

Students __.

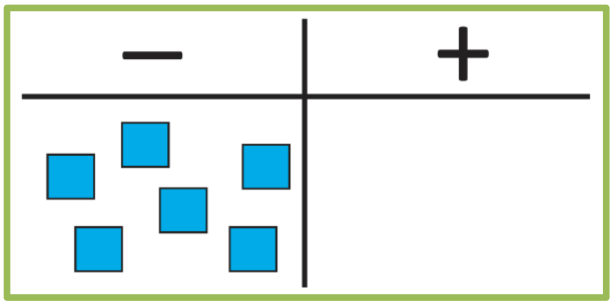
Teacher **__ plus __ equals __. Let's say that together.**

Students	__ plus __ equals __.
Teacher	**Nice job! Using the mat and cubes helps you add integers. How can you use the mat and cubes to add integers?**
Students	You show the first addend. Then, you add the second addend. You create zero pairs until all the cubes are positive or negative. The sum is the remaining cubes.

Example

2 + (-7)

EXAMPLE WITH POSITIVE AND NEGATIVE MAT



Teacher	**Let's add integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on adding with this positive and negative mat and these cubes.** (Show mat and cubes.)
Teacher	**With the mat, we'll place positive integers on the positive side. Where will we place positive integers?**
Students	Positive side.

Teacher	**And we'll place negative integers on the negative side. Where will we place negative integers?**
Students	Negative side.
	(Show problem.)
Teacher	**What numbers are we adding?**
Students	2 plus -7.
Teacher	**So, let's start at the first addend. What's the first addend?**
Students	2.
Teacher	**Let's show the first addend with the cubes. How do we show 2?**
Students	Show 2 cubes on the positive side.
Teacher	**Yes, we'll show 2 cubes on the positive side of the mat.**
	(Show cubes.)
Teacher	**Now, let's add. What number do we add?**
Students	-7.
Teacher	**Let's add the second addend to the first. How do we show -7?**
Students	Show 7 cubes on the negative side.
Teacher	**Yes, we'll show 7 cubes on the negative side of the mat.**
	(Show cubes.)
Teacher	**Are the cubes on the same side of the mat?**
Students	No!
Teacher	**When we have cubes on both the positive side and negative side, we'll make zero pairs until we only have all positive cubes or all negative cubes. What we will make?**
Students	Zero pairs.
Teacher	**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students	When you add a positive number and its opposite. The sum is zero.
Teacher	**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students	0.
Teacher	**Let's make zero pairs. I see a positive and a negative** (place cubes side-by-side)**. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. We can remove this pair from our workspace.**
	(Remove zero pair.)
Teacher	**Can we make another zero pair?**
Students	Yes!
Teacher	**Let's make another zero pair. I see a positive and a negative** (place cubes side-by-side)**. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. We can remove this pair from our workspace.**
	(Remove zero pair.)
Teacher	**Can we make another zero pair?**
Students	No!
Teacher	**All of the cubes are on the same side of the mat. How many cubes?**

Students	-5.
Teacher	**So, what's 2 plus -7?**
Students	-5.
Teacher	**2 plus -7 equals -5. Let's say that together.**
Students	2 plus -7 equals -5.
Teacher	**Nice job! Using the mat and cubes helps you add integers. How can you use the mat and cubes to add integers?**
Students	You show the first addend. Then, you add the second addend. You create zero pairs until all the cubes are positive or negative. The sum is the remaining cubes.

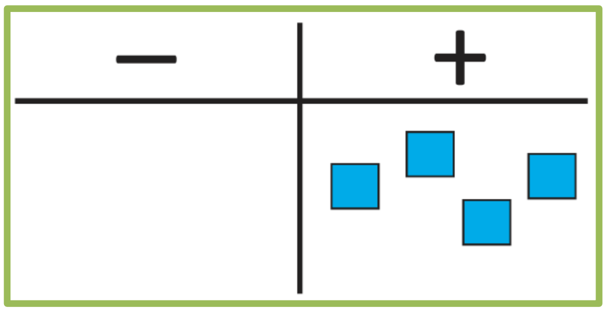
(6) Subtraction with Positive and Negative Mat with Cubes

Routine

Materials:

- Module 18 Problem Sets
- Module 18 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes or paperclips

ROUTINE WITH POSITIVE AND NEGATIVE MAT



Teacher	**Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on subtracting with this positive and negative mat and these cubes.** (Show mat and cubes.)
Teacher	**With the mat, we'll place positive integers on the positive side. Where will we place positive integers?**
Students	Positive side.
Teacher	**And we'll place negative integers on the negative side. Where will we place negative integers?**

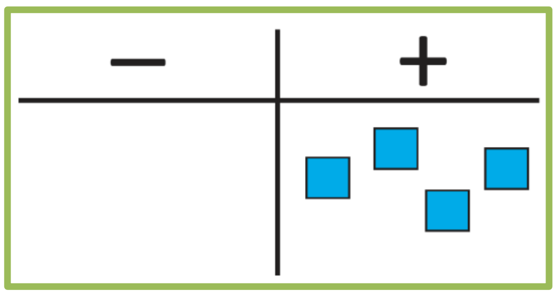
Students	Negative side. (Show problem.)	
Teacher	**What numbers are we subtracting?**	
Students	__ minus __.	
Teacher	**So, let's start at the minuend. What's the minuend?**	
Students	__.	
Teacher	**Let's show the minuend with the cubes. How do we show __** (minuend)**?**	
Students	Show __ cubes on the positive/negative side.	
Teacher	**Yes, we'll show __ cubes on the positive/negative side of the mat.** (Show cubes.)	
Teacher	**Now, let's subtract. What's the subtrahend?**	
Students	__.	
Teacher	**We need to subtract how many __ positive/negative cubes?**	
Students	__.	
Teacher	**So, look at the mat. Do you have enough positive/negative cubes to subtract __ positive/negative cubes?**	
Students	*OPTION 1:*	Yes.
	OPTION 2:	No.
Teacher	***OPTION 2:***	**We don't have enough positive/negative cubes to subtract the minuend. We can make zero pairs until we have enough cubes to subtract the __** (subtrahend) **cubes. What we will make?**
Students		Zero pairs.
Teacher		**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students		When you add a positive number and its opposite. The sum is zero.
Teacher		**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students		0.
Teacher		**Let's make zero pairs for our workspace. Let's add one zero pair. What did we create?**
Students		A zero pair.
Teacher		**We made a zero pair. Let's bring that zero pair to the workspace.** (Add zero pair.)
Teacher		**Look at the cubes. Do we have enough __** (positive/negative) **cubes to subtract __** (subtrahend) **cubes?**
Students		Yes/no.
Teacher		**We keep making zero pairs until we have enough positive/negative cubes to subtract __** (subtrahend) **cubes.** (Continue making zero pairs until there are enough positive/negative cubes to subtract.)
Teacher	**Now, let's subtract the subtrahend. That means we'll subtract or take away __ cubes.** (Subtract.)	

Teacher	**How many cubes remaining?**
Students	__.
Teacher	**So, what's __ minus __?**
Students	__.
Teacher	**__ minus __ equals __. Let's say that together.**
Students	__ minus__ equals __.
Teacher	**Nice job! Using the mat and cubes helps you subtract integers. How can you use the mat and cubes to subtract integers?**
Students	You show the minuend. Then, you subtract the subtrahend. If you don't have enough minuend counters to subtract, you can bring in zero pairs. Then, you subtract. The difference is the remaining cubes.

Example

1 – (-3)

EXAMPLE WITH POSITIVE AND NEGATIVE MAT



Teacher	**Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)

Teacher	**Let's work on subtracting with this positive and negative mat and these cubes.**
	(Show mat and cubes.)
Teacher	**With the mat, we'll place positive integers on the positive side. Where will we place positive integers?**
Students	Positive side.
Teacher	**And we'll place negative integers on the negative side. Where will we place negative integers?**
Students	Negative side.
	(Show problem.)
Teacher	**What numbers are we subtracting?**
Students	1 minus -3.
Teacher	**So, let's start at the minuend. What's the minuend?**
Students	1.
Teacher	**Let's show the minuend with the cubes. How do we show 1?**
Students	Show 1 cube on the positive side of the mat.
Teacher	**Yes, we'll show 1 cube on the positive side of the mat.**
	(Show cube.)
Teacher	**Now, let's subtract. What's the subtrahend?**
Students	-3.
Teacher	**We need to subtract how many negative cubes?**
Students	3.
Teacher	**So, look at the mat. Do you have enough negative cubes to subtract 3 negative cubes?**
Students	No.
Teacher	**We don't have enough negative cubes to subtract the minuend. We can make zero pairs until we have enough cubes to subtract the 3 negative cubes. What we will make?**
Students	Zero pairs.
Teacher	**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students	When you add a positive number and its opposite. The sum is zero.
Teacher	**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students	0.
Teacher	**Let's make zero pairs for our workspace. Let's add one zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**
	(Add zero pair.)
Teacher	**Look at the cubes. Do we have enough negative cubes to subtract 3 cubes?**
Students	No.
Teacher	**Let's add another zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**

	(Add zero pair.)
Teacher	**Look at the cubes. Now we have 2 negative cubes. Do we have enough negative cubes to subtract 3 cubes?**
	No.
Teacher	**Let's add another zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**
	(Add zero pair.)
Teacher	**Look at the cubes. Now we have 3 negative cubes. Do we have enough negative cubes to subtract 3 cubes?**
Students	Yes!
Teacher	**Now, let's subtract the subtrahend. That means we'll subtract or take away 3 negative cubes.**
	(Subtract.)
Teacher	**How many cubes remaining?**
Students	4.
Teacher	**So, what's 1 minus -3?**
Students	4.
Teacher	**1 minus -3 equals 4. Let's say that together.**
Students	1 minus -3 equals 4.
Teacher	**Nice job! Using the mat and cubes helps you subtract integers. How can you use the mat and cubes to subtract integers?**
Students	You show the minuend. Then, you subtract the subtrahend. If you don't have enough minuend counters to subtract, you can bring in zero pairs. Then, you subtract. The difference is the remaining cubes.

D. Problems for Use During Instruction

See Module 18 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 18 Vocabulary Cards.

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Module 18:

Addition and Subtraction of Integers

Problem Sets

A.

$3 + (-8)$

A.

$6 + (-4)$

A.

$7 + (-2)$

A.

$5 + (-10)$

A.

$9 + (-1)$

A.

$2 + (-5)$

A.

$6 + (-3)$

A.

$7 + (-8)$

A.

$4 + (-2)$

A.

$17 + (-12)$

A.

5 + (-5)

A.

11 + (-15)

A.

8 + (-4)

A.

$14 + (-8)$

A.

$$7 + (-1)$$

A.

$12 + (-3)$

A.

$1 + (-6)$

A.

$4 + (-5)$

A.

$2 + (-3)$

A.

0 + (-9)

B.

$(-1) + 5$

B.

$(-3) + 6$

B.

(-9) + 2

B.

(-4) + 3

B.

$(-7) + 8$

B.

(-8) + 6

B.

$(-5) + 4$

B.

$(-6) + 10$

B.

$(-6) + 3$

B.

$$(-10) + 5$$

B.

$(-2) + 2$

B.

$(-7) + 8$

B.

$(-19) + 13$

B.

(-12) + 3

B.

$(-14) + 6$

B.

$$(-11) + 8$$

B.

$(-15) + 4$

B.

(-5) + 4

B.

(-2) + 0

B.

$$(-8) + 1$$

c.

$$(-2) + (-3)$$

c.

$$(-6) + (-1)$$

c.

$$(-8) + (-4)$$

c.

$$(-9) + (-9)$$

c.

$$(-5) + (-7)$$

c.

$$(-4) + (-2)$$

c.

$$(-11) + (-6)$$

c.

$$(-3) + (-4)$$

c.

$$(-1) + (-10)$$

c.

$$(-7) + (-12)$$

c.

$$(-9) + (-1)$$

c.

$$(-8) + (-6)$$

c.

$$(-10) + (-9)$$

c.

$$(-2) + (-15)$$

c.

$$(-16) + (-3)$$

c.

$$(-7) + (-14)$$

c.

$$(-12) + (-4)$$

c.

$$(-13) + (-9)$$

c.

$$(-17) + (-4)$$

c.

(-16) + (-8)

D.

$3 - (-8)$

D.

$6 - (-4)$

D.

8 – (-2)

D.

$5 - (-10)$

D.

$9 - (-1)$

D.

$2 - (-5)$

D.

$6 - (-3)$

D.

7 – (-8)

D.

$4 - (-2)$

D.

17 – (-12)

D.

$5 - (-5)$

D.

$11 - (-15)$

D.

$8 - (-4)$

D.

14 – (-8)

D.

$$7 - (-1)$$

D.

$$12 - (-3)$$

D.

$$1 - (-6)$$

D.

$4 - (-5)$

D.

$2 - (-3)$

D.

$0 - (-9)$

E.

$$(-1) - 5$$

E.

$(-3) - 6$

E.

$$(-9) - 2$$

E.

$(-4) - 3$

E.

$$(-7) - 8$$

E.

$$(-8) - 6$$

E.

$(-5) - 4$

E.

$$(-6) - 10$$

E.

$$(-6) - 3$$

E.

(-10) – 5

E.

$$(-2) - 2$$

E.

$$(-7) - 8$$

E.

$(-19) - 13$

E.

$(-12) - 3$

E.

$(-14) - 6$

E.

(-11) – 8

E.

$(-15) - 4$

E.

$$(-6) - 3$$

E.

$$(-2) - 0$$

E.

$(-8) - 1$

F.

$$(-2) - (-3)$$

F.

$$(-6) - (-1)$$

F.

$(-8) - (-4)$

F.

$(-9) - (-9)$

F.

$(-5) - (-7)$

F.

$$(-4) - (-2)$$

F.

$(-11) - (-6)$

F.

$$(-3) - (-4)$$

F.

$$(-1) - (-10)$$

F.

$$(-7) - (-12)$$

F.

$$(-9) - (-1)$$

F.

$(-8) - (-6)$

F.

$(-10) - (-9)$

F.

$$(-2) - (-15)$$

F.

$$(-16) - (-3)$$

F.

$$(-7) - (-14)$$

F.

$$(-12) - (-4)$$

F.

$$(-13) - (-9)$$

F.

$$(-17) - (-4)$$

F.

$$(-16) - (-8)$$

Module 18:

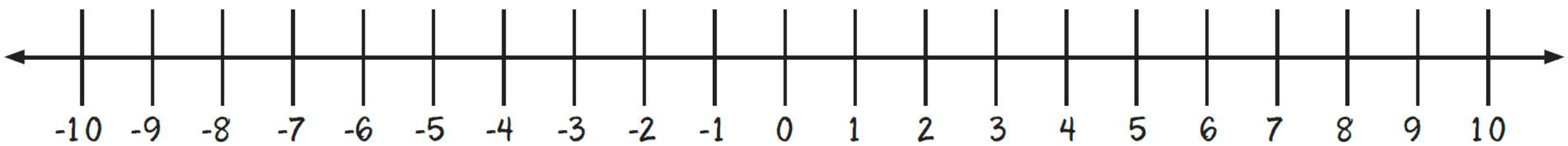
Addition and Subtraction of Integers

Vocabulary Cards

absolute value
addend
difference
integer
minuend
negative number
number line
opposites
positive number
subtrahend
sum
zero pair

absolute value

The distance of a number from 0 on a number line.



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addend

Any numbers added together.

6 + 2 = 8

6 and 2 are addends

difference

The result of subtracting one number from another number.

$$6 - 4 = 2$$

2 is the difference

---

integer

A positive or negative whole number.

-3 -2 -1 1 2 3

minuend

The number from which another number is subtracted.

9 – 4 = 5

9 is the minuend

---

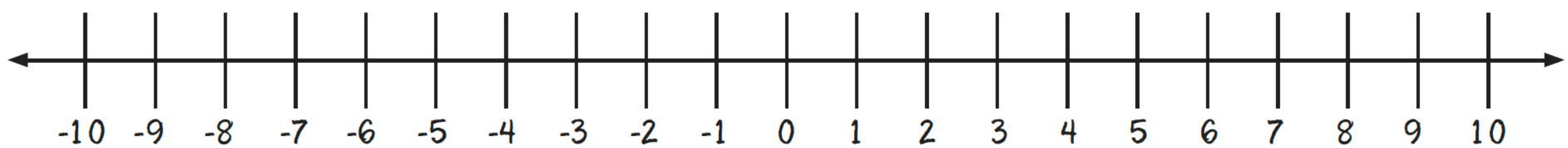
negative number

Any number less than 0.

-3 -2 -1

number line

A straight line with numbers placed at equal intervals along its length.



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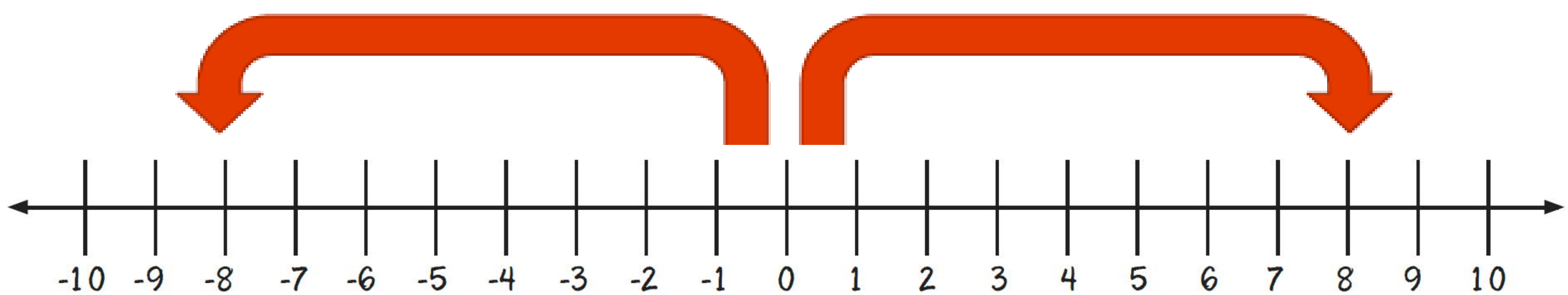
opposites

Two numbers that are equal distance from 0 on a number line.

-8 and 8 are opposites



positive number

Any number greater than 0.

1 2 3

---

subtrahend

The number to be subtracted.

9 – 4 = 5

4 is the subtrahend

sum

The result of adding two or more numbers or the total number when you combine sets.

$$7 + 2 + 1 = 10$$

10 is the sum

---

zero pair

A pair of numbers with a sum of 0.

$$-7 + 7 = 0$$

Instructional Routines for Mathematics Intervention

MODULE 19

Multiplication and Division of Integers

Module 19: Multiplication and Division of Integers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
absolute value	The distance of a number from 0 on a number line.
divide/division	To separate into equal groups.
dividend	The number that is to be divided in a division problem.
divisor	The number that the dividend is divided by.
factor	A number that you multiply with another number to get the product.
integer	A positive or negative whole number.
multiply/multiplication	The process of adding a number to itself a number of times.
negative number	Any number less than 0.
number line	A straight line with numbers placed at equal intervals along its length.
opposites	Two numbers that are equal distance from 0 on a number line.
positive number	Any number greater than 0.
product	The result of multiplying two or more factors.
quotient	The number that results when one number is divided by another number.
zero pair	A pair of numbers with a sum of 0.

B. Background Information

In this module, we focus on multiplication and division of integers. An integer is a positive or negative whole number. We use the following different models to help students understand multiplication and division of integers:

(1) Multiplication with a Number Line
(2) Division with a Number Line
(3) Multiplication with a Quadrant Mat and Cubes
(4) Division with a Quadrant Mat and Cubes

When referring to integers, be sure to emphasize that numbers without a negative symbol (-) are assumed positive. So:

7 is "positive seven" or "seven."
-7 is "negative seven."

C. Routines and Examples

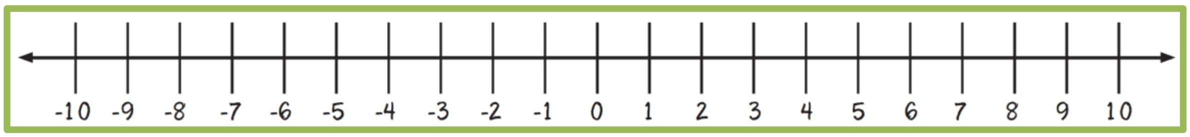
(1) Multiplication with a Number Line

Routine

Materials:

- Module 19 Problem Sets
- Module 19 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A number line and a manipulative with a face (e.g., duck or dinosaur)

ROUTINE WITH NUMBER LINE



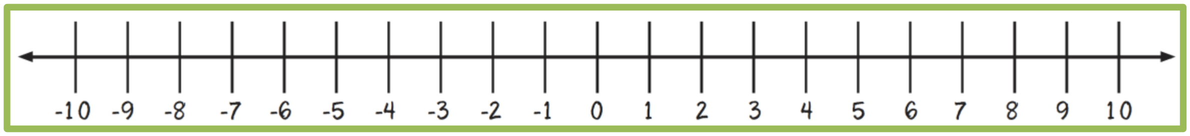
Teacher	**Let's multiply integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on multiplying with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we multiplying?**
Students	__ times __.
Teacher	**So, let's start with the first factor. What's the first factor?**
Students	__.
Teacher	**Let's place the duck on the number line at zero. Where do we place the duck?**
Students	At zero.

Teacher	**If the first factor is positive, the duck will face the increasing numbers on the number line. When does the duck face the increasing numbers?**
Students	When the first factor is positive.
Teacher	**If the first factor is negative, the duck will face the decreasing numbers on the number line. When does the duck face the decreasing numbers?**
Students	When the first factor is negative.
Teacher	**So, which way will the duck face in this problem?**
Students	Increasing/decreasing.
Teacher	**Yes, the first factor is positive/negative, so the duck faces the increasing/decreasing numbers.**
Students	(Place duck on zero. Make sure duck is facing increasing/decreasing numbers on the number line.)
Teacher	**Now, let's multiply. What is the second factor?**
Students	__.
Teacher	**If the second factor is positive, the duck will move forward from its position. When does the duck move forward?**
Students	When the second factor is positive.
Teacher	**If the second factor is negative, the duck will move backward from its position. When does the duck move backward?**
Students	When the second factor is negative.
Teacher	**So, which direction should we move?**
Students	Forward/backward.
Teacher	**Because the second factor is positive/negative, we move forward/backward. The second factor is __ so we'll move by jumps of __** (second factor)**. Let's do that together. Count with me.**
Students	__, __, __, …
Teacher	**So, our duck shows the product. What's __ times __?**
Students	__.
Teacher	**Yes. __ times __ equals __. Using this number line helps you understand what it means to multiply integers. How can you use the number line to multiply integers?**
Students	Start at zero. The duck faces increasing numbers with a positive factor and decreasing numbers with a negative factor. Then, the duck jumps the second factor by moving forward if it's a positive factor or backward if it's a negative factor.

Example

-3 × (-2)

EXAMPLE WITH NUMBER LINE



Teacher	**Let's multiply integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on multiplying with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we multiplying?**
Students	-3 times -2.
Teacher	**So, let's start with the first factor. What's the first factor?**
Students	-3.
Teacher	**Let's place the duck on the number line at zero. Where do we place the duck?**
Students	At zero.
Teacher	**If the first factor is positive, the duck will face the increasing numbers on the number line. When does the duck face the increasing numbers?**
Students	When the first factor is positive.
Teacher	**If the first factor is negative, the duck will face the decreasing numbers on the number line. When does the duck face the decreasing numbers?**
Students	When the first factor is negative.
Teacher	**So, which way will the duck face in this problem?**
Students	Decreasing.
Teacher	**Yes, the first factor is negative, so the duck faces the decreasing numbers.**

Students	(Place duck on zero. Make sure duck is facing decreasing numbers on the number line.)
Teacher	**Now, let's multiply. What is the second factor?**
Students	-2.
Teacher	**If the second factor is positive, the duck will move forward from its position. When does the duck move forward?**
Students	When the second factor is positive.
Teacher	**If the second factor is negative, the duck will move backward from its position. When does the duck move backward?**
Students	When the second factor is negative.
Teacher	**So, which direction should we move?**
Students	Backward.
Teacher	**Because the second factor is negative, we move backward. The second factor is -2 so we'll move by jumps of 2. Let's do that together. Count with me.**
Students	2, 4, 6.
Teacher	**So, our duck shows the product. What's the product?**
Students	6.
Teacher	**What's -3 times -2?**
Students	6.
Teacher	**Yes. -3 times -2 equals 6. Using this number line helps you understand what it means to multiply integers. How can you use the number line to multiply integers?**
Students	Start at zero. The duck faces increasing numbers with a positive factor and decreasing numbers with a negative factor. Then, the duck jumps the second factor by moving forward if it's a positive factor or backward if it's a negative factor.

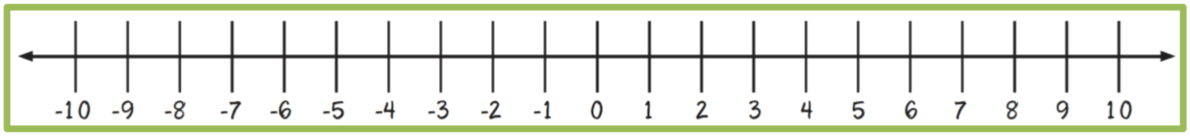
(2) Division with a Number Line

Routine

Materials:

- Module 19 Problem Sets
- Module 19 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A number line and a manipulative with a face (e.g., duck or dinosaur)

ROUTINE WITH NUMBER LINE



Teacher	**Let's divide integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on dividing with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we dividing?**
Students	__ divided by __.
Teacher	**So, let's start by thinking about the divisor. What's the divisor?**
Students	__.
Teacher	**Let's place the duck on the number line at zero. Where do we place the duck?**
Students	At zero.
Teacher	**If the divisor is positive, the duck will walk forward. When will the duck walk forward?**
Students	When the divisor is positive.

Teacher	**If the divisor is negative, the duck will walk backward. When will the duck walk backward?**
Students	When the divisor is negative.
Teacher	**Now, let's think about the dividend. The duck starts at zero and moves to the dividend. What's the dividend?**
Students	__.
Teacher	**The duck needs to move toward the dividend. If the duck will walk forward – because the divisor is positive – then face the duck toward the dividend. When do you face the duck toward the dividend?**
Students	When the divisor is positive and the duck will walk forward.
Teacher	**If the duck will walk backward – because the divisor is negative – then face the duck away from the dividend. When do you face the duck away from the dividend?**
Students	When the divisor is negative and the duck will walk backward.
Teacher	**So, which way will the duck face in this problem?**
Students	Toward the dividend/away from the dividend.
Teacher	**Yes, the dividend is positive/negative and the duck needs to walk forward/backward** (of the divisor)**, so the duck faces/doesn't face the dividend.**
Students	(Place the duck on zero. Make sure the duck is facing toward the dividend if the divisor is positive. Make sure the duck is facing away from the dividend if the divisor is negative.)
Teacher	**Now, let's divide. What's the divisor?**
Students	__.
Teacher	**So, the duck will jump the number of spaces in the divisor. If the divisor is 2, the ducks jumps in groups of 2. If the divisor is -5, the duck jumps in groups of 5. What would happen if the divisor is 10? How would the duck jump?**
Students	By 10.
Teacher	**So, the duck will jump the number of spaces in the divisor. And the jumps will be forward/backward because the divisor is positive/negative. Let's do that together. Count with me.**
Students	__, __, __, …
Teacher	**How many jumps did the duck make?**
Students	__.
Teacher	**__ is the quotient. Let's decide whether that's positive or negative. Is the duck facing the increasing numbers or decreasing numbers?**
Students	Increasing/decreasing.
Teacher	**If the duck faces the increasing numbers, then the quotient is positive. When is the quotient positive?**
Students	When the duck faces the increasing numbers.
Teacher	**If the duck faces the decreasing numbers, then the quotient is negative. When is the quotient negative?**
Students	When the duck faces the decreasing numbers.
Teacher	**What's the quotient?**

Students __.

Teacher That's right. __ divided by __ equals __. Let's say that together.

Students __ divided by __ equals __.

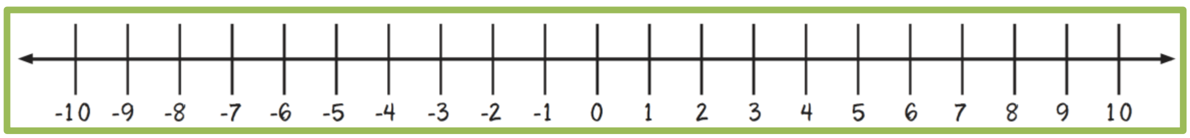
Teacher Yes. __ divided by __ equals __. Using this number line helps you understand what it means to divide integers. How can you use the number line to divide integers?

Students Start at zero. If the divisor is positive, the duck will jump forward. If the divisor is negative, the duck will jump backward. We jump in groups of the divisor. That's the quotient. If the duck is facing the increasing numbers, the quotient is positive. If the duck is facing the decreasing numbers, the quotient is negative.

Example

12 ÷ (-3)

ROUTINE WITH NUMBER LINE



Teacher Let's divide integers. An integer is a positive or negative whole number. What's an integer?

Students A positive or negative whole number.

Teacher Let's think about a positive number. How do you know a number is positive?

Students It has a positive sign or it doesn't have a sign in front of the number.

Teacher We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.
(Draw +.)

Teacher We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?

Students When there is not a negative sign directly in front of the number.

Teacher How do you know a number is negative?

Students It has a negative sign.

Teacher We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.
(Draw -.)

Teacher Let's work on dividing with this number line.
(Show number line.)
(Show problem.)

Teacher What numbers are we dividing?

Students 12 divided by -3.

Teacher So, let's start by thinking about the divisor. What's the divisor?

Students -3.

Teacher Let's place the duck on the number line at zero. Where do we place the duck?

Students At zero.
Teacher If the divisor is positive, the duck will walk forward. When will the duck walk forward?
Students When the divisor is positive.
Teacher If the divisor is negative, the duck will walk backward. When will the duck walk backward?
Students When the divisor is negative.
Teacher Now, let's think about the dividend. The duck starts at zero and moves to the dividend. What's the dividend?
Students 12.
Teacher The duck needs to move toward the dividend. If the duck will walk forward – because the divisor is positive – then face the duck toward the dividend. When do you face the duck toward the dividend?
Students When the divisor is positive and the duck will walk forward.
Teacher If the duck will walk backward – because the divisor is negative – then face the duck away from the dividend. When do you face the duck away from the dividend?
Students When the divisor is negative and the duck will walk backward.
Teacher So, which way will the duck face in this problem?
Students Away from the dividend.
Teacher Yes, the dividend is positive and the duck needs to walk backward because the divisor is negative, so the duck doesn't face the dividend.
Students (Place the duck on zero. Make sure the duck is facing away from the dividend if the divisor is negative.)
Teacher Now, let's divide. What's the divisor?
Students -3.
Teacher So, the duck will jump the number of spaces in the divisor. What's the divisor?
Students -3.
Teacher So, the duck will jump in groups of 3. And the jumps will be backward because the divisor is negative. Let's do that together. Count with me.
Students 3, 6, 9, 12.
Teacher How many jumps did the duck make?
Students 4.
Teacher The duck made 3 jumps. Is the duck facing the increasing numbers or decreasing numbers?
Students Decreasing.
Teacher If the duck faces the increasing numbers, then the quotient is positive. When is the quotient positive?
Students When the duck faces the increasing numbers.
Teacher If the duck faces the decreasing numbers, then the quotient is negative. When is the quotient negative?
Students When the duck faces the decreasing numbers.
Teacher What's the quotient?

Students	-4.
Teacher	**That's right. 12 divided by -3 equals -4. Let's say that together.**
Students	12 divided by -3 equals -4.
Teacher	**Using this number line helps you understand what it means to divide integers. How can you use the number line to divide integers?**
Students	Start at zero. If the divisor is positive, the duck will jump forward. If the divisor is negative, the duck will jump backward. We jump in groups of the divisor. That's the quotient. If the duck is facing the increasing numbers, the quotient is positive. If the duck is facing the decreasing numbers, the quotient is negative.

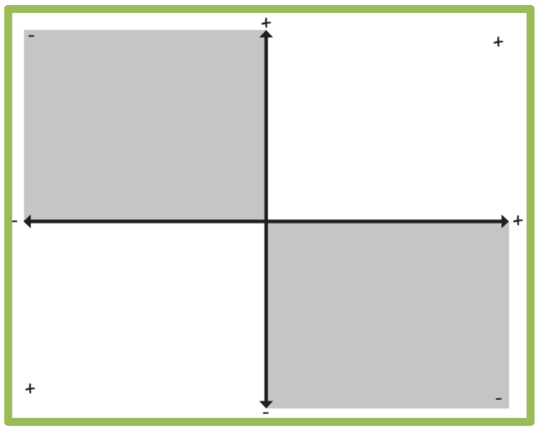
(3) Multiplication with Quadrant Mat and Cubes

Routine

Materials:

- Module 19 Problem Sets
- Module 19 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes

ROUTINE WITH QUADRANT MAT



Teacher	**Let's multiply integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on multiplying with this quadrant mat and these cubes.** (Show mat and cubes.)
Teacher	**On the mat, we have a horizontal axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the horizontal axis?**
Students	Line across the mat.

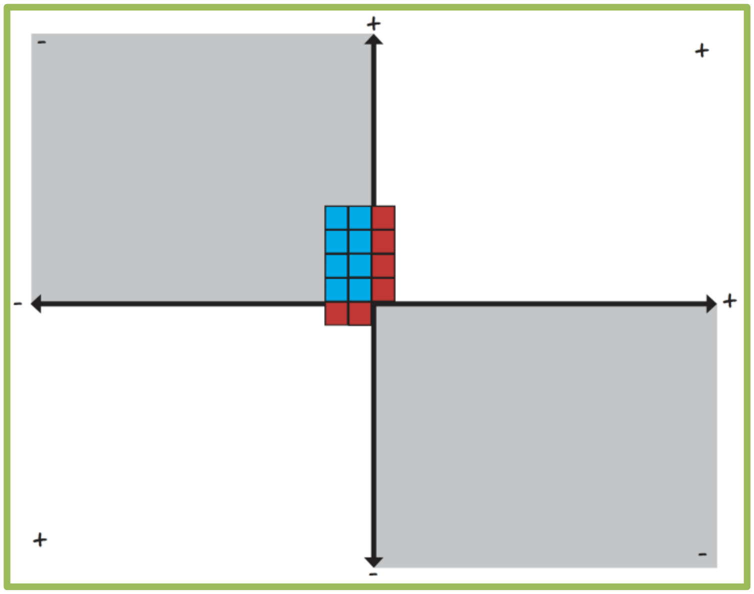
Teacher	**On the mat, we have a vertical axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the vertical axis?**
Students	Line up and down on the mat.
	(Show problem.)
Teacher	**What numbers are we multiplying?**
Students	__ times __.
Teacher	**So, let's start at the first factor. What's the first factor?**
Students	__.
Teacher	**Let's show the first factor with the cubes. We'll place the first factor on the horizontal axis on the positive side if the factor is positive and the negative side if the factor is negative. How do we show the first factor?**
Students	Show __ cubes on the positive/negative side of the horizontal axis.
Teacher	**Yes, we'll show __ cubes on the positive/negative side of the horizontal axis.**
	(Show cubes.)
Teacher	**Now, let's multiply. What number do we multiply?**
Students	__.
Teacher	**Let's show the second factor with the cubes. We'll place the second factor on the vertical axis on the positive side if the factor is positive and the negative side if the factor is negative. How do we show the second factor?**
Students	Show __ cubes on the positive/negative side of the vertical axis.
Teacher	**Yes, we'll show __ cubes on the positive/negative side of the vertical axis.**
	(Show cubes.)
Teacher	**Now, let's multiply. That means we multiply each of the cubes on the horizontal axis by each of the cubes on the vertical axis. Let me show you what I mean. On the horizontal axis, we have 1 cube. Let's multiply that cube by 1, 2, 3,… cubes on the vertical axis. I'll place the cubes in the rectangular area created by the multiplication. Where do I place the cubes?**
Students	In the rectangular area created by the multiplication.
	(Create area with cubes.)
Teacher	**Let's keep multiplying each cube on the horizontal axis until we've multiplied all the cubes.**
	(Create area with cubes.)
Teacher	**We've created an area with our multiplication. How many cubes are in that area?**
Students	__.
Teacher	**Is the area in a positive quadrant or negative quadrant?**
Students	Positive/negative.
Teacher	**So, what's __ times __?**
Students	__.
Teacher	**__ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Nice job! Using the quadrant mat and cubes helps you multiply integers. How can you use the quadrant mat and cubes to multiply integers?**

Students	Show the first factor on the horizontal axis. Show the second factor on the vertical axis. Multiply the cubes to create an area.

Example

-2 × (-4)

ROUTINE WITH QUADRANT MAT



Teacher	**Let's multiply integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on multiplying with this quadrant mat and these cubes.** (Show mat and cubes.)

Teacher	**On the mat, we have a horizontal axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the horizontal axis?**
Students	Line across the mat.
Teacher	**On the mat, we have a vertical axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the vertical axis?**
Students	Line up and down on the mat.
	(Show problem.)
Teacher	**What numbers are we multiplying?**
Students	-2 times 4.
Teacher	**So, let's start at the first factor. What's the first factor?**
Students	-2.
Teacher	**Let's show the first factor with the cubes. We'll place the first factor on the horizontal axis on the positive side if the factor is positive and the negative side if the factor is negative. How do we show the first factor?**
Students	Show 2 cubes on the negative side of the horizontal axis.
Teacher	**Yes, we'll show 2 cubes on the negative side of the horizontal axis.**
	(Show cubes.)
Teacher	**Now, let's multiply. What number do we multiply?**
Students	4.
Teacher	**Let's show the second factor with the cubes. We'll place the second factor on the vertical axis on the positive side if the factor is positive and the negative side if the factor is negative. How do we show the second factor?**
Students	Show 4 cubes on the positive side of the vertical axis.
Teacher	**Yes, we'll show 4 cubes on the positive side of the vertical axis.**
	(Show cubes.)
Teacher	**Now, let's multiply. That means we multiply each of the cubes on the horizontal axis by each of the cubes on the vertical axis. Let me show you what I mean. On the horizontal axis, we have 1 cube. Let's multiply that cube by 1, 2, 3, 4 cubes on the vertical axis. I'll place the cubes in the rectangular area created by the multiplication. Where do I place the cubes?**
Students	In the rectangular area created by the multiplication.
	(Create area with cubes.)
Teacher	**Let's keep multiplying each cube on the horizontal axis until we've multiplied all the cubes. On the horizontal axis, we have a 2nd cube. Let's multiply that cube by 1, 2, 3, 4 cubes on the vertical axis. I'll place the cubes in the rectangular area created by the multiplication.**
	(Create area with cubes.)
Teacher	**We've created a rectangular area with our multiplication. How many cubes are in that area?**
Students	8.
Teacher	**Is the area in a positive quadrant or negative quadrant?**
Students	Negative.
Teacher	**So, what's -2 times 4?**
Students	-8.

Teacher	**-2 times 4 equals -8. Let's say that together.**
Students	-2 times 4 equals -8.
Teacher	**Nice job! Using the quadrant mat and cubes helps you multiply integers. How can you use the quadrant mat and cubes to multiply integers?**
Students	Show the first factor on the horizontal axis. Show the second factor on the vertical axis. Multiply the cubes to create an area.

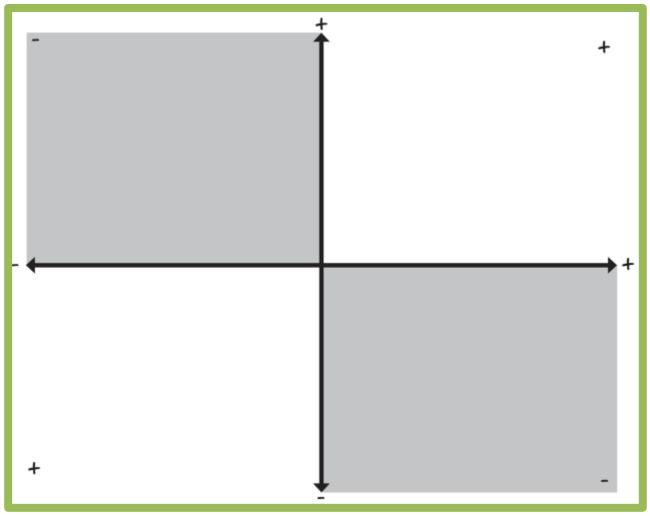
(4) Division with Quadrant Mat and Cubes

Routine

Materials:

- Module 19 Problem Sets
- Module 19 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes

ROUTINE WITH QUADRANT MAT



Teacher	**Let's divide integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on dividing with this quadrant mat and these cubes.** (Show mat and cubes.)

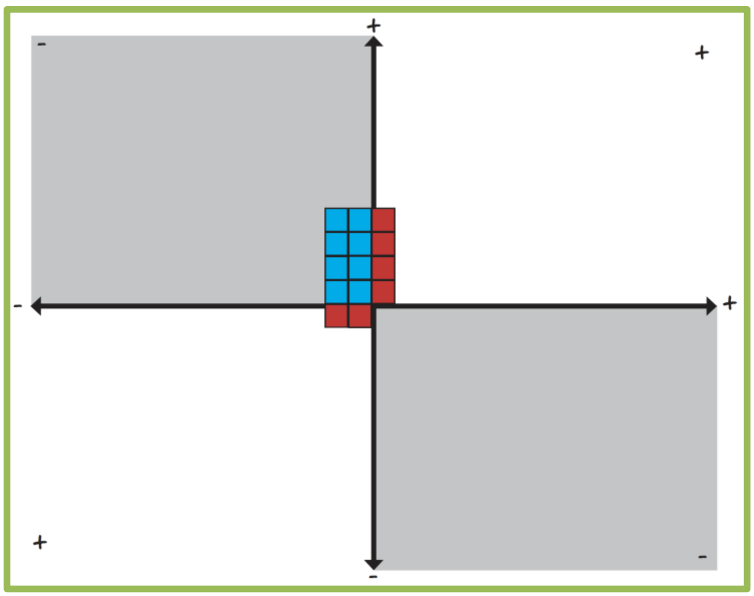
Teacher	**On the mat, we have a horizontal axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the horizontal axis?**
Students	Line across the mat.
Teacher	**On the mat, we have a vertical axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the vertical axis?**
Students	Line up and down on the mat.
	(Show problem.)
Teacher	**What numbers are we dividing?**
Students	__ divided by __.
Teacher	**So, let's start with the dividend. What's the dividend?**
Students	__.
Teacher	**Let's show the dividend with the cubes. We'll place the dividend in a positive quadrant if the dividend is positive. When do we place the dividend in a positive quadrant?**
Students	When the dividend is positive.
Teacher	**We'll place the dividend in a negative quadrant if the dividend is negative. When do we place the dividend in a negative quadrant?**
Students	When the dividend is negative.
Teacher	**Yes, we'll show __ cubes in a positive/negative quadrant.**
	(Show cubes.)
Teacher	**Now, let's divide. What number do we divide by? What's the divisor?**
Students	__.
Teacher	**Let's show the divisor with the cubes. We'll place the divisor on the positive side of an axis is the divisor is positive and the negative side of an axis if the divisor is negative. How do we show the divisor?**
Students	Show __ cubes on the positive/negative side of an axis.
	(Show cubes.)
Teacher	**You may have to move the dividend cubes to be near the divisor. For example, if you place the dividend cubes in the upper-right positive quadrant but the divisor is negative, you move the dividend cubes to the bottom-left positive quadrant. Do we need to move the dividend cubes?**
Students	Yes/no.
	(Move cubes if necessary.)
Teacher	**Now, let's divide. Let's see how many groups we can make with the divisor. So, we'll create groups of __** (divisor) **with the dividend. Let me show you what I mean. I can make 1 group. I'll place the cubes in a row by the divisor. Where do I place the cubes?**
Students	In a row by the divisor.
	(Show division into groups with cubes.)
Teacher	**Let's keep dividing until we've dividend all the cubes.**
	(Show division into groups with cubes.)
Teacher	**Now, let's determine our quotient by seeing how many groups we created. We created 1, 2, 3, … groups. How many?**
Students	__.

Teacher	**I'll place cubes on the axis to show the groups.** (Place cubes on axis.)
Teacher	**The cubes on the axis are the quotient. Is the quotient positive or negative? Look at the placement of the cubes on the axis.**
Students	Positive/negative.
Teacher	**So, what's __ divided by __?**
Students	__.
Teacher	**__ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**Nice job! Using the quadrant mat and cubes helps you divide integers. How can you use the quadrant mat and cubes to divide integers?**
Students	Place the dividend cubes in a quadrant. Show the divisor cubes on one of the axes. Make groups of the divisor. Place the quotient cubes on the axis.

Example

-8 ÷ (-2)

ROUTINE WITH QUADRANT MAT



Teacher	**Let's divide integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)

Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on dividing with this quadrant mat and these cubes.** (Show mat and cubes.)
Teacher	**On the mat, we have a horizontal axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the horizontal axis?**
Students	Line across the mat.
Teacher	**On the mat, we have a vertical axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the vertical axis?**
Students	Line up and down on the mat. (Show problem.)
Teacher	**What numbers are we dividing?**
Students	-8 divided by -2.
Teacher	**So, let's start with the dividend. What's the dividend?**
Students	-8.
Teacher	**Let's show the dividend with the cubes. We'll place the dividend in a positive quadrant if the dividend is positive. When do we place the dividend in a positive quadrant?**
Students	When the dividend is positive.
Teacher	**We'll place the dividend in a negative quadrant if the dividend is negative. When do we place the dividend in a negative quadrant?**
Students	When the dividend is negative.
Teacher	**Yes, we'll show 8 cubes in a negative quadrant.** (Show cubes.)
Teacher	**Now, let's divide. What number do we divide by? What's the divisor?**
Students	-2.
Teacher	**Let's show the divisor with the cubes. We'll place the divisor on the positive side of an axis is the divisor is positive and the negative side of an axis if the divisor is negative. How do we show the divisor?**
Students	Show 2 cubes on the negative side of an axis. (Show cubes.)
Teacher	**You may have to move the dividend cubes to be near the divisor. For example, if you place the dividend cubes in the upper-right positive quadrant but the divisor is negative, you move the dividend cubes to the bottom-left positive quadrant. Do we need to move the dividend cubes?**
Students	No.
Teacher	**Now, let's divide. Let's see how many groups we can make with the divisor. So, we'll create groups of 2 with the dividend. Let me show you what I**

	mean. I can make 1 group. I'll place the cubes in a row by the divisor. Where do I place the cubes?
Students	In a row by the divisor. (Show division into groups with cubes.)
Teacher	**Let's keep dividing until we've dividend all the cubes. I can make 2, 3, 4 groups.** (Show division into groups with cubes.)
Teacher	**Now, let's determine our quotient by seeing how many groups we created. We created 1, 2, 3, 4 groups. How many?**
Students	4.
Teacher	**I'll place 1, 2, 3, 4 cubes on the axis to show the groups.** (Place cubes on axis.)
Teacher	**The cubes on the axis are the quotient. Is the quotient positive or negative? Look at the placement of the cubes on the axis.**
Students	Positive.
Teacher	**So, what's -8 divided by -2?**
Students	4.
Teacher	**-8 divided by -2 equals 4. Let's say that together.**
Students	-8 divided by -2 equals 4.
Teacher	**Nice job! Using the quadrant mat and cubes helps you divide integers. How can you use the quadrant mat and cubes to divide integers?**
Students	Place the dividend cubes in a quadrant. Show the divisor cubes on one of the axes. Make groups of the divisor. Place the quotient cubes on the axis.

D. Problems for Use During Instruction

See Module 19 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 19 Vocabulary Cards.

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Module 19:

Multiplication and Division of Integers

Problem Sets

A.

$3 \times (-8)$

A.

$$6 \times (-4)$$

A.

$$7 \times (-2)$$

A.

$5 \times (-10)$

A.

$9 \times (-3)$

A.

$$2 \times (-5)$$

A.

$$6 \times (-3)$$

A.

$$7 \times (-8)$$

A.

$4 \times (-9)$

A.

$$11 \times (-12)$$

A.

$$5 \times (-5)$$

A.

$11 \times (-10)$

A.

$8 \times (-4)$

A.

$13 \times (-8)$

A.

$7 \times (-4)$

A.

$12 \times (-3)$

A.

$9 \times (-6)$

A.

$4 \times (-5)$

A.

$$2 \times (-3)$$

A.

$0 \times (-9)$

B.

$$(-6) \times 5$$

B.

$$(-3) \times 6$$

B.

$$(-9) \times 2$$

B.

$$(-4) \times 3$$

B.

$$(-7) \times 8$$

B.

$$(-5) \times 6$$

B.

$$(-7) \times 4$$

B.

$$(-6) \times 10$$

B.

$$(-3) \times 3$$

B.

$$(-10) \times 5$$

B.

$(-2) \times 8$

B.

$$(-7) \times 7$$

B.

$$(-11) \times 13$$

B.

$$(-12) \times 3$$

B.

$$(-14) \times 6$$

B.

$$(-11) \times 8$$

B.

$(-15) \times 4$

B.

$$(-8) \times 8$$

B.

$$(-2) \times 0$$

B.

$$(-8) \times 1$$

c.

$$(-2) \times (-3)$$

c.

$$(-6) \times (-5)$$

c.

$$(-8) \times (-4)$$

c.

$$(-9) \times (-9)$$

c.

$$(-5) \times (-7)$$

c.

$$(-4) \times (-6)$$

c.

$$(-11) \times (-6)$$

c.

$$(-3) \times (-4)$$

c.

$$(-8) \times (-10)$$

c.

$$(-7) \times (-12)$$

c.

$$(-9) \times (-4)$$

c.

$$(-8) \times (-6)$$

c.

$$(-12) \times (-9)$$

c.

$$(-3) \times (-15)$$

c.

$$(-16) \times (-2)$$

c.

$$(-7) \times (-11)$$

c.

$$(-12) \times (-4)$$

c.

$$(-13) \times (-3)$$

c.

$$(-12) \times (-5)$$

c.

$$(-16) \times (-2)$$

D.

$9 \div (-3)$

D.

$6 \div (-2)$

D.

$5 \div (-5)$

D.

$$14 \div (-2)$$

D.

$$45 \div (-5)$$

D.

$18 \div (-2)$

D.

$49 \div (-7)$

D.

$54 \div (-6)$

D.

$21 \div (-3)$

D.

$32 \div (-4)$

D.

$18 \div (-3)$

D.

$40 \div (-5)$

D.

$12 \div (-6)$

D.

$48 \div (-8)$

D.

$$72 \div (-9)$$

D.

$$63 \div (-7)$$

D.

$16 \div (-8)$

D.

$20 \div (-5)$

D.

$10 \div (-2)$

D.

$$18 \div (-6)$$

E.

$(-28) \div 7$

E.

$$(-40) \div 8$$

E.

$(-12) \div 4$

E.

$$(-14) \div 2$$

E.

$$(-49) \div 7$$

E.

$(-63) \div 9$

E.

$(-20) \div 2$

E.

$(-16) \div 8$

E.

$$(-18) \div 9$$

E.

$(-14) \div 1$

E.

$$(-21) \div 3$$

E.

$$(-48) \div 6$$

E.

$$(-44) \div 4$$

E.

$$(-81) \div 9$$

E.

$(-56) \div 7$

E.

$$(-25) \div 5$$

E.

$$(-12) \div 6$$

E.

$(-64) \div 8$

E.

$$(-15) \div 3$$

E.

$(-72) \div 9$

F.

$$(-18) \div (-9)$$

F.

$(-12) \div (-3)$

F.

$$(-24) \div (-6)$$

F.

$$(-40) \div (-5)$$

F.

$(-72) \div (-8)$

F.

$$(-36) \div (-6)$$

F.

$$(-20) \div (-4)$$

F.

(-70) ÷ (-7)

F.

$$(-21) \div (-3)$$

F.

$$(-45) \div (-9)$$

F.

$$(-27) \div (-3)$$

F.

$(-15) \div (-5)$

F.

$$(-16) \div (-4)$$

F.

$$(-10) \div (-5)$$

F.

$(-30) \div (-6)$

F.

$$(-32) \div (-8)$$

F.

$$(-99) \div (-9)$$

F.

$$(-24) \div (-2)$$

F.

$$(-36) \div (-3)$$

F.

$$(-27) \div (-9)$$

Module 19:

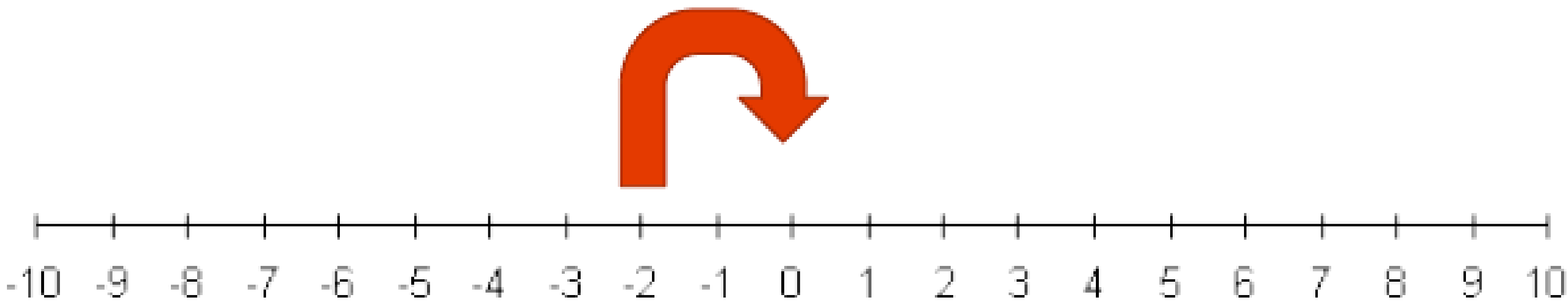
Multiplication and Division of Integers

Vocabulary Cards

absolute value
divide/division
dividend
divisor
factor
integer
multiply/multiplication
negative number
number line
opposites
positive number
product
quotient
zero pair

absolute value

The distance of a number from 0 on a number line.



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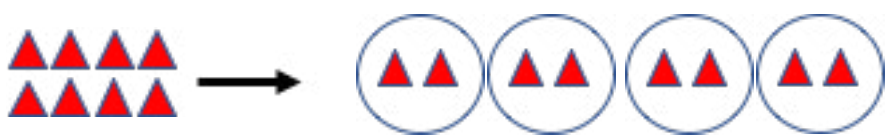
divide/division

To separate into equal groups.



8 ÷ 4 = 2

dividend

The number that is to be divided in a division problem.

$$16 \div 8 = 2$$

16 is the dividend

---

divisor

The number that the dividend is divided by.

$$16 \div 8 = 2$$

8 is the divisor

factor

A number that you multiply with another number to get the product.

$$2 \times 8 = 16$$

2 and 8 are the factors

---

integer

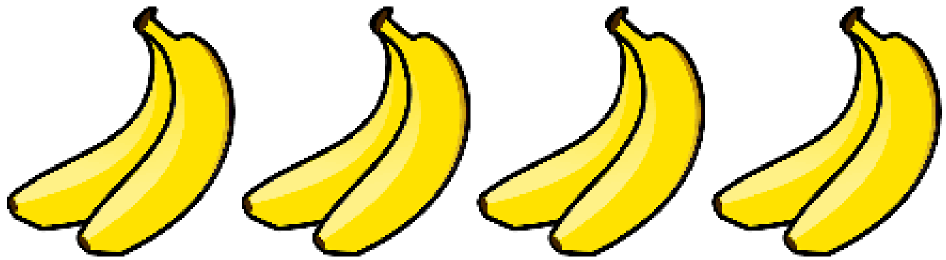
A positive or negative whole number.

-3 -2 -1 1 2 3

multiply/multiplication

The process of adding a number to itself a number of times.

$4 \times 2 = 8$



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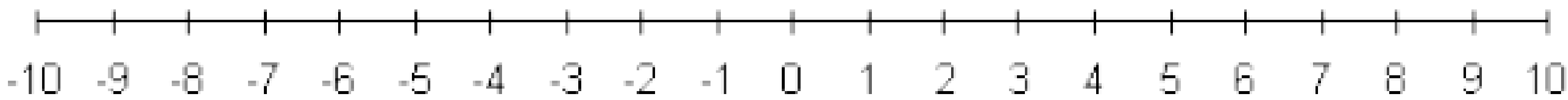
negative number

Any number less than 0.

-3 -2 -1

number line

A straight line with numbers placed at equal intervals along its length.



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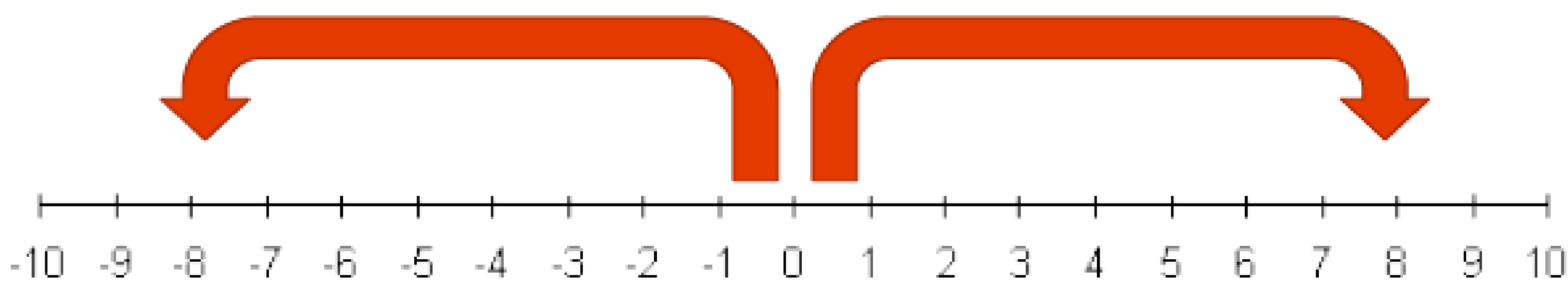
opposites

Two numbers that are equal distance from 0 on a number line.

-8 and 8 are opposites



positive number

Any number greater than 0.

1 2 3

---

product

The result of multiplying two or more factors.

$$2 \times 8 = 16$$

16 is the product

quotient

The number that results when one number is divided by another number.

$$16 \div 8 = 2$$

2 is the quotient

---

zero pair

A pair of numbers with a sum of 0.

$$-7 + 7 = 0$$

Instructional Routines for Mathematics Intervention

MODULE 20

Functions and Ordered Pairs

Module 20: Functions and Ordered Pairs

Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
coordinate plane	A two-dimensional plane formed at the intersection of the *x*-axis and *y*-axis.
equation	A mathematical statement that two expressions are the same or equal; must have an equal sign.
expression	A combination of variables, numbers, and/or operations that represents a mathematical relationship; does not have an equal sign.
function	A relationship between two quantities in which every output corresponds to exactly one input.
function table	A table that displays a set of inputs and outputs in such a way that each input has a unique output.
input variable	The *x* of an equation; the information put in to find the output.
ordered pair	A pair of numbers used to locate a point on a coordinate plane.
origin	A point where the *x*-axis and *y*-axis intersect. The origin has the coordinates (0, 0).
output variable	The *y* of an equation; the information gained after the input is plugged into an equation.
quadrant	The *x*- and *y*-axes divide the coordinate plane into four regions called quadrants.
x-axis	The horizontal number line on a coordinate plane.
y-axis	The vertical number line on a coordinate plane.

B. Background Information

In this module, we focus primarily on functions. The secondary focus is ordered pairs and graphing related to functions. We include routines and examples for the following:

(1) Function Tables with Rules and Expressions
(2) Using the Rule in Function Tables
(3) Function Tables with Rules and Equations
(4) Function Tables and Ordered Pairs
(5) Graphing Ordered Pairs

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C. Routines and Examples

(1) Function Tables with Rules and Expressions

Routine

Materials:

- Module 20 Problem Sets
- Module 20 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching

ROUTINE

Teacher	**Let's determine the rule for a function table and write an expression that represents the rule. First, let's talk about a function table. A function table has two columns** (for tables presented vertically)**/two rows** (for tables presented horizontally)**. What does a function table have?**
Students	Two columns/two rows.
Teacher	**And the columns/rows show *x* and *y*. What do the columns/rows show?**
Students	*x* and *y.*
Teacher	**We can use *x* and *y* to determine a rule about the relationship between *x* and *y.* What can we determine?**
Students	A rule about the relationship between *x* and *y.*
Teacher	**Using that rule, we can write an expression that represents the rule. What's an expression?**
Students	Numbers and operator symbols.
Teacher	**That's right. An expression is made of numbers and at least one operator symbol – like the plus sign, minus sign, multiplication symbol, or division symbol. An expression doesn't have an equal sign. Let's get started.** (Show function table.)
Teacher	**This is a function table. Tell me what you notice about the function table.**
Students	The information is organized in columns/rows. Each column/row represents *x* or *y*.
Teacher	**With this function table, first, we will determine the rule. That's the relationship between *x* and *y.* Look at the *x* column/row. What do you notice about the numbers in the *x* column/row?**
Students	(Comments on pattern.)
Teacher	**The numbers in the *x* column/row are …** (fill in information here about positive and negative numbers and other important aspects). **Now, look at the *y* column/row. What do you notice about the numbers in the *y* column/row?**
Students	(Comments on pattern.)
Teacher	**The numbers in the *y* column/row are …** (fill in information here about positive and negative numbers and other important aspects). **Now, let's look at the relationship between each pair of *x* and *y* values. Look carefully. Do you see**

	any relationship between *x* and *y* that is the same for every pair of numbers in the function table?
Students	(Comments on rule.)
Teacher	**Let's test out that rule. You said that if we start with *x* and then add/subtract/multiply/divide __, then we arrive at *y.* Let's see if that works. If *x* equals __ and we add/subtract/multiply/divide __** (rule)**, then *y* should be __. Is that true for the first pair of *x* and *y?***
Students	Yes.
Teacher	**Now, we have to make sure the rule works for every pair of *x* and *y* in the function table. What do we have to do?**
Students	See if the rule works for every pair.
Teacher	**If the rule only works for one or two pairs, then it isn't the correct rule for this function table. Let's see if the rule works for every pair of *x* and *y.* Does the rule work?**
Students	Yes!
Teacher	**So, we determined a rule about the relationship between *x* and *y* in this function table. What's the rule?**
Students	(Explains rule.)
Teacher	**Let's write the rule.** (Write rule. For example, + 6 or – 2 or ÷ 5.)
Teacher	**Now, let's write an expression for the rule. We'll use *x* in our expression. What happens to *x* with our rule?**
Students	(Explains rule.)
Teacher	**Using our rule, our expression would be *x* +/–/×/÷ __** (rule). **Let's write our expression.** (Write expression. For example, *x* + 6 or *x* – 2 or x ÷ 5.)
Teacher	**What's our expression?**
Students	__.
Teacher	**Excellent. We used this function table to do two things. First, we determined the rule that described the relationship between *x* and *y*. Second, we used that rule to write an expression that represented the rule. How did we determine the rule?**
Students	We looked at each pair of *x* and *y* and found the relationship that was the same for each pair.
Teacher	**How did we write the expression?**
Students	We used the rule to write an expression about *x*.
Teacher	**Great work! How can you use the function table to determine a rule and expression?**
Students	Look at all the pairs of *x* and *y.* Determine the change between *x* and *y* and see if that change is the same for all pairs. Then, use the rule to write an expression about *x*.

Example

x	*y*
2	8
5	20
8	32

EXAMPLE

Teacher	**Let's determine the rule for a function table and write an expression that represents the rule. First, let's talk about a function table. A function table has two columns** (for tables presented vertically)**/two rows** (for tables presented horizontally)**. What does a function table have?**
Students	Two columns/two rows.
Teacher	**Look at this function table. What do you notice about this table?** (Show table.)
Students	*x* is in a column and *y* is in a column.
Teacher	**In this function table, *x* and *y* are at the top of each column. We can use *x* and *y* to determine a rule about the relationship between *x* and *y*. What can we determine?**
Students	A rule about the relationship between *x* and *y*.
Teacher	**Using that rule, we can write an expression that represents the rule. What's an expression?**
Students	Numbers and operator symbols.
Teacher	**That's right. An expression is made of numbers and at least one operator symbol – like the plus sign, minus sign, multiplication symbol, or division symbol. An expression doesn't have an equal sign. What does an expression not have?**
Students	An equal sign.
Teacher	**With this function table, let's determine the rule. That's the relationship between *x* and *y*. Look at the *x* column. What do you notice about the numbers in the *x* column?**
Students	All the *x* values are positive.
Teacher	**The numbers in the *x* column are all positive. Now, look at the *y* column. What do you notice about the numbers in the *y* column?**
Students	These *y* values are also positive. Each *y* value is greater than the corresponding *x* value.
Teacher	**The numbers in the *y* column are all positive. I also see that each *y* value is greater than the corresponding *x* value. Now, let's look at the relationship between each pair of *x* and *y* values. Look carefully. Do you see any relationship between *x* and *y* that is the same for every pair of numbers in the function table?**
Students	If you multiply *x* times 4, the product equals *y*.

Teacher	**Let's test out that rule. You said that if we start with *x* and multiply *x* times 4, the product equals *y.* If *x* equals 2 and we multiply by 4, then *y* should be 8. Is that true for the first pair of *x* and *y?***
Students	Yes.
Teacher	**Now, we have to make sure the rule works for every pair of *x* and *y* in the function table. What do we have to do?**
Students	See if the rule works for every pair.
Teacher	**If the rule only works for one or two pairs, then it isn't the correct rule for this function table. Let's see if the rule works for every pair of *x* and *y.* Does the rule work?**
Students	Yes!
Teacher	**So, we determined a rule about the relationship between *x* and *y* in this function table. What's the rule?**
Students	Times 4.
Teacher	**Let's write the rule. Our rule is times 4.** (Write rule: × 4.)
Teacher	**Now, let's write an expression for the rule. We'll use *x* in our expression. What happens to *x* with our rule?**
Students	When we multiply *x* times 4, the product is *y.*
Teacher	**Using our rule, our expression would be *x* times 4. Let's write our expression.** (Write expression: *x* × 4.)
Teacher	**What's our expression?**
Students	*x* times 4.
Teacher	**Super. We used this function table to do two things. First, we determined the rule that described the relationship between *x* and *y*. Second, we used that rule to write an expression that represented the rule. How did we determine the rule?**
Students	We looked at each pair of *x* and *y* and found the relationship that was the same for each pair.
Teacher	**How did we write the expression?**
Students	We used the rule to write an expression about *x*.
Teacher	**Great work! How can you use the function table to determine a rule and expression?**
Students	Look at all the pairs of *x* and *y.* Determine the change between *x* and *y* and see if that change is the same for all pairs. Then, use the rule to write an expression about *x*.

(2) Using the Rule in Function Tables

Routine

Materials:

- Module 20 Problem Sets
- Module 20 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching

ROUTINE

Teacher	**Let's determine the rule for a function table and write an expression that represents the rule. First, let's talk about a function table. A function table has two columns** (for tables presented vertically)**/two rows** (for tables presented horizontally)**. What does a function table have?**
Students	Two columns/two rows.
Teacher	**And the columns/rows show *x* and *y*. What do the columns/rows show?**
Students	*x* and *y*.
Teacher	**We can use *x* and *y* to determine a rule about the relationship between *x* and *y*. What can we determine?**
Students	A rule about the relationship between *x* and *y*.
Teacher	**Using that rule, we can write an expression that represents the rule. What's an expression?**
Students	Numbers and operator symbols.
Teacher	**That's right. An expression is made of numbers and at least one operator symbol – like the plus sign, minus sign, multiplication symbol, or division symbol. An expression does not have an equal sign. Let's get started.** (Show function table.)
Teacher	**This is a function table. Tell me what you notice about the function table.**
Students	The information is organized in columns/rows. Each column/row represents *x* or *y*.
Teacher	**With this function table, first, we will determine the rule. That's the relationship between *x* and *y*. Look at the *x* column/row. What do you notice about the numbers in the *x* column/row?**
Students	(Comments on pattern.)
Teacher	**The numbers in the *x* column/row are …** (fill in information here about positive and negative numbers and other important aspects). **Now, look at the *y* column/row. What do you notice about the numbers in the *y* column/row?**
Students	(Comments on pattern.)
Teacher	**The numbers in the *y* column/row are …** (fill in information here about positive and negative numbers and other important aspects). **Now, let's look at the relationship between each pair of *x* and *y* values. Look carefully. Do you see any relationship between *x* and *y* that is the same for every pair of numbers in the function table?**
Students	(Comments on rule.)

Teacher	**Let's test out that rule. You said that if we start with *x* and then add/subtract/multiply/divide __, then we arrive at *y.* Let's see if that works. If *x* equals __ and we add/subtract/multiply/divide __** (rule)**, then *y* should be __. Is that true for the first pair of *x* and *y?***
Students	Yes.
Teacher	**Now, we have to make sure the rule works for every pair of *x* and *y* in the function table. What do we have to do?**
Students	See if the rule works for every pair.
Teacher	**If the rule only works for one or two pairs, then it isn't the correct rule for this function table. Let's see if the rule works for every pair of *x* and *y.* Does the rule work?**
Students	Yes!
Teacher	**So, we determined a rule about the relationship between *x* and *y* in this function table. What's the rule?**
Students	(Explains rule.)
Teacher	**Let's write the rule.** (Write rule. For example, + 6 or – 2 or ÷ 5.)
Teacher	**Now, let's write an expression for the rule. We'll use *x* in our expression. What happens to *x* with our rule?**
Students	(Explains rule.)
Teacher	**Using our rule, our expression would be *x* +/–/×/÷ __** (rule). **Let's write our expression.** (Write expression. For example, *x* + 6 or *x* – 2 or x ÷ 5.)
Teacher	**What's our expression?**
Students	__.
Teacher	**Great work. Now, this function table has missing information. It's our job to fill in the missing information using our rule. What information is missing?**
Students	*x* or *y*.
Teacher	**In this table, *x*/*y* is missing. What's the rule or the relationship between *x* and *y*?** (Explains rule.)
Teacher	**Let's use that rule to fill in the missing information.** (Fill in missing information in table.)
Teacher	**Awesome. How can you fill in missing information in a function table?**
Students	Look at all the pairs of *x* and *y.* Determine the change between *x* and *y* and see if that change is the same for all pairs. Then, use the rule to write an expression about *x*. Finally, use the rule or expression to figure out any missing *x* or *y* values.

Example

x	2	4	7	10
y	-3	-1	__	5

EXAMPLE

Teacher	**Let's determine the rule for a function table and write an expression that represents the rule. First, let's talk about a function table. A function table has two columns** (for tables presented vertically)**/two rows** (for tables presented horizontally)**. What does a function table have?**
Students	Two columns/two rows.
Teacher	**Look at this function table. Tell me what you notice about this function table.** (Show table.)
Students	The information is organized by rows. Each row represents *x* or *y.*
Teacher	**We can use *x* and *y* to determine a rule about the relationship between *x* and *y.* What can we determine?**
Students	A rule about the relationship between *x* and *y.*
Teacher	**Using that rule, we can write an expression that represents the rule. What's an expression?**
Students	Numbers and operator symbols.
Teacher	**That's right. An expression is made of numbers and at least one operator symbol – like the plus sign, minus sign, multiplication symbol, or division symbol. An expression does not have an equal sign.**
Teacher	**This is a function table. Tell me what you notice about the function table.**
Students	The information is organized in rows. Each row represents *x* or *y*.
Teacher	**With this function table, first, we will determine the rule. That's the relationship between *x* and *y.* Look at the *x* row. What do you notice about the numbers in the *x* row?**
Students	The numbers are positive. The numbers are greater than the corresponding *y* values.
Teacher	**The numbers in the *x* row are positive. I also see that each *x* value is greater than the corresponding *y* value. Now, look at the *y* row. What do you notice about the numbers in the *y* row?**
Students	The numbers are a mix of positive and negative numbers. Each *y* value is less than the corresponding *x* value.
Teacher	**The numbers in the *y* row are less than each of the corresponding *y* values. Now, let's look at the relationship between each pair of *x* and *y* values. Look carefully. Do you see any relationship between *x* and *y* that is the same for every pair of numbers in the function table?**
Students	If you subtract 5 from *x*, the difference is *y.*
Teacher	**Let's test out that rule. You said that if we start with *x* and then subtract 5, then we arrive at *y.* Let's see if that works. If *x* equals 2 and we subtract 5, then *y* should be -3. Is that true for the first pair of *x* and *y?***

Students	Yes.
Teacher	**Now, we have to make sure the rule works for every pair of *x* and *y* in the function table. What do we have to do?**
Students	See if the rule works for every pair.
Teacher	**If the rule only works for one or two pairs, then it isn't the correct rule for this function table. Let's see if the rule works for every pair of *x* and *y.* Does the rule work?**
Students	Yes!
Teacher	**So, we determined a rule about the relationship between *x* and *y* in this function table. What's the rule?**
Students	Subtract 5.
Teacher	**Let's write the rule.** (Write rule: – 5.)
Teacher	**Now, let's write an expression for the rule. We'll use *x* in our expression. What happens to *x* with our rule?**
Students	*x* minus 5.
Teacher	**Using our rule, our expression would be *x* – 5. Let's write our expression.** (Write expression: *x* – 5.)
Teacher	**What's our expression?**
Students	*x* minus 5.
Teacher	**Now, this function table has missing information. It's our job to fill in the missing information using our rule. What information is missing?**
Students	*y*.
Teacher	**In this table, *y* is missing. What's the rule or the relationship between *x* and *y*?**
Students	*x* minus 5.
Teacher	**Let's use that rule to fill in the missing information. If *x* equals 7 and you subtract 5, what would *y* be?**
Students	2.
Teacher	**Let's write 2 in the blank for *y.*** (Write 2.)
Teacher	**Great job. How can you fill in missing information in a function table?**
Students	Look at all the pairs of *x* and *y.* Determine the change between *x* and *y* and see if that change is the same for all pairs. Then, use the rule to write an expression about *x*. Finally, use the rule or expression to figure out any missing *x* or *y* values.

(3) Function Tables with Rules and Equations

Routine

Materials:

- Module 20 Problem Sets
- Module 20 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching

ROUTINE

Teacher	**Let's determine the rule for a function table and write an equation that represents the rule. First, let's talk about a function table. A function table has two columns** (for tables presented vertically)**/two rows** (for tables presented horizontally)**. What does a function table have?**
Students	Two columns/two rows.
Teacher	**And the columns/rows show *x* and *y*. What do the columns/rows show?**
Students	*x* and *y*.
Teacher	**We can use *x* and *y* to determine a rule about the relationship between *x* and *y*. What can we determine?**
Students	A rule about the relationship between *x* and *y*.
Teacher	**Using that rule, we can write an equation that represents the rule. What's an equation?**
Students	Numbers and operator symbols with an equal sign.
Teacher	**That's right. An equation is made of numbers and at least one operator symbol – like the plus sign, minus sign, multiplication symbol, or division symbol as well as the equal sign. Let's get started.** (Show function table.)
Teacher	**This is a function table. Tell me what you notice about the function table.**
Students	The information is organized in columns/rows. Each column/row represents *x* or *y*.
Teacher	**With this function table, first, we will determine the rule. That's the relationship between *x* and *y*. Look at the *x* column/row. What do you notice about the numbers in the *x* column/row?**
Students	(Comments on pattern.)
Teacher	**The numbers in the *x* column/row are …** (fill in information here about positive and negative numbers and other important aspects). **Now, look at the *y* column/row. What do you notice about the numbers in the *y* column/row?**
Students	(Comments on pattern.)
Teacher	**The numbers in the *y* column/row are …** (fill in information here about positive and negative numbers and other important aspects). **Now, let's look at the relationship between each pair of *x* and *y* values. Look carefully. Do you see any relationship between *x* and *y* that is the same for every pair of numbers in the function table?**
Students	(Comments on rule.)

Teacher	**Let's test out that rule. You said that if we start with *x* and then add/subtract/multiply/divide __, then we arrive at *y*. Let's see if that works. If *x* equals __ and we add/subtract/multiply/divide __** (rule)**, then *y* should be __. Is that true for the first pair of *x* and *y*?**
Students	Yes.
Teacher	**Now, we have to make sure the rule works for every pair of *x* and *y* in the function table. What do we have to do?**
Students	See if the rule works for every pair.
Teacher	**If the rule only works for one or two pairs, then it isn't the correct rule for this function table. Let's see if the rule works for every pair of *x* and *y*. Does the rule work?**
Students	Yes!
Teacher	**So, we determined a rule about the relationship between *x* and *y* in this function table. What's the rule?**
Students	(Explains rule.)
Teacher	**Let's write the rule.** (Write rule. For example, + 6 or – 2 or ÷ 5.)
Teacher	**Now, let's write an equation for the rule. We'll use both *x* and *y* in our equation. For a rule with addition or subtraction, let's write the equation as $y = x +/- a$. In this equation, *a* represents the number in the rule. How do we write an equation for a rule with addition and subtraction?**
Students	$y = x +/- a$.
Teacher	**For a rule with multiplication or division, let's write the equation as $y = ax$ or $y = x \div a$. In this equation, *a* represents the number in the rule. How do we write an equation for a rule with multiplication or division?**
Students	For multiplication, $y = ax$. For division, $y = x \div a$.
Teacher	**Using our rule, our equation would be *y* = __. Let's write our equation.** (Write equation.)
Teacher	**What's our equation?**
Students	*y* = __.
Teacher	**Nice work. We used this function table to do two things. First, we determined the rule that described the relationship between *x* and *y*. Second, we used that rule to write an equation that represented the rule. How did we determine the rule?**
Students	We looked at each pair of *x* and *y* and found the relationship that was the same for each pair.
Teacher	**How did we write the equation?**
Students	We used the rule to write an equation about *x* and *y*.
Teacher	**So, how can you use the function table to determine a rule and equation?**
Students	Look at all the pairs of *x* and *y*. Determine the change between *x* and *y* and see if that change is the same for all pairs. Then, use the rule to write an equation showing the relationship between *x* and *y*.

Example

x	*y*
6	1
36	6
60	10

EXAMPLE

Teacher	**Let's determine the rule for a function table and write an equation that represents the rule. First, let's talk about a function table. A function table has two columns** (for tables presented vertically)**/two rows** (for tables presented horizontally)**. What does a function table have?**
Students	Two columns/two rows.
Teacher	**Look at this function table. What do you notice about this table?** (Show function table.)
Students	*x* and *y* are in columns.
Teacher	**We can use *x* and *y* to determine a rule about the relationship between *x* and *y.* What can we determine?**
Students	A rule about the relationship between *x* and *y.*
Teacher	**Using that rule, we can write an equation that represents the rule. What's an equation?**
Students	Numbers and operator symbols with an equal sign.
Teacher	**That's right. An equation is made of numbers and at least one operator symbol as well as the equal sign. Look at the function table. First, we will determine the rule. That's the relationship between *x* and *y.* Look at the *x* column. What do you notice about the numbers in the *x* column?**
Students	Each *x* is greater than each *y.*
Teacher	**The numbers in the *x* column are greater than each corresponding *y* value. Now, look at the *y* column. What do you notice about the numbers in the *y* column?**
Students	Each *y* value is less than each corresponding *x* value.
Teacher	**The numbers in the *y* column are less than each corresponding *x* value. Now, let's look at the relationship between each pair of *x* and *y* values. Look carefully. Do you see any relationship between *x* and *y* that is the same for every pair of numbers in the function table?**
Students	If you divide each *x* by 6, the quotient is *y.*
Teacher	**Let's test out that rule. You said that if we start with *x* and then divide by 6, then we arrive at *y.* Let's see if that works. If *x* equals 6 and we divide by 6, then *y* should be 1. Is that true for the first pair of *x* and *y?***
Students	Yes.
Teacher	**Now, we have to make sure the rule works for every pair of *x* and *y* in the function table. What do we have to do?**
Students	See if the rule works for every pair.

Teacher **If the rule only works for one or two pairs, then it isn't the correct rule for this function table. Let's see if the rule works for every pair of *x* and *y.* Does the rule work?**

Students Yes!

Teacher **So, we determined a rule about the relationship between *x* and *y* in this function table. What's the rule?**

Students Divide by 6.

Teacher **Let's write the rule.**

(Write rule: ÷ 6.)

Teacher **Now, let's write an equation for the rule. We'll use both *x* and *y* in our equation. For a rule with addition or subtraction, let's write the equation as $y = x +/- a$. In this equation, *a* represents the number in the rule. How do we write an equation for a rule with addition and subtraction?**

Students $y = x +/- a$.

Teacher **For a rule with multiplication or division, let's write the equation as $y = ax$ or $y = x \div a$. In this equation, *a* represents the number in the rule. How do we write an equation for a rule with multiplication or division?**

Students For multiplication, $y = ax$. For division, $y = x \div a$.

Teacher **Using our rule, our equation would be $y = x \div 6$. Let's write our equation.**

(Write: $y = x \div 6$.)

Teacher **What's our equation?**

Students $y = x \div 6$.

Teacher **Nice work. We used this function table to do two things. First, we determined the rule that described the relationship between *x* and *y*. Second, we used that rule to write an equation that represented the rule. How did we determine the rule?**

Students We looked at each pair of *x* and *y* and found the relationship that was the same for each pair.

Teacher **How did we write the equation?**

Students We used the rule to write an equation about *x* and *y.*

Teacher **So, how can you use the function table to determine a rule and equation?**

Students Look at all the pairs of *x* and *y.* Determine the change between *x* and *y* and see if that change is the same for all pairs. Then, use the rule to write an equation showing the relationship between *x* and *y.*

(4) Function Tables and Ordered Pairs

Routine

Materials:

- Module 20 Problem Sets
- Module 20 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching

ROUTINE

Teacher	**Let's use this function table to determine ordered pairs. First, what's a function table?**
Students	It's a table that shows the relationship between *x* and *y.*
Teacher	**Yes. A function table shows the relationship between *x* and *y.* Second, what's an ordered pair?**
Students	It's a way to write *x* and *y.*
Teacher	**An ordered pair shows the relationship between one *x* and *y* pair. We write an ordered pair in parentheses. What do we write in parentheses?**
Students	An ordered pair.
Teacher	**And we write the ordered pair as *x* comma *y.* How do we write the ordered pair?**
Students	*x* comma *y.*
Teacher	**Let's get started.** (Show function table.)
Teacher	**This is a function table. In this table, each *x* is paired with a *y.* The *x* represents the first number in an ordered pair. What does the *x* represent?**
Students	The first number in the ordered pair.
Teacher	**The *x* tells you how many spaces from the origin of a coordinate plane you move horizontally. What does *x* represent?**
Students	How many spaces from the origin you move horizontally or across.
Teacher	**The *y* represents the second number in an ordered pair. What does the *y* represent?**
Students	The second number in the ordered pair.
Teacher	**The *y* tells you how many spaces from the *x* of a coordinate plane you move vertically or up and down. What does y represent?**
Students	How many spaces from *x* you move vertically.
Teacher	**So, let's write all the ordered pairs we have in this function table. What's one ordered pair?**
Students	(*x*, *y*).
Teacher	**Yes. Let's write that ordered pair.** (Write ordered pair.)
Teacher	**What's another ordered pair from the function table?**
Students	(*x*, *y*).
Teacher	**Yes. Let's write that ordered pair.**

	(Write ordered pair.)
Teacher	**Let's write all the ordered pairs from the function table.**
	(Write ordered pairs.)
Teacher	**Let's read our ordered pairs. We read them from left to right, like "three, four" or "negative seven, five."**
Students	(Reads ordered pairs.)
Teacher	**Super! We used this function table to write ordered pairs. We wrote each ordered pair in parentheses as *x* comma *y.* How did we write the ordered pairs?**
Students	We wrote each ordered pair in parentheses as *x* comma *y.*
Teacher	**Great. How could you explain ordered pairs to a friend?**
Students	In an ordered pair, *x* represents the number of spaces from the origin of a coordinate plane that you move horizontally. The *y* represents the number of spaces from *x* of a coordinate plane that you move vertically.

Example

x	-1	1	5
y	-3	-1	3

EXAMPLE

Teacher	**Let's use this function table to determine ordered pairs. First, what's a function table?**
Students	It's a table that shows the relationship between *x* and *y.*
Teacher	**Yes. A function table shows the relationship between *x* and *y.* Second, what's an ordered pair?**
Students	It's a way to write *x* and *y.*
Teacher	**An ordered pair shows the relationship between one *x* and *y* pair. We write an ordered pair in parentheses. What do we write in parentheses?**
Students	An ordered pair.
Teacher	**And we write the ordered pair as *x* comma *y.* How do we write the ordered pair?**
Students	*x* comma *y.*
Teacher	**Let's get started.**
	(Show function table.)
Teacher	**This is a function table. In this table, each *x* is paired with a *y.* The *x* represents the first number in an ordered pair. What does the *x* represent?**
Students	The first number in the ordered pair.
Teacher	**The *x* tells you how many spaces from the origin of a coordinate plane you move horizontally or across. What does *x* represent?**
Students	How many spaces from the origin you move horizontally or across.

Teacher	**The *y* represents the second number in an ordered pair. What does the *y* represent?**
Students	The second number in the ordered pair.
Teacher	**The *y* tells you how many spaces from the *x* of a coordinate plane you move vertically or up and down. What does y represent?**
Students	How many spaces from *x* you move vertically or up and down.
Teacher	**So, let's write all the ordered pairs we have in this function table. What's one ordered pair?**
Students	(-1, -3).
Teacher	**Yes. Let's write that ordered pair.** (Write: (-1, -3).)
Teacher	**What's another ordered pair from the function table?**
Students	(1, -1).
Teacher	**Yes. Let's write that ordered pair.** (Write: (1, -1).)
Teacher	**Let's write all the ordered pairs from the function table.** (Write: (5, 3).)
Teacher	**Let's read our ordered pairs. We read them from left to right.**
Students	Negative 1, negative 3. 1, negative 1. 5, 3.
Teacher	**Great work! We used this function table to write ordered pairs. We wrote each ordered pair in parentheses as *x* comma *y*. How did we write the ordered pairs?**
Students	We wrote each ordered pair in parentheses as *x* comma *y.*
Teacher	**Great. How could you explain ordered pairs to a friend?**
Students	In an ordered pair, *x* represents the number of spaces from the origin of a coordinate plane that you move horizontally. The *y* represents the number of spaces from *x* of a coordinate plane that you move vertically.

(5) Graphing Ordered Pairs

Routine

Materials:

- Module 20 Problem Sets
- Module 20 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching

ROUTINE

Teacher	**Let's graph ordered pairs on a coordinate plane. What's an ordered pair?**
Students	It shows the relationship between one *x* and *y* pair.
Teacher	**Yes. An ordered pair shows the relationship between one *x* and *y* pair. Today, we'll plot or mark ordered pairs on this coordinate plane.** (Show coordinate plane.)
Teacher	**What do you notice about this coordinate plane?**
Students	It has an origin. It has one/four quadrants. It has an *x*-axis and a *y*-axis.
Teacher	**This is a coordinate plane. The coordinate plane has an origin. The origin is where the *x*-axis and *y*-axis intersect. What's the origin?**
Students	Where the *x*-axis and *y*-axis intersect.
Teacher	**Speaking of axes, this** (point) **is the *x*-axis. The *x*-axis is a line that runs horizontal or across. What's the *x*-axis?**
Students	A horizontal line.
Teacher	**This** (point) **is the *y*-axis. The *y*-axis is a line that runs vertical or up and down. What's the *y*-axis?**
Students	A vertical line.
Teacher	**The *x*-axis and *y*-axis are the *axes*. Say that with me.**
Students	Axes.
Teacher	**The axes create different quadrants. For example, sometimes, if we're only focused on positive numbers, our coordinate plane will have one quadrant. If we're focused on both positive and negative numbers, our coordinate plane will show four quadrants. A quadrant is the space created by the *x*-axis and *y*-axis from the origin. How many quadrants are in this coordinate plane?**
Students	One/four.
Teacher	**Let's get started.** (Show ordered pair.)
Teacher	**This is an ordered pair. Read this ordered pair.**
Students	(__, __).
Teacher	**The first number in an ordered pair represents *x*. What does the first number represent?**
Students	*x*.
Teacher	**The first number tells you how many spaces from the origin of a coordinate plane you move horizontally or across. What does *x* or the first number represent?**

Students How many spaces from the origin you move horizontally or across.
Teacher So, let's start at the origin. Where's the origin on this coordinate plane?
(Describes origin.)
Teacher The origin is the place where the *x*-axis and *y*-axis intersect. It's helpful to think of the origin as the ordered pair (0, 0). How can we interpret the origin as an ordered pair?
Students (0, 0).
Teacher If *x* is positive, we'll move forward from the origin along the *x*-axis. How do we move if *x* is positive?
Students Forward.
Teacher If *x* is negative, we'll move backward from the origin along the *x*-axis. How do we move if *x* is negative?
Students Backward.
Teacher Let's mark *x* on this coordinate plane. Starting at the origin, let's move our pencil 1, 2, 3, … spaces horizontally from the origin along the *x*-axis.
(Move pencil *x* spaces.)
Teacher Now, I leave my pencil where it is and turn my attention to the second number in the ordered pair. The second number in an ordered pair represents *y*. What does the second number represent?
Students *y*.
Teacher The second number tells you how many spaces from the *x* of a coordinate plane you move vertically or up and down. What does *y* or the second number represent?
Students How many spaces from *x* you move vertically or up and down.
Teacher So, let's start at *x*. Where's *x* on this coordinate plane?
Students (Describes *x*.)
Teacher If *y* is positive, we'll move up from *x* along the *y*-axis. How do we move if *y* is positive?
Students Up.
Teacher If *y* is negative, we'll move down from *x* along the *y*-axis. How do we move if *y* is negative?
Students Down.
Teacher Let's mark our ordered pair on this coordinate plane. Starting at *x*, let's move our pencil 1, 2, 3, … spaces vertically from *x* along the *y*-axis.
(Move pencil *y* spaces. Draw dot at location of ordered pair.)
Teacher So, we marked __ (ordered pair) **on the coordinate plane. Let's label this dot with our ordered pair.**
(Write ordered pair.)
Teacher We used this coordinate plane to mark or plot an ordered pair. How did we plot the ordered pair?
Students We moved *x* spaces horizontally from the origin and then *y* spaces vertically from *x*.
Teacher Great. How could you explain plotting an ordered pair on a coordinate plane to a friend?

Students	We started at the origin. We moved *x* spaces horizontally from the origin. Then, we moved *y* spaces vertically from the *x*. We drew a dot and labeled the ordered pair.

Example

(-4, 5)

Example

Teacher	**Let's graph ordered pairs on a coordinate plane. What's an ordered pair?**
Students	It shows the relationship between one *x* and *y* pair.
Teacher	**Yes. An ordered pair shows the relationship between one *x* and *y* pair. Today, we'll plot or mark ordered pairs on this coordinate plane.** (Show coordinate plane.)
Teacher	**What do you notice about this coordinate plane?**
Students	It has an origin. It has four quadrants. It has an *x*-axis and a *y*-axis.
Teacher	**This is a coordinate plane. The coordinate plane has an origin. The origin is where the *x*-axis and *y*-axis intersect. What's the origin?**
Students	Where the *x*-axis and *y*-axis intersect.
Teacher	**Speaking of axes, this** (point) **is the *x*-axis. The *x*-axis is a line that runs horizontal or across. What's the *x*-axis?**
Students	A horizontal line.
Teacher	**This** (point) **is the *y*-axis. The *y*-axis is a line that runs vertical or up and down. What's the *y*-axis?**
Students	A vertical line.
Teacher	**The *x*-axis and *y*-axis are the *axes*. Say that with me.**
Students	Axes.
Teacher	**The axes create different quadrants. For example, sometimes, if we're only focused on positive numbers, our coordinate plane will have one quadrant. If we're focused on both positive and negative numbers, our coordinate plane will show four quadrants. A quadrant is the space created by the *x*-axis and *y*-axis from the origin. How many quadrants in this coordinate plane?**
Students	Four.
Teacher	**Let's get started.** (Show ordered pair.)
Teacher	**This is an ordered pair. Read this ordered pair.**
Students	(-4, 5).
Teacher	**The first number in an ordered pair represents *x*. What does the first number represent?**
Students	*x*.
Teacher	**The first number tells you how many spaces from the origin of a coordinate plane you move horizontally or across. What does *x* or the first number represent?**
Students	How many spaces from the origin you move horizontally or across.

Teacher	**So, let's start at the origin. Where's the origin on this coordinate plane?**
	(Describes origin.)
Teacher	**The origin is the place where the *x*-axis and *y*-axis intersect. It's helpful to think of the origin as the ordered pair (0, 0). How can we interpret the origin as an ordered pair?**
Students	(0, 0).
Teacher	**If *x* is positive, we'll move forward from the origin along the *x*-axis. How do we move if *x* is positive?**
Students	Forward.
Teacher	**If *x* is negative, we'll move backward from the origin along the *x*-axis. How do we move if *x* is negative?**
Students	Backward.
Teacher	**Let's mark *x* on this coordinate plane. Is x positive or negative?**
Students	Negative.
Teacher	**Because -4 is negative, we'll move backward from the origin. Starting at the origin, let's move our pencil -1, -2, -3, -4 horizontally from the origin along the *x*-axis.**
	(Move pencil backward 4 spaces from origin.)
Teacher	**Now, I leave my pencil where it is and turn my attention to the second number in the ordered pair. The second number in an ordered pair represents *y*. What does the second number represent?**
Students	*y*.
Teacher	**The second number tells you how many spaces from the *x* of a coordinate plane you move vertically or up and down. What does *y* or the second number represent?**
Students	How many spaces from *x* you move vertically or up and down.
Teacher	**So, let's start at *x*. Where's *x* on this coordinate plane?**
Students	(-4.)
Teacher	**If *y* is positive, we'll move up from *x* along the *y*-axis. How do we move if *y* is positive?**
Students	Up.
Teacher	**If *y* is negative, we'll move down from *x* along the *y*-axis. How do we move if *y* is negative?**
Students	Down.
Teacher	**Because 5 is positive, we'll move up vertically from -4. Starting at -4, let's move our pencil 1, 2, 3, 4, 5 spaces vertically from -4.**
	(Move pencil up 5 spaces from -4. Draw dot at location of ordered pair.)
Teacher	**So, we marked (-4, 5) on the coordinate plane. Let's label this dot with our ordered pair.**
	(Write (-4, 5).)
Teacher	**We used this coordinate plane to mark or plot an ordered pair. How did we plot the ordered pair?**
Students	We moved -4 spaces horizontally from the origin and then 5 spaces vertically from -4.

Teacher	**Great. How could you explain plotting an ordered pair on a coordinate plane to a friend?**
Students	We started at the origin. We moved *x* spaces horizontally from the origin. Then, we moved *y* spaces vertically from the *x*. We drew a dot and labeled the ordered pair.

D. Problems for Use During Instruction

See Module 20 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 20 Vocabulary Cards.

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Module 20:
Functions and Ordered Pairs

Problem Sets

A.

x	*y*
2	4
4	6
6	8

A.

x	*y*
1	5
3	7
4	8

A.

x	y
3	10
4	11
5	12

A.

x	*y*
2	7
3	8
5	10

A.

x	*y*
1	9
4	12
7	15

A.

x	*y*
2	0
3	1
17	15

A.

x	*y*
10	2
13	5
18	10

A.

x	*y*
6	1
8	3
11	6

A.

x	*y*
14	11
18	15
22	19

A.

x	*y*
1	0
2	1
3	2

A.

x	y
1	3
2	6
7	21

A.

x	*y*
0	0
5	45
6	54

A.

x	y
4	4
6	6
7	7

A.

x	*y*
2	20
3	30
5	50

A.

x	*y*
1	5
3	15
4	20

A.

x	*y*
9	1
27	3
54	6

A.

x	*y*
14	2
28	4
49	7

A.

x	*y*
56	7
72	9
88	11

A.

x	*y*
3	1
6	2
9	3

A.

x	y
30	3
40	4
60	6

B.

x	2	4	6
y	4	6	8

B.

x	1	3	4
y	5	7	8

B.

x	3	4	5
y	10	11	12

B.

x	2	3	5
y	7	8	10

B.

x	1	4	7
y	9	12	15

B.

x	2	3	17
y	0	1	15

B.

x	10	13	18
y	2	5	10

B.

x	6	8	11
y	1	3	6

B.

x	14	18	22
y	11	15	18

B.

x	1	2	3
y	0	1	2

B.

x	1	2	7
y	3	6	21

B.

x	0	5	6
y	0	45	54

B.

x	4	6	7
y	4	6	7

B.

x	2	3	5
y	20	30	50

B.

x	1	3	4
y	5	15	20

B.

x	9	27	54
y	1	3	6

B.

x	14	28	49
y	2	4	7

B.

x	56	72	88
y	7	9	11

B.

x	3	6	9
y	1	2	3

B.

x	30	40	60
y	3	4	6

c.

x	*y*
2	5
5	8
6	9
9	___

c.

x	*y*
1	7
3	___
5	11
7	13

c.

x	*y*
4	___
7	12
10	15
14	19

c.

x	*y*
12	22
___	25
18	28
20	30

c.

x	*y*
5	19
6	20
____	21
10	24

c.

x	y
10	6
13	9
18	14
___	24

c.

x	y
13	3
29	___
58	48
65	55

c.

x	y
23	___
35	28
49	42
61	54

c.

x	***y***
___	11
34	25
47	38
59	50

c.

x	*y*
11	0
12	1
___	11
26	15

c.

x	*y*
___	4
2	8
3	12
4	16

c.

x	*y*
0	0
2	___
3	6
5	10

c.

x	*y*
2	22
3	33
4	44
___	55

c.

x	***y***
3	___
4	24
7	54
12	72

c.

x	*y*
2	16
___	32
5	40
6	48

c.

x	*y*
4	___
20	5
32	8
36	9

c.

x	*y*
16	2
___	3
56	7
64	8

c.

x	y
4	2
8	4
18	___
20	10

c.

x	y
10	2
___	3
20	4
25	5

c.

x	***y***
9	3
15	5
21	7
27	___

D.

x	2	5	6	9
y	5	8	9	___

D.

x	1	3	5	7
y	7	___	11	13

D.

x	4	7	10	14
y	___	12	15	19

D.

x	12	___	18	20
y	22	25	28	30

D.

x	5	6	__	10
y	19	20	21	24

D.

x	10	13	18	__
y	6	9	14	24

D.

x	13	29	58	65
y	3	___	48	55

D.

x	23	35	49	61
y	___	28	42	54

D.

x	___	34	47	59
y	11	25	38	50

D.

x	11	12	___	26
y	0	1	11	15

D.

x	___	2	3	4
y	4	8	12	16

D.

x	0	2	3	5
y	0	___	6	10

D.

x	2	3	4	___
y	22	33	44	55

D.

x	3	4	7	12
y	___	24	54	72

D.

x	2	__	5	6
y	16	32	40	48

D.

x	4	20	32	36
y	___	5	8	9

D.

x	16	___	56	64
y	2	3	7	8

D.

x	4	8	18	20
y	2	4	___	10

D.

x	10	___	20	25
y	2	3	4	5

D.

x	9	15	21	27
y	3	5	7	___

E.

x	*y*
-1	3
-2	2
-3	1

E.

x	*y*
-9	-6
-8	-5
-7	-4

E.

x	*y*
-6	0
-12	-6
-18	-12

E.

x	y
0	-2
1	-1
2	0

E.

x	*y*
-8	-5
4	1
7	4

E.

x	y
2	-3
7	2
13	8

E.

x	*y*
-4	-16
-5	-20
6	24

E.

x	*y*
-5	0
-7	0
-16	0

E.

x	y
8	4
-8	-4
-12	-6

E.

x	*y*
-6	-1
-12	-2
18	3

F.

x	-1	-2	-3
y	3	2	1

F.

x	-9	-8	-7
y	-6	-5	-4

F.

x	-6	-12	-18
y	0	-6	-12

F.

x	0	1	2
y	-2	-1	0

F.

x	-8	4	7
y	-5	1	4

F.

x	2	7	13
y	-3	2	8

F.

x	-4	-5	6
y	-16	-20	24

F.

x	-5	-7	-16
y	0	0	0

F.

x	8	-8	-12
y	4	-4	-6

F.

x	-6	-12	18
y	-1	-2	3

G.

x	*y*
___	-10
-10	0
0	10
10	20

G.

x	*y*
-16	-8
-8	0
-4	4
0	___

G.

x	*y*
-45	-30
___	-15
-15	0
15	30

G.

x	*y*
-1	-6
0	___
1	-4
2	-3

G.

x	*y*
18	9
-9	0
___	-12
-18	-27

G.

x	*y*
26	13
14	1
-23	___
-54	-67

G.

x	*y*
-7	-49
-9	-63
-10	-70
___	-77

G.

x	*y*
3	___
4	48
5	60
6	72

G.

x	y
___	-11
-63	-9
-42	-6
-21	-3

G.

x	*y*
-8	___
-6	3
-4	2
-2	1

H.

x	___	-10	0	10
y	-10	0	10	20

H.

x	-16	-8	-4	0
y	-8	0	4	___

H.

x	-45	___	-15	15
y	-30	-15	0	30

H.

x	-1	0	1	2
y	-6	___	-4	-3

H.

x	18	-9	___	-18
y	9	0	-12	-27

H.

x	26	14	-23	-54
y	13	1	___	-67

H.

x	-7	-9	-10	___
y	-49	-63	-70	-77

H.

x	3	4	5	6
y	___	48	60	72

H.

x	___	-63	-42	-21
y	-11	-9	-6	-3

H.

x	-8	-6	-4	-2
y	___	3	2	1

1.

(5, 8)

1.

(9, 5)

I.

(7, 7)

I.

(0, 4)

1.

(6, 2)

1.

(5, 0)

1.

(9, 9)

1.

(3, 2)

1.

(0, 7)

I.

(1, 1)

1.

(3, 6)

1.

(7, 15)

I.

(8, 11)

1.

(8, 10)

I.

(12, 0)

I.

(4, 4)

1.

(6, 4)

1.

(2, 5)

1.

(4, 3)

1.

(13, 3)

J.

(-5, 8)

J.

(-9, 5)

J.

(7, -7)

J.

(0, -4)

J.

(-6, 2)

J.

(-5, 0)

J.

(9, -9)

J.

(-3, 2)

J.

(0, -7)

J.

(1, -1)

J.

(3, -6)

J.

(7, -15)

J.

(8, -11)

J.

(-8, 10)

J.

(-12, 0)

J.

(-4, 4)

J.

(6, -4)

J.

(-2, 5)

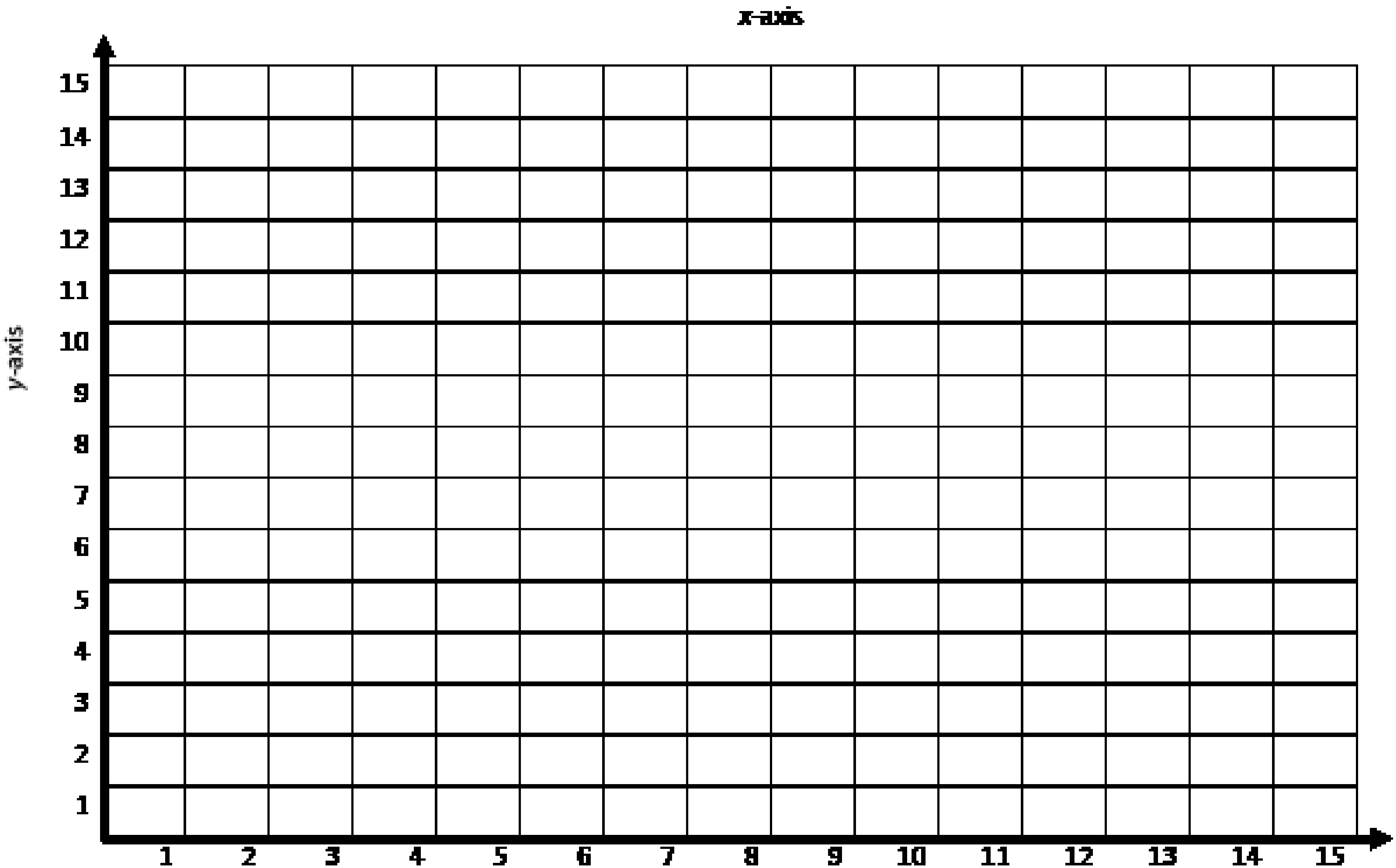
J.

(4, -3)

J.

(-13, 3)

K.



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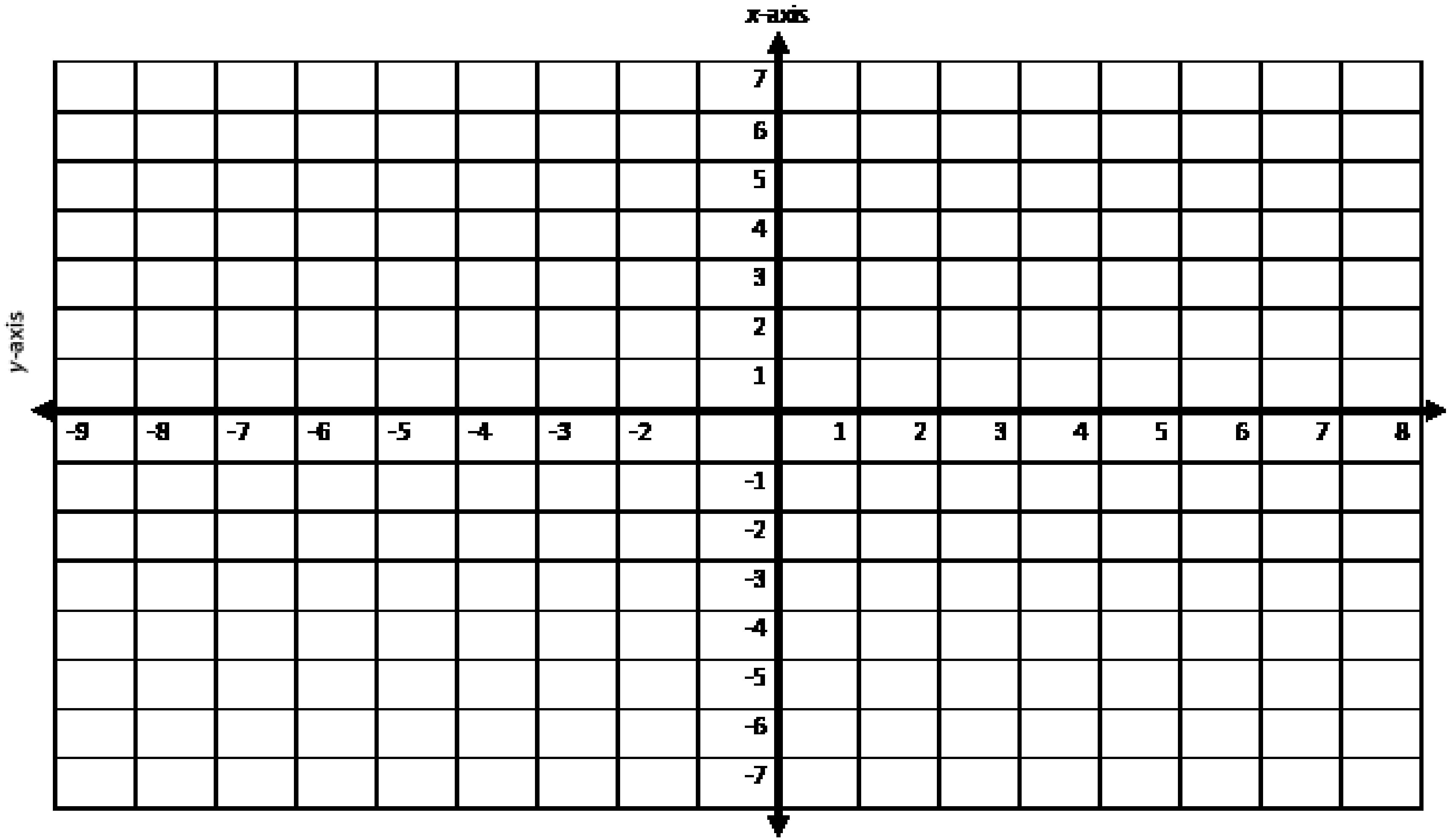


Module 20:

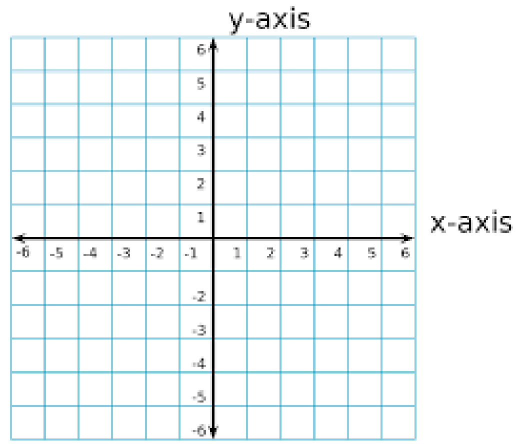
Functions and Ordered Pairs

Vocabulary Cards

coordinate plane
equation
expression
function
function table
input variable
ordered pair
origin
output variable
quadrant
***x*-axis**
***y*-axis**

coordinate plane

A two-dimensional plane formed at the intersection of the *x*-axis and *y*-axis.



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equation

A mathematical statement that two expressions are the same or equal; must have an equal sign.

$$5x + 9 = 24$$

5x + 9 = 24 is an equation

(<u>DOES</u> have an = sign)

expression

A combination of variables, numbers, and/or operations that represents a mathematical relationship; does not have an equal sign.

$5x + 9$ 24

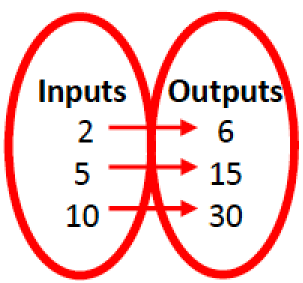
5x + 9 and 24 are expressions

(DOES NOT have an = sign)

---

function

A relationship between two quantities in which every input corresponds to exactly one output.



x	*y*
2	6
5	15
10	30

function table

A table that displays a set of inputs and outputs in such a way that each input has a unique output.

x	*y*
1	5
3	7
4	8

x	1	3	4
y	5	7	8

---

input variable

The *x* of an equation; the information put in to find the output.

In the equation $x + 1 = y$, x is the input variable

ordered pair

A pair of numbers used to locate a point on a coordinate plane.

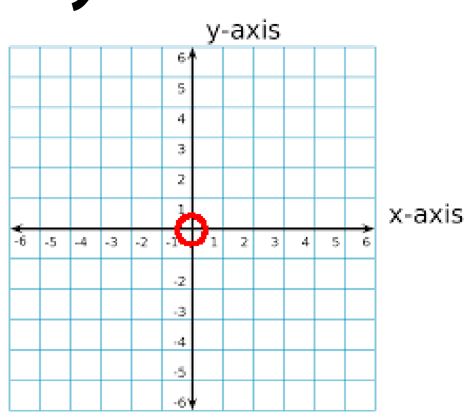
Examples:

(-4, 3) (0, 2) (6, -1)

---

origin

A point where the *x*-axis and *y*-axis intersect. The origin has the coordinates (0, 0).



output variable

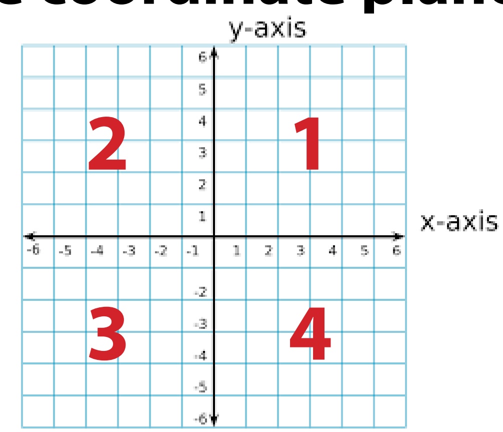
The *y* of an equation; the information gained after the input is plugged into an equation.

In the equation $x + 1 = y$, y is the output variable

---

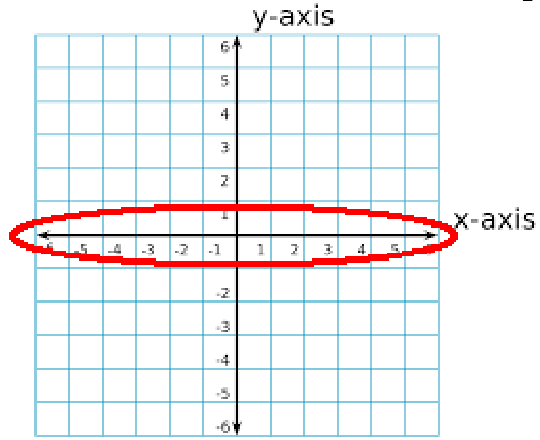
quadrant

The *x*- and *y*-axes divide the coordinate plane into four regions called quadrants.



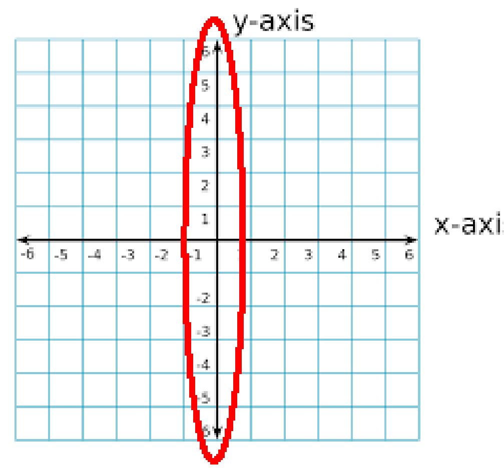
x-axis

The horizontal number line on a coordinate plane.



y-axis

The vertical number line on a coordinate plane.



Instructional Routines for Mathematics Intervention

MODULE 21

Ratios, Proportions, Rates, and Percentages

Module 21: Ratios, Proportions, Rates, and Percentages
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
coefficient	A number that is multiplied by a variable.
constant	A term that does not change; a number on its own.
denominator	The term in a fraction that tells the number of equal parts in a whole.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent fractions	Fractions that have different numerators and denominators that represent the same value or proportion of the whole.
equivalent ratios	Ratios that have the same fractional number, value, or measure.
fraction	A number representing part of a whole or set.
improper fraction	Any fraction in which the numerator is greater than the denominator.
least common multiple	The common multiple with the least value.
like fractions	Fractions that have the same denominator.
lowest terms	A fraction is simplified to lowest terms when there is no number other than 1 that will evenly divide into both the numerator and denominator.
mixed number	A whole number and a fraction combined.
multiple	The product of a number and any integer.
numerator	The term in a fraction that tells how many parts of a fraction.
percentage	A rate of an amount per hundred.
proper fraction	A fraction where the numerator is less than the denominator.
proportion	An equation that states that two ratios are equal.
rate	A comparison of two quantities that have different units of measure.
ratio	A comparison of two quantities that have the same unit of measure.
unit rate	A ratio that is written as a number to one.
unlike fractions	Fractions that have different denominators.
variable	A symbol for an unknown value, which is usually represented by a letter.

B. Background Information

In this module, we focus on representing (1) ratios, (2) proportions, (3) rates, and (4) percentages.

C. Routines and Examples

(1) Representing Ratios

Routine

Materials:

- Module 21 Problem Sets
- Module 21 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like blocks or shapes

ROUTINE WITH GEOMETRIC SHAPES

Teacher	**Let's show different ratios. What's a ratio?**
Students	An expression in which we compare one quantity to another.
Teacher	**A ratio is an expression. In a ratio, we compare how much of one amount we have compared to another amount. With ratios, we can compare parts to parts or parts to a whole. How can we compare ratios?**
Students	Parts to parts or parts to a whole.
Teacher	**So, let's show different ratios. We'll use these geometric shapes.** (Show manipulatives.) (Show ratio.)
Teacher	**What's this ratio?**
Students	__ to __.
Teacher	**When we read ratios, make sure to say "to" between the numbers. So, __** (read numbers and emphasize "to")**. Let's say that together.**
Students	__ to __.
Teacher	**Let's show this ratio by comparing parts to parts. What's the first number in the ratio?**
Students	__.
Teacher	**So, let's show __** (first number) **of the shapes. Let's show __ squares. How many?**
Students	__. (Show using shapes.)
Teacher	**Now, what's the second number in the ratio?**
Students	__.
Teacher	**So, let's show __** (second number) **of the shapes. Let's show __ triangles. How many?**

Students	__. (Show using shapes.)
Teacher	**With this ratio, __** (first number) **are squares and __** (second number) **are triangles. The ratio of squares to triangles is __ to __. Say that with me.**
Students	__ to __.
Teacher	**We write our ratio using the colon. I write __ to __ as __** (first number) **colon __** (second number). **Let's write the ratio.** (Write ratio.)
Teacher	**We also can write a ratio as a fraction. The first number in the ratio will be the numerator and the second number will be the denominator. How do we write a ratio as a fraction?**
Students	Write the first number as the numerator and second number as the denominator.
Teacher	**Let's write this ratio as a fraction.** (Write fraction.)
Teacher	**What's the fraction?**
Students	__.
Teacher	**If we write a fraction for a part to part ratio, we don't read the fraction as a fraction. We can write it as a fraction but we don't read it as a fraction. Should we read this as a fraction?**
Students	No.
Teacher	**Now, let's think about the ratio in a different way. Another way to show a ratio is to compare parts to the whole or set. What's another way to show a ratio?**
Students	To compare parts to the whole or set.
Teacher	**Let's use the squares and triangles from before. Altogether, we have 1, 2, 3…** (count) **shapes. How many squares are in this set?**
Students	__.
Teacher	**How many shapes are there in the set?**
Students	__.
Teacher	**If there are __ squares, the ratio of squares to all of the shapes is __ to __. What's the ratio?**
Students	__ to __.
Teacher	**Let's write that ratio.** (Write ratio.)
Teacher	**Let's write this ratio as a fraction.** (Write fraction.)
Teacher	**What's the fraction?**
Students	__.
Teacher	**Now, how many triangles are in this set?**
Students	__.
Teacher	**If there are __ triangles, the ratio of triangles to all of the shapes is __ to __. What's the ratio?**
Students	__ to __.

Teacher	**Let's write that ratio.** (Write ratio.)
Teacher	**Let's write this ratio as a fraction.** (Write fraction.)
Teacher	**What's the fraction?**
Students	__.
Teacher	**One way to show ratios is to compare parts to parts. How do you show parts to parts?**
Students	Show the number of squares and compare the number of squares to the number of triangles.
Teacher	**When we compare parts to parts, we show two different objects and compare one object, like squares, to another object, like triangles. What other objects could you use to compare parts to parts?**
Students	Cats and dogs, blue cubes and red cubes, cereal and candy.
Teacher	**Another way to show ratios is to compare parts to a whole or the set. How do you compare parts to a set?**
Students	Show the number of squares and compare the number of squares to the number of all of the shapes in a set.
Teacher	**When we compare parts to a whole or a set, we show two different objects and compare one object, like squares, to all of the objects in the set. We use the same objects but we think about the ratio in a different way. Let's review. What's a ratio?**
Students	An expression in which we compare one quantity to another.
Teacher	**How do you write a ratio as a fraction?**
Students	Write the first number as the numerator and the second number as the denominator.
Teacher	**Great work! Using these objects helps you understand the different ratios. How can you use objects to show a ratio?**
Students	You could show ratios using shapes like squares and triangles. To compare parts to parts, you compare the squares to the triangles. To compare parts to the whole, you compare one shape, like squares, to all of the shapes.

Example

4 : 3

EXAMPLE WITH COLORED CUBES

Teacher	**Let's show different ratios. What's a ratio?**
Students	An expression in which we compare one quantity to another.
Teacher	**A ratio is an expression. In a ratio, we compare how much of one amount we have compared to another amount. With ratios, we can compare parts to parts or parts to a whole. How can we compare ratios?**
Students	Parts to parts or parts to a whole.

Teacher	**So, let's show different ratios. We'll use these colored cubes.** (Show manipulatives.) (Show ratio.)
Teacher	**What's this ratio?**
Students	4 to 3.
Teacher	**When we read ratios, make sure to say "to" between the numbers. So, 4 *to* 3. Let's say that together.**
Students	4 *to* 3.
Teacher	**Let's show this ratio by comparing parts to parts. What's the first number in the ratio?**
Students	4.
Teacher	**So, let's show 4 of the colored cubes. Let's use the blue cubes. Let's show 4 blue cubes. How many?**
Students	4. (Show using cubes.)
Teacher	**Now, what's the second number in the ratio?**
Students	3.
Teacher	**So, let's show 3 of the colored cubes. Let's use the yellow cubes. Let's show 3 yellow cubes. How many?**
Students	3. (Show using cubes.)
Teacher	**With this ratio, 4 are blue and 3 are yellow. The ratio of blue to yellow is 4 to 3. Say that with me.**
Students	4 to 3.
Teacher	**We write our ratio using the colon. I write 4 to 3 as 4 colon 3. Let's write the ratio.** (Write ratio.)
Teacher	**We also can write a ratio as a fraction. The first number in the ratio will be the numerator and the second number will be the denominator. How do we write a ratio as a fraction?**
Students	Write the first number as the numerator and second number as the denominator.
Teacher	**Let's write this ratio as a fraction.** (Write fraction.)
Teacher	**What's the fraction?**
Students	$\frac{4}{3}$.
Teacher	**If we write a fraction for a part to part ratio, we don't read the fraction as four-thirds. We can write it as a fraction but we don't read it as a fraction. Should we read this as a fraction?**
Students	No.
Teacher	**Now, let's think about the ratio in a different way. Another way to show a ratio is to compare parts to the whole or set. What's another way to show a ratio?**

Students	To compare parts to the whole or set.
Teacher	**Let's use the blue and yellow cubes from before. Altogether, we have 1, 2, 3, 4, 5, 6, 7 cubes. How many blue cubes are in this set?**
Students	4.
Teacher	**And how many cubes are there in the set altogether?**
Students	7.
Teacher	**If there are 4 blue cubes, the ratio of blue cubes to all of the cubes is 4 to 7. What's the ratio?**
Students	4 to 7.
Teacher	**Let's write that ratio.** (Write ratio.)
Teacher	**Let's write this ratio as a fraction.** (Write fraction.)
Teacher	**What's the fraction?**
Students	$\frac{4}{7}$.
Teacher	**We can read this as four-sevenths. How can we read this fraction?**
Students	Four-sevenths.
Teacher	**Now, how many yellow cubes are in this set?**
Students	3.
Teacher	**If there are 3 yellow cubes, the ratio of yellow cubes to all of the cubes is 3 to 7. What's the ratio?**
Students	3 to 7.
Teacher	**Let's write that ratio.** (Write ratio.)
Teacher	**Let's write this ratio as a fraction.** (Write fraction.)
Teacher	**What's the fraction?**
Students	$\frac{3}{7}$.
Teacher	**We can read this as three-sevenths. How can we read this fraction?**
Students	Three-sevenths.
Teacher	**One way to show ratios is to compare parts to parts. How do you show parts to parts?**
Students	Show the number of blue cubes and compare the number of blue cubes to the number of yellow cubes.
Teacher	**When we compare parts to parts, we show two different objects and compare one object, like blue cubes, to another object, like yellow cubes. What other objects could you use to compare parts to parts?**
Students	Cats and dogs, squares and triangles, cereal and candy.
Teacher	**Another way to show ratios is to compare parts to a whole or the set. How do you compare parts to a set?**
Students	Show the number of blue cubes and compare the number of blue cubes to the number of all of the cubes.

Teacher	**When we compare parts to a whole or a set, we show two different objects and compare one object, like blue cubes, to all of the cubes in the set. We use the same objects but we think about the ratio in a different way. Let's review. What's a ratio?**
Students	An expression in which we compare one quantity to another.
Teacher	**How do you write a ratio as a fraction?**
Students	Write the first number as the numerator and the second number as the denominator.
Teacher	**Great work! Using these objects helps you understand the different ratios. How can you use objects to show a ratio?**
Students	You could show ratios using cubes like blue cubes and yellow cubes. To compare parts to parts, you compare the blue cubes to the yellow cubes. To compare parts to the whole, you compare one color, like blue cubes, to all the cubes.

(2) Representing Proportions

Routine

Materials:

- Module 21 Problem Sets
- Module 21 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes or fraction tiles

ROUTINE WITH FRACTION TILES

Teacher	**Let's look at a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**A proportion shows two equal ratios. Most often, a proportion has an unknown. We use an equation to solve for the unknown within a proportion. Look at this proportion. What do you notice?** (Show proportion.)
Students	Two fractions.
Teacher	**This proportion does have two fractions. Remember, fractions also can be used to represent the ratios within a proportion. This proportion has an unknown. We will solve for the unknown with these fraction tiles.** (Show manipulatives.)
Teacher	**So, __** (first fraction) **is equal to __** (second fraction)**. Let's read that together.**
Students	__ is equal to __.
Teacher	**Which fraction does not have an unknown?**
Students	__.
Teacher	**Let's show that fraction with the fraction tiles. First, I show the denominator divided into equal parts. Then, we show the numerator with the equal parts.** (Show fraction with fraction tiles.)
Teacher	**Now, which fraction does have an unknown?**
Students	__.
Teacher	**What's unknown – the numerator or the denominator?**
Students	Numerator/denominator.
Teacher	**If the numerator is the unknown, we'll use the denominator to divide another whole into equal parts. We'll learn how many of the numerator parts are equal to the other fraction in our proportion. Let's do an example when the numerator is unknown. I take another whole and divide that whole into the equal parts of the denominator. What's the denominator?**
Students	__.
Teacher	**Let's divide the whole into __ equal parts.** (Show denominator with fraction tiles.)
Teacher	**Now, let's place this fraction near our other fraction so we can compare. I like to place them one above the other. Let's compare the fractions. How many**

	equal parts of this unknown fraction are equivalent to the numerator of the known fraction?
Students	__.
Teacher	**Yes. The numerator would be __. What's the numerator of the unknown fraction?**
Students	__.
Teacher	**Let's write in __ for the unknown.** (Write unknown.)
Teacher	**If the denominator is the unknown, we'll use the numerator and place in numerator parts compared to a whole. Let's do an example when the denominator is unknown. I take another whole. What's the numerator?**
Students	__.
Teacher	**Now, let's place this whole so we can compare it to our other fraction in the proportion. I like to place them one above the other. Let's compare the fractions. How many equal numerator parts of this unknown fraction could be used to be equivalent to the numerator of the know fraction?**
Students	__.
Teacher	**Let's show the numerator with __ equal parts.** (Show numerator with fraction tiles.)
Teacher	**We use one-__**(denominator) **parts to show the numerator. That means the denominator is __. Let's write in __** (denominator) **for the unknown.** (Write unknown.)
Teacher	**Let's read the proportion. __ is equal to __. Let's say that together.**
Students	__ is equal to __.
Teacher	**Let's review. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**Using objects helps you understand the how to solve for the unknown in a proportion. How can you use objects to solve for an unknown in a proportion?**
Students	You could use fraction tiles to show the known fraction. Then, you could use another set of fraction tiles to compare fractions to determine the unknown in an equivalent fraction.

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's look at a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**A proportion shows two equal ratios. Often, a proportion has an unknown, and we use an equation to solve for the unknown within a proportion. Look at this proportion. What do you notice?** (Show proportion.)
Students	Two fractions.

Teacher	**This proportion does have two fractions. Remember, fractions can be used to represent ratios. This proportion has an unknown. We will solve for the unknown using multiplication and division. Let's read the proportion.**
Students	__ is equal to __.
Teacher	**So, __** (first fraction) **is equal to __** (second fraction)**. Where's the unknown in this proportion?**
Students	__.
Teacher	**We have to determine the unknown in this proportion. The unknown is marked by *x/y/a.* We can determine the unknown by isolating the unknown. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will need to multiply and divide. What will we do?**
Students	Multiply and divide.
Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	__.
Teacher	**The denominator of the first fraction is __. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by __** (first denominator) **and have a denominator of __** (first denominator)**, __ divided by __ equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.** (Cross out.)
Teacher	**Now, multiply the second numerator by __** (first denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.
Teacher	**We now have a numerator of __ in the second fraction. Let's write __.** (Write.)
Teacher	**Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by __** (second denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.

Teacher	**We now have a numerator of __ in the first fraction. Let's write __.** (Write.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.** (Cross out.)
Teacher	**Using multiplication, we've changed our proportion to the equation __ equals __. What's the equation?**
Students	__ equals __.
Teacher	**Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**
Students	It's the constant multiplied by a variable.
Teacher	**A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**
Students	Divide each side of the equation by the coefficient.
Teacher	**What's the coefficient?**
Students	__.
Teacher	**Let's divide each side of the equation by __** (coefficient)**. Whatever we do to one side of the equal sign we also have to do to the other. What's __ divided by __?**
Students	__. (Write.)
Teacher	**So, the variable equals __. What's the value of the unknown?**
Students	__.
Teacher	**Now, there is another way to solve for an unknown. Where is the unknown in this problem?**
Students	Numerator/denominator.
Teacher	**The unknown is in the numerator/denominator, so look at the denominator/numerator. Look at __** (first denominator/numerator) **and __** (second denominator/numerator)**. What do you notice about the relationship between __ and __?**
Students	(Describes relationship.)
Teacher	**Yes! I see that if you multiply/divide by __ with the first denominator/numerator, that equals the second denominator/numerator. It's like a rule in a function! Let's apply that rule to the numerator/denominator. What are the numerator/denominators in each fraction?**
Students	__/*x* and *x*/__.
Teacher	**Let's solve for *x* using the same rule. How could we solve for *x*?**
Students	Multiply/divide.

Teacher	**Using the same rule as the denominator/numerator, *x* would be __. What's *x*?**
Students	__.
Teacher	**Let's check. Does the rule work with the relationship between the numerators?**
Students	Yes.
Teacher	**Does the rule work with the relationship between the denominators?**
Students	Yes.
Teacher	**So, another way to solve for an unknown is to determine the rule between the numerators/denominators and use that to solve for *x*. Which method do you prefer?**
Students	(Explains preferred method.)
Teacher	**Let's review. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**What's one way we solved for an unknown in a proportion?**
Students	We multiplied the first denominator by the first numerator and the second numerator. Then, we multiplied the second denominator by the first numerator and the second numerator. Then, we divided by a coefficient to solve for the unknown.
Teacher	**What's another way we solved for an unknown in a proportion?**
Students	We determined the rule of the relationship between the numerators/denominators and applied that rule to determine the unknown.

Example

$$\frac{x}{12} = \frac{2}{3}$$

EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's look at a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**A proportion shows two equal ratios. Often, a proportion has an unknown, and we use an equation to solve for the unknown with a proportion. Look at this proportion. What do you notice?** (Show proportion.)
Students	Two fractions.
Teacher	**This proportion does have two fractions. Remember, fractions can be used to represent ratios. This proportion has an unknown. We will solve for the unknown using multiplication and division. Let's read the proportion.**
Students	$\frac{x}{12}$ is equal to $\frac{2}{3}$.
Teacher	**Where's the unknown in this proportion?**
Students	In the first fraction.

Teacher	**We have to determine the unknown in this proportion. The unknown is marked by *x*. We can determine the unknown by isolating the unknown. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will need to multiply and divide. What will we do?**
Students	Multiply and divide.
Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	12.
Teacher	**The denominator of the first fraction is 12. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write × 12.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by 12 and have a denominator of 12, 12 divided by 12 equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.** (Cross out 12 and 12.)
Teacher	**Now, multiply the second numerator by 12.** (Write × 12.) **What's the product of 12 times 2?**
Students	24.
Teacher	**We now have a numerator of 24 in the second fraction. Let's write 24.** (Write 24.)
Teacher	**Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by 3.** (Write × 3.) **What's the product of 3 times *x*?**
Students	3*x*.
Teacher	**We now have a numerator of 3*x* in the first fraction. Let's write 3*x*.** (Write 3*x*.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write × 3.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.**

	(Cross out 3 and 3.)
Teacher	**Using multiplication, we've changed our proportion to the equation 3*x* equals 24. What's the equation?**
Students	3*x* equals 24.
Teacher	**Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**
Students	It's the constant multiplied by a variable.
Teacher	**A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**
Students	Divide each side of the equation by the coefficient.
Teacher	**What's the coefficient?**
Students	3.
Teacher	**Let's divide each side of the equation by 3. Whatever we do to one side of the equal sign we also have to do to the other. What's 24 divided by 3?**
Students	8.
	(Write 8.)
Teacher	**So, the variable equals 8. What's the value of the unknown?**
Students	8.
Teacher	**That's right. $\frac{8}{12}$ is equal to $\frac{2}{3}$. Say that with me.**
Students	$\frac{8}{12}$ is equal to $\frac{2}{3}$.
Teacher	**Now, there is another way to solve for an unknown. Where is the unknown in this problem?**
Students	Numerator.
Teacher	**The unknown is in the numerator, so look at the denominators. Look at 12 and 3. What do you notice about the relationship between 12 and 3?**
Students	If you divide 12 by 4, you get 3.
Teacher	**Yes! I see that if you divide by 4, 12 divided by 4 equals the second denominator. It's like a rule in a function! Let's apply that rule to the numerator. What are the numerators in each fraction?**
Students	*x* and 2.
Teacher	**Let's solve for *x* using the same rule. How could we solve for *x*?**
Students	Figure out what you can divide by 4 to get 2.
Teacher	**Using the same rule as the denominator, *x* would be 8. 8 divided by 4 equals 2. What's *x*?**
Students	8.
Teacher	**Let's check. Does the rule work with the relationship between the numerators?**
Students	Yes.
Teacher	**Does the rule work with the relationship between the denominators?**
Students	Yes.

Teacher	**So, another way to solve for an unknown is to determine the rule between the numerators or denominators and use that to solve for *x*. Which method do you prefer?**
Students	(Explains preferred method.)
Teacher	**Let's review. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**What's one way we solved for an unknown in a proportion?**
Students	We multiplied the first denominator by the first numerator and the second numerator. Then, we multiplied the second denominator by the first numerator and the second numerator. Then, we divided by a coefficient to solve for the unknown.
Teacher	**What's another way we solved for an unknown in a proportion?**
Students	We determined the rule of the relationship between the denominators and applied that rule to determine the numerators.

(3) Representing Rates

Routine

Materials:

- [Module 21 Problem Sets]
- [Module 21 Vocabulary Cards]
    - If necessary, review Vocabulary Cards before teaching

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Today, let's work on rates. A rate is a ratio that compares two different units. What's a rate?**
Students	A ratio that compares two different units.
Teacher	**Units might be *miles* a car can drive per *gallon* of gas. *Miles* and *gallons* are the two different units. Can you share two other units that might be used to show a rate?**
Students	(Shares example.)
Teacher	**Another example might be *dollars* per *package* of strawberries. *Dollars* and *packages* are the two different units. Look at this problem.** (Show problem.)
Teacher	**Often, when solving problems about rate, we use a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**When determining the rate, we'll interpret each fraction in a proportion in the same way. We'll use the same unit for the numerator. We'll then use the other unit for the denominator. How will we think of the two different units with the numerator and denominator?**
Students	The numerator will represent one unit. The denominator will represent the other unit.
Teacher	**In this problem, we have to figure out the unit rate. That is, what is the value for 1 of __** (unit). **What is the unit rate?**
Students	The value for 1 of something.
Teacher	**The unit rate is the value for 1 of __** (unit)**. We can use a proportion to determine the unit rate. 1 divided by *x* can be used in the proportion to represent the unit rate. What can be used to represent the unit rate?**
Students	1 divided by *x*.
Teacher	**We have to determine 1 of *x* in this proportion. We can do this by isolating the unknown or *x*. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will need to multiply and divide. What will we do?**
Students	Multiply and divide.
Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	__.

Teacher	**The denominator of the first fraction is __. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by __** (first denominator) **and have a denominator of __** (first denominator)**, __ divided by __ equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.** (Cross out.)
Teacher	**Now, multiply the second numerator by __** (first denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.
Teacher	**We now have a numerator of __ in the second fraction. Let's write __.** (Write.)
Teacher	**Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by __** (second denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.
Teacher	**We now have a numerator of __ in the first fraction. Let's write __.** (Write.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.** (Cross out.)
Teacher	**Using multiplication, we've changed our proportion to the equation __ equals __. What's the equation?**
Students	__ equals __.
Teacher	**Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**
Students	It's the constant multiplied by a variable.

Teacher	**A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**
Students	Divide each side of the equation by the coefficient.
Teacher	**What's the coefficient?**
Students	__.
Teacher	**Let's divide each side of the equation by __** (coefficient)**. Whatever we do to one side of the equal sign we also have to do to the other. What's __ divided by __?**
Students	__. (Write.)
Teacher	**So, the variable equals __. That's the unit rate. One *x* equals __. What's the unit rate?**
Students	__.
Teacher	**Now, there is another way to solve for an unknown to determine the unit rate. Where is the unknown in this problem?**
Students	Numerator/denominator.
Teacher	**The unknown is in the numerator/denominator, so look at the denominators/numerators. Look at __** (first denominator/numerator) **and __** (second denominator/numerator)**. What do you notice about the relationship between __ and __?**
Students	(Describes relationship.)
Teacher	**Yes! I see that if you multiply/divide by __ with the first denominator/numerator, that equals the second denominator/numerator. It's like a rule in a function! Let's apply that rule to the numerator/denominator. What's the numerator/denominator in each fraction?**
Students	__/*x* and *x*/__.
Teacher	**Let's solve for *x* using the same rule. How could we solve for *x*?**
Students	Multiply/divide.
Teacher	**Using the same rule as the denominator/numerator, *x* would be __. What's *x*?**
Students	__.
Teacher	**Let's check. Does the rule work with the relationship between the numerators?**
Students	Yes.
Teacher	**Does the rule work with the relationship between the denominators?**
Students	Yes.
Teacher	**So, another way to solve for an unknown is to determine the rule between the numerators/denominators and use that to solve for *x*. Which method do you prefer?**
Students	(Explains preferred method.)
Teacher	**Let's review. What's the unit rate?**
Students	The value for 1 of something.

Teacher	**How did we determine the unit rate for an unknown in a proportion?**
Students	We first multiplied each denominator times each numerator. Then, we divided by the coefficient to solve for the unknown. Or, we determined the rule between numerators and applied that rule to the denominators.

Example

$$\frac{7}{301} = \frac{1}{x}$$

EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Today, let's work on rates. A rate is a ratio that compares two different units. What's a rate?**
Students	A ratio that compares two different units.
Teacher	**Units might be *miles* a car can drive per *gallon* of gas. *Miles* and *gallons* are the two different units. Can you share two other units that might be used to show a rate?**
Students	(Shares example.)
Teacher	**Another example might be *dollars* per *package* of strawberries. *Dollars* and *packages* are the two different units. Look at this problem.** (Show problem.)
Teacher	**Often, when solving problems about rate, we use a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**When determining the rate, we'll interpret each fraction in a proportion in the same way. We'll use the same unit for the numerator. We'll then use the other unit for the denominator. How will we think of the two different units with the numerator and denominator?**
Students	The numerator will represent one unit. The denominator will represent the other unit.
Teacher	**In this problem, we have to figure out the unit rate. That is, what is the value for 1 of *x*. What is the unit rate?**
Students	The value for 1 of something.
Teacher	**The unit rate is the value for 1 of *x*. We can use a proportion to determine the unit rate. 1 divided by *x* can be used in the proportion to represent the unit rate. What can be used to represent the unit rate?**
Students	1 divided by *x*.
Teacher	**We have to determine 1 of *x* in this proportion. We can do this by isolating the unknown or *x*. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will need to multiply and divide. What will we do?**
Students	Multiply and divide.

Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	301.
Teacher	**The denominator of the first fraction is 301. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write × 301.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by 301 and have a denominator of 301, 301 divided by 301 equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.** (Cross out 301.)
Teacher	**Now, multiply the second numerator by 301.** (Write.) **What's the product of 1 times 301?**
Students	301.
Teacher	**We now have a numerator of 301 in the second fraction. Let's write 301.** (Write 301.)
Teacher	**Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by *x*.** (Write × *x*.) **What's the product of 7 times *x*?**
Students	7*x*.
Teacher	**We now have a numerator of 7*x* in the first fraction. Let's write 7*x*.** (Write.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write × *x*.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.** (Cross out *x*.)
Teacher	**Using multiplication, we've changed our proportion to the equation 7*x* equals 301. What's the equation?**
Students	7*x* equals 301.
Teacher	**Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**
Students	It's the constant multiplied by a variable.

Teacher	**A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**
Students	Divide each side of the equation by the coefficient.
Teacher	**What's the coefficient?**
Students	7.
Teacher	**Let's divide each side of the equation by 7. Whatever we do to one side of the equal sign we also have to do to the other. What's 301 divided by 7?**
Students	43. (Write.)
Teacher	**So, the variable equals 43. That's the unit rate. One *x* equals 43. What's the unit rate?**
Students	43.
Teacher	**Now, there is another way to solve for an unknown. Where is the unknown in this problem?**
Students	Denominator.
Teacher	**The unknown is in the denominator, so look at the numerators. What do you notice about the relationship between 7 and 1?**
Students	If you divide 7 by 7, that equals 1.
Teacher	**Yes! I see that if you divide 7 by 7, that equals 1. It's like a rule in a function! Let's apply that rule to the denominator. What's the denominator in the first fraction?**
Students	301.
Teacher	**Let's solve for *x* using the same rule. How could we solve for *x*?**
Students	Divide by 7.
Teacher	**Using the same rule as the numerator, divide 301 by 7. What's *x*?**
Students	43.
Teacher	**Let's check. Does the rule work with the relationship between the numerators?**
Students	Yes.
Teacher	**Does the rule work with the relationship between the denominators?**
Students	Yes.
Teacher	**So, another way to solve for an unknown is to determine the rule between the numerators and use that to solve for *x*. Which method do you prefer?**
Students	(Explains preferred method.)
Teacher	**Let's review. What's the unit rate?**
Students	The value for 1 of something.
Teacher	**How did we determine the unit rate for an unknown in a proportion?**
Students	We first multiplied each denominator times each numerator. Then, we divided by the coefficient to solve for the unknown. Or, we determined the rule between numerators and applied that rule to the denominators.

(4) Representing Percentages

Routine

Materials:

- [Module 21 Problem Sets]
- [Module 21 Vocabulary Cards]
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 Blocks

ROUTINE WITH BASE-10 BLOCKS

Teacher	**Today, let's work on percentages. A percentage is just a rate that tells how many of something per hundred. What's a percentage?**
Students	A rate of an amount per hundred.
Teacher	**We can show percentages in different ways. Today, let's use these Base-10 blocks.** (Show manipulatives.)
Teacher	**Look at this flat. How many units are in this flat?**
Students	100.
Teacher	**A percentage is how many per hundred. So, if we have 100 cubes in the flat, the flat can represent the hundred. Let's leave the flat on the table. Now, let's focus on the percentage. Look at this problem.** (Show problem.)
Teacher	**What's the percentage?**
Students	__%.
Teacher	**In this problem, the percentage is __. So, we can show this percentage by showing __ Base-10 blocks on top of the flat.** (Show percentage.)
Teacher	**So, what percentage did we show?**
Students	__%.
Teacher	**Is __ less or greater than 50%?**
Students	Less/greater.
Teacher	**Is __ less or greater than 100?**
Students	Less/greater.
Teacher	**You can use these blocks to help you understand the value of the percentage. Let's review.**
Teacher	**What's a percentage?**
Students	A rate of an amount per hundred.
Teacher	**How can you use Base-10 blocks to show a percentage?**
Students	Show the hundred flat. Then place the percentage, using Base-10 blocks, on top of the flat.

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Today, let's work on percentages. A percentage is just a rate that tells how many of something per hundred. What's a percentage?**
Students	A rate of an amount per hundred.
	(Show problem.)
Teacher	**When determining the percentage of something, we use a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**So, in our proportion, we want to determine the percentage of a fraction or ratio. We can show this as __** (fraction) **is equal to *x* divided by 100. How can we represent the percentage?**
Students	*x* divided by 100.
Teacher	**In this problem, we have to figure out the percentage. That is, what is the value for *x* per 100. We can do this by isolating the unknown. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will multiply and divide. What will we do?**
Students	Multiply and divide.
Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	__.
Teacher	**The denominator of the first fraction is __. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by __** (first denominator) **and have a denominator of __** (first denominator)**, __ divided by __ equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.**
	(Cross out.)
Teacher	**Now, multiply the second numerator by __** (first denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.
Teacher	**We now have a numerator of __ in the second fraction. Let's write __.**
	(Write.)
Teacher	**Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**

Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by __** (second denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.
Teacher	**We now have a numerator of __ in the first fraction. Let's write __.** (Write.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.** (Cross out.)
Teacher	**Using multiplication, we've changed our proportion to the equation __ equals __. What's the equation?**
Students	__ equals __.
Teacher	**Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**
Students	It's the constant multiplied by a variable.
Teacher	**A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**
Students	Divide each side of the equation by the coefficient.
Teacher	**What's the coefficient?**
Students	__.
Teacher	**Let's divide each side of the equation by __** (coefficient)**. Whatever we do to one side of the equal sign we also have to do to the other. What's __ divided by __?**
Students	__. (Write.)
Teacher	**So, the variable equals __. That's the percentage. What's the percentage?**
Students	__.
Teacher	**Let's review. What's a percentage?**
Students	A rate of an amount per hundred.
Teacher	**How did we determine the percentage in a proportion?**
Students	We first multiplied each denominator times each numerator. Then, we divided by the coefficient to solve for the unknown.

Example

$$\frac{32}{40} = \frac{x}{100}$$

EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Today, let's work on percentages. A percentage is just a rate that tells how many of something per hundred. What's a percentage?**
Students	A rate of an amount per hundred. (Show problem.)
Teacher	**When determining the percentage of something, we use a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**So, in our proportion, we want to determine the percentage of a fraction or ratio. We can show this as $\frac{32}{40}$ is equal to *x* divided by 100. How can we represent the percentage?**
Students	*x* divided by 100.
Teacher	**In this problem, we have to figure out the percentage. That is, what is the value for *x* per 100. We can do this by isolating the unknown. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will multiply and divide. What will we do?**
Students	Multiply and divide.
Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	40.
Teacher	**The denominator of the first fraction is 40. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write × 40.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by 40 and have a denominator of 40, 40 divided by 40 equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.** (Cross out 40.)
Teacher	**Now, multiply the second numerator by 40.** (Write × 40.) **What's the product of *x* times 40?**
Students	40*x*.
Teacher	**We now have a numerator of 40*x* in the second fraction. Let's write 40*x*.**

	(Write 40*x*.)
Teacher	**Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by 100.** (Write × 100.) **What's the product of 32 times 100?**
Students	3,200.
Teacher	**We now have a numerator of 3,200 in the first fraction. Let's write 3,200.** (Write 3,200.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write × 100.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.** (Cross out 100.)
Teacher	**Using multiplication, we've changed our proportion to the equation 3,200 equals 40*x*. What's the equation?**
Students	3,200 equals 40*x*.
Teacher	**Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**
Students	It's the constant multiplied by a variable.
Teacher	**A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**
Students	Divide each side of the equation by the coefficient.
Teacher	**What's the coefficient?**
Students	40.
Teacher	**Let's divide each side of the equation by 40. Whatever we do to one side of the equal sign we also have to do to the other. What's 3,200 divided by 40?**
Students	80. (Write.)
Teacher	**So, the variable equals 80. That's the percentage. What's the percentage?**
Students	80%.
Teacher	**Let's review. What's a percentage?**
Students	A rate of an amount per hundred.
Teacher	**How did we determine the percentage in a proportion?**
Students	We first multiplied each denominator times each numerator. Then, we divided by the coefficient to solve for the unknown.

D. Problems for Use During Instruction

See Module 21 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 21 Vocabulary Cards.

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Module 21:

Ratios, Proportions, Rates, and Percentages

Problem Sets

A.

2 : 3

A.

1 : 4

A.

1 : 7

A.

1 : 2

A.

6 : 6

A.

2 : 1

A.

7 : 3

A.

2 : 9

A.

3 : 4

A.

5 : 8

A.

1 : 5

A.

9 : 8

A.

4 : 6

A.

7 : 8

A.

1 : 2

A.

15 : 6

A.

10 : 1

A.

11 : 3

A.

2 : 30

A.

3 : 20

A.

2 : 15

A.

6 : 32

A.

5 : 11

A.

4 : 24

A.

1 : 20

A.

3 : 15

A.

9 : 90

A.

50 : 150

A.

10 : 300

A.

20 : 600

B.

$$\frac{?}{4} = \frac{3}{12}$$

B.

$$\frac{x}{6} = \frac{15}{18}$$

B.

$$\frac{?}{3} = \frac{16}{24}$$

B.

$$\frac{x}{6} = \frac{2}{12}$$

B.

$$\frac{?}{20} = \frac{4}{5}$$

B.

$$\frac{x}{3} = \frac{9}{12}$$

B.

$$\frac{?}{8} = \frac{1}{2}$$

B.

$$\frac{x}{16} = \frac{5}{8}$$

B.

$$\frac{?}{100} = \frac{1}{4}$$

B.

$$\frac{x}{100} = \frac{1}{10}$$

B.

$$\frac{1}{?} = \frac{3}{12}$$

B.

$$\frac{2}{x} = \frac{12}{24}$$

B.

$$\frac{1}{?} = \frac{10}{30}$$

B.

$$\frac{3}{x} = \frac{6}{20}$$

B.

$$\frac{1}{?} = \frac{2}{6}$$

B.

$$\frac{28}{x} = \frac{4}{6}$$

B.

$$\frac{36}{?} = \frac{4}{5}$$

B.

$$\frac{9}{x} = \frac{1}{6}$$

B.

$$\frac{25}{?} = \frac{1}{4}$$

B.

$$\frac{20}{x} = \frac{2}{10}$$

B.

$$\frac{1}{4} = \frac{?}{12}$$

B.

$$\frac{3}{5} = \frac{x}{15}$$

B.

$$\frac{4}{5} = \frac{?}{35}$$

B.

$$\frac{2}{6} = \frac{x}{48}$$

B.

$$\frac{1}{4} = \frac{?}{8}$$

B.

$$\frac{16}{20} = \frac{x}{15}$$

B.

$$\frac{21}{35} = \frac{?}{5}$$

B.

$$\frac{4}{40} = \frac{x}{10}$$

B.

$$\frac{40}{100} = \frac{?}{25}$$

B.

$$\frac{100}{125} = \frac{x}{25}$$

B.

$$\frac{4}{8} = \frac{28}{?}$$

B.

$$\frac{1}{4} = \frac{3}{x}$$

B.

$$\frac{1}{4} = \frac{5}{?}$$

B.

$$\frac{2}{3} = \frac{12}{x}$$

B.

$$\frac{6}{12} = \frac{18}{?}$$

B.

$$\frac{16}{20} = \frac{4}{x}$$

B.

$$\frac{8}{80} = \frac{1}{?}$$

B.

$$\frac{8}{16} = \frac{3}{x}$$

B.

$$\frac{200}{500} = \frac{100}{?}$$

B.

$$\frac{50}{100} = \frac{5}{x}$$

c.

$$\frac{5}{45} = \frac{1}{x}$$

c.

$$\frac{8}{80} = \frac{1}{x}$$

c.

$$\frac{3}{27} = \frac{1}{x}$$

c.

$$\frac{8}{56} = \frac{1}{x}$$

c.

$$\frac{15}{45} = \frac{1}{x}$$

c.

$$\frac{12}{144} = \frac{1}{x}$$

c.

$$\frac{7}{182} = \frac{1}{x}$$

c.

$$\frac{6}{210} = \frac{1}{x}$$

c.

$$\frac{13}{195} = \frac{1}{x}$$

c.

$$\frac{8}{400} = \frac{1}{x}$$

c.

$$\frac{16}{64} = \frac{1}{x}$$

c.

$$\frac{3}{165} = \frac{1}{x}$$

c.

$$\frac{7}{539} = \frac{1}{x}$$

c.

$$\frac{5}{60} = \frac{1}{x}$$

c.

$$\frac{2}{166} = \frac{1}{x}$$

c.

$$\frac{4}{76} = \frac{1}{x}$$

c.

$$\frac{25}{300} = \frac{1}{x}$$

c.

$$\frac{12}{150} = \frac{1}{x}$$

c.

$$\frac{7}{427} = \frac{1}{x}$$

c.

$$\frac{4}{932} = \frac{1}{x}$$

D.

50%

D.

12%

D.

75%

D.

24%

D.

96%

D.

37%

D.

8%

D.

42%

D.

62%

D.

79%

E.

$$\frac{32}{40} = \frac{x}{100}$$

E.

$$\frac{4}{5} = \frac{x}{100}$$

E.

$$\frac{18}{20} = \frac{x}{100}$$

E.

$$\frac{4}{50} = \frac{x}{100}$$

E.

$$\frac{6}{25} = \frac{x}{100}$$

E.

$$\frac{8}{16} = \frac{x}{100}$$

E.

$$\frac{28}{70} = \frac{x}{100}$$

E.

$$\frac{1}{5} = \frac{x}{100}$$

E.

$$\frac{14}{20} = \frac{x}{100}$$

E.

$$\frac{18}{30} = \frac{x}{100}$$

E.

$$\frac{7}{35} = \frac{x}{100}$$

E.

$$\frac{22}{44} = \frac{x}{100}$$

E.

$$\frac{9}{12} = \frac{x}{100}$$

E.

$$\frac{12}{80} = \frac{x}{100}$$

E.

$$\frac{24}{40} = \frac{x}{100}$$

E.

$$\frac{3}{4} = \frac{x}{100}$$

E.

$$\frac{1}{2} = \frac{x}{100}$$

E.

$$\frac{11}{25} = \frac{x}{100}$$

E.

$$\frac{17}{25} = \frac{x}{100}$$

E.

$$\frac{3}{5} = \frac{x}{100}$$

Module 21:
Ratios, Proportions, Rates, and Percentages

Vocabulary Cards

- coefficient
- constant
- denominator
- equal sign
- equivalent fractions
- equivalent ratios
- fraction
- improper fraction
- least common multiple
- like fractions
- lowest terms
- mixed number
- multiple
- numerator
- percentage
- proper fraction
- proportion
- rate
- ratio
- unit rate
- unlike fractions
- variable

coefficient

A number that is multiplied by a variable.

$$5x + 9 = 24$$

5 is a coefficient

---

constant

A term that does not change; a number on its own.

$$5x + 9 = 24$$

9 and 24 are constants

denominator

The term in a fraction that tells the number of equal parts in a whole.

2 / 3 $\quad \frac{2}{3}$ **In these fractions, 3 is the denominator.**

---

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

12 + 8 = 20

= is the equal sign

equivalent fractions

Fractions that have different numerators and denominators that represent the same value or proportion of the whole.

$$\frac{1}{4} = \frac{2}{8} \qquad \frac{2}{3} = \frac{8}{12}$$

---

equivalent ratios

Ratios that have the same fractional number, value, or measure.

$$1 : 7 = 2 : 14 \qquad 2 : 5 = 4 : 10$$

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \qquad \frac{10}{12} \qquad \frac{8}{3}$$

---

improper fraction

Any fraction in which the numerator is greater than the denominator.

$$\frac{9}{4} \qquad \frac{17}{12} \qquad \frac{10}{3}$$

least common multiple

The common multiple with the least value.

6: 6, 12, 18, 24, 30

8: 8, 16, 24, 32, 40

With multiples of 6 and 8, the least common multiple is 24.

---

like fractions

Fractions that have the same denominator.

$$\frac{1}{4} \qquad \frac{2}{4} \qquad \frac{3}{4}$$

lowest terms

A fraction is reduced to lowest terms when there is no number other than 1 that will evenly divide into both the numerator and denominator.

$$\frac{2}{8} = \frac{1}{4}$$

lowest terms

$$\frac{3}{9} = \frac{1}{3}$$

lowest terms

---

mixed number

A whole number and a fraction combined.

$$1\frac{1}{6} \qquad 4\frac{5}{12} \qquad 12\frac{4}{3}$$

multiple

The product of a number and any integer.

4: 4, 8, 12, 16, 20

---

numerator

The term in a fraction that tells how many parts of a fraction.

2 / 3 $\frac{2}{3}$ **In these fractions, 2 is the numerator.**

percentage

A rate of an amount per hundred.

$$\frac{3}{4} = \frac{x}{100} = 75\%$$

---

proper fraction

A fraction where the numerator is less than the denominator.

$$\frac{3}{4} \qquad \frac{5}{6} \qquad \frac{8}{21}$$

proportion

An equation that states that two ratios are equal.

$$\frac{2}{3} = \frac{4}{6} \qquad \frac{5}{15} = \frac{1}{3}$$

---

rate

A comparison of two quantities that have different units of measure.

$$\frac{\text{25 feet}}{\text{3 hours}} \qquad \frac{\text{4 words}}{\text{2 minutes}}$$

↓ rate ↓ rate

ratio

A comparison of two quantities that have the same unit of measure.

♀♀♀♀♂♂♂

The ratio of ♀ to ♂ is 4 : 3 or 4/3.

---

unit rate

A ratio that is written as a number to one.

$$\frac{60 \text{ miles}}{3 \text{ hours}} = \frac{20 \text{ miles}}{1 \text{ hour}}$$

↓ rate

↓ unit rate

unlike fractions

Fractions that have different denominators.

$$\frac{1}{2} \qquad \frac{1}{3} \qquad \frac{1}{7}$$

---

variable

A symbol for an unknown value, which is usually represented by a letter.

$$5x + 9 = 24$$

x is a variable

Instructional Routines for Mathematics Intervention

MODULE 22

Representing Expressions and Equations

Module 22: Representing Expressions and Equations
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
base	A number that is multiplied by an exponent.
coefficient	A number that is multiplied by a variable.
constant	A term that does not change; a number on its own.
equation	A mathematical statement that two expressions are the same or equal; must have an equal sign.
exponent	The power to which a number is raised.
expression	A combination of variables, numbers, and/or operations that represents a mathematical relationship; does not have an equal sign.
grouping	A combination of variables, numbers, and/or operations grouped together in parentheses or brackets.
inequality	An algebraic relation showing that a quantity is greater or less than another quantity.
like terms	Terms that have the same variable or constant and can be combined.
operator	A symbol $(+, -, \times, \div)$ that represents a mathematical operation.
term	A single number or a variable, or numbers or variables multiplied together.
variable	A symbol for an unknown value, which is usually represented by a letter.

B. Background Information

In this module, we focus on early algebraic concepts:

(1) Order of Operations

(2) Representing Expressions

(3) Representing Equations

C. Routines and Examples

(1) Order of Operations

Routine

Materials:

- Module 22 Problem Sets
- Module 22 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching

ROUTINE

Teacher	**Let's learn about the order of operations. What's an operation?**
Students	Add, subtract, multiply, or divide.
Teacher	**The operations we'll focus on today are adding, subtracting, multiplying, and dividing. When you see an expression like 2 + 3, you see a plus sign and add. You don't have to think about the order of operations. But if you see an expression like 2[(8 × 5) ÷ 4] – (3 + 5), we have to think about the order in which we'll do the operations. We don't always work left to right. Look at this problem.** (Show problem.)
Teacher	**Let's read this problem together.**
Students	__.
Teacher	**We'll simplify expressions and solve equations by applying the order of operations. Our order of operations will be Grouping, Exponents, Multiplication and Division, then Addition and Subtraction. Let's start with Grouping. What will we do first with the order of the operations?**
Students	Grouping.
Teacher	**Grouping means we will do all the math within groups. A group might be presented within parentheses or brackets. How could a group be presented?**
Students	In parentheses or brackets.
Teacher	**When we simplify an expression and solve an equation, we'll first do the math within groups presented with parentheses or brackets. The second step for applying the order of the operations is to do the math for any exponents. What will we do next for the order of the operations?**
Students	Exponents.
Teacher	**An exponent is attached to a base and describes the power to which a base should be raised. What's an example of an exponent?**
Students	$\_\_^2$.
Teacher	**Great. $\_\_^2$ is an example of an exponent. So is $\_\_^5$. The third step for applying the order of the operations is to do any multiplication and division. What's the third step?**

Students	Do multiplication and division.
Teacher	**We'll multiply or divide any parts of the expression or equation. The fourth step for applying the order of the operations is to do any addition or subtraction. What's the fourth step?**
Students	Do addition and subtraction.
Teacher	**Yes. We'll add or subtract any parts of the expression or equation. So, let's review. To simplify expressions or solve equations you apply the order of the operations. We do the Grouping, then Exponents, then Multiplication and Division, then Addition and Subtraction. What's the order of the operations?**
Students	Grouping, Exponents, Multiplication and Division, Addition and Subtraction.
Teacher	**Now, let's practice. Let's simplify this expression. What should we think about first?**
Students	Grouping.
Teacher	**Are there any groupings with brackets or parentheses?**
Students	Yes/no.
Teacher	*IF YES:* **There is a grouping. Let's do the math within each of the groups.** (Write.)
Teacher	**What's the second step for applying the order of the operations?**
Students	Exponents.
Teacher	**Are there any exponents?**
Students	Yes/no.
Teacher	*IF YES:* **There is an exponent. Let's do the math for each of the bases and exponents.** (Write.)
Teacher	**What's the third step for applying the order of the operations?**
Students	Multiplication and Division.
Teacher	**Is there any multiplication or division for us to do?**
Students	Yes/no.
Teacher	*IF YES:* **There is multiplication or division. Let's do the math for the multiplication and division. Let's work the problem left to right doing all the multiplication and division.** (Write.)
Teacher	**What's the fourth step for applying the order of the operations?**
Students	Addition and Subtraction.
Teacher	**Is there any addition or subtraction for us to do?**
Students	Yes/no.
Teacher	*IF YES:* **There is addition or subtraction. Let's do the math for the addition or subtraction. Let's work the problem left to right doing all the addition and subtraction.** (Write.)
Teacher	**Look at the problem. Did we simplify the expression or solve the equation?**
Students	Yes!
Teacher	**We followed the order of the operations to simplify or solve. Let's review. What's the order of the operations?**

Students	Grouping, Exponents, Multiplication and Division, Addition and Subtraction.
Teacher	**When do you use the order of the operations?**
Students	Whenever you have an expression or equation with more than one operator symbol.
Teacher	**How could you explain the order of operations to a friend?**
Students	First, you do the math for any groupings with brackets and parentheses. Then, you do the math for any exponents. Then, you do any of the multiplication and division. Finally, you do any of the addition and subtraction.

Example

18 ÷ 6 × (4 + 3) – 6

EXAMPLE

Teacher	**Let's learn about the order of operations. What's an operation?**
Students	Add, subtract, multiply, or divide.
Teacher	**The operations we'll focus on today are adding, subtracting, multiplying, and dividing. When you see an expression with multiple operations, we have to think about the order in which we'll do the operations. We don't always work left to right. Look at this problem.** (Show problem.)
Teacher	**Let's read this problem together.**
Students	18 ÷ 6 × (4 + 3) – 6.
Teacher	**We'll simplify expressions and solve equations by applying the order of operations. Our order of operations will be Grouping, Exponents, Multiplication and Division, then Addition and Subtraction. Let's start with Grouping. What will we do first with the order of the operations?**
Students	Grouping.
Teacher	**Grouping means we will do all the math within groups. A group might be presented within parentheses or brackets. How could a group be presented?**
Students	In parentheses or brackets.
Teacher	**When we simplify an expression and solve an equation, we'll first do the math within groups presented with parentheses or brackets. The second step for applying the order of the operations is to do the math for any exponents. What will we do next for the order of the operations?**
Students	Exponents.
Teacher	**An exponent is attached to a base and describes the power to which a base should be raised. What's an example of an exponent?**
Students	3^2.
Teacher	**Great. 3^2 is an example of an exponent. So is 2^5. The third step for applying the order of the operations is to do any multiplication and division. What's the third step?**
Students	Do multiplication and division.

Teacher	**We'll multiply or divide any parts of the expression or equation. The fourth step for applying the order of the operations is to do any addition or subtraction. What's the fourth step?**
Students	Do addition and subtraction.
Teacher	**Yes. We'll add or subtract any parts of the expression or equation. So, let's review. To simplify expressions or solve equations you apply the order of the operations. We do the Grouping, then Exponents, then Multiplication and Division, then Addition and Subtraction. What's the order of the operations?**
Students	Grouping, Exponents, Multiplication and Division, Addition and Subtraction.
Teacher	**Now, let's practice. Let's simplify this expression. What should we think about first?**
Students	Grouping.
Teacher	**Are there any groupings with brackets or parentheses?**
Students	Yes.
Teacher	**There is a grouping. Let's do the math within the parentheses. What's 4 + 3?**
Students	7.
Teacher	**Let's write 7 below the parentheses.** (Write 7.)
Teacher	**What's the second step for applying the order of the operations?**
Students	Exponents.
Teacher	**Are there any exponents?**
Students	No.
Teacher	**There are no exponents. What's the third step for applying the order of the operations?**
Students	Multiplication and Division.
Teacher	**Is there any multiplication or division for us to do?**
Students	Yes.
Teacher	**There is multiplication or division. Let's work the problem left to right doing all the multiplication and division. What's the first multiplication or division we need to do?**
Students	18 ÷ 6.
Teacher	**What's 18 divided by 6?**
Students	3.
Teacher	**Let's write 3 below the division to keep track of the quotient.** (Write 3.)
Teacher	**Is there more multiplication or division?**
Students	Yes.
Teacher	**What do we need to do?**
Students	3 × 7.
Teacher	**What's 3 times 7?**
Students	21.
Teacher	**Let's write 21 to keep track of the product.** (Write 21.)
Teacher	**What's the fourth step for applying the order of the operations?**

Students	Addition and Subtraction.
Teacher	**Is there any addition or subtraction for us to do?**
Students	Yes.
Teacher	**There is addition or subtraction. Let's work the problem left to right doing all the addition and subtraction. What do we need to do?**
Students	21 – 6.
Teacher	**Yes. What's 21 minus 6?**
Students	15. (Write 15.)
Teacher	**Look at the problem. Did we simplify the expression or solve the equation?**
Students	Yes!
Teacher	**We followed the order of the operations to simplify or solve. Let's review. What's the order of the operations?**
Students	Grouping, Exponents, Multiplication and Division, Addition and Subtraction.
Teacher	**When do you use the order of the operations?**
Students	Whenever you have an expression or equation with more than one operator symbol.
Teacher	**How could you explain the order of operations to a friend?**
Students	First, you do the math for any groupings with brackets and parentheses. Then, you do the math for any exponents. Then, you do any of the multiplication and division. Finally, you do any of the addition and subtraction.

(2) Representing Expressions

Routine

Materials:

- Module 22 Problem Sets
- Module 22 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A manipulative like algebra tiles

ROUTINE WITH MANIPULATIVES

Teacher	**Let's show different expressions. What's an expression?**
Students	Numbers and operator symbols.
Teacher	**An expression has numbers and operator symbols. An expression does not have an equal sign or inequality symbol. What's not in an expression?**
Students	Equal sign or inequality symbol.
Teacher	**Let's represent different expressions with these algebra tiles.** (Show manipulatives.)
Teacher	**With the algebra tiles, we'll interpret this unit to represent a constant. What's a constant?**
Students	A number or value that does not change.

Teacher	**Yes. A constant is a number or value that does not change.**
Teacher	**We'll use this unit to show the constant. The unit has a positive side. That's brown. What color is the positive side?**
Students	Brown.
Teacher	**The unit also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**With the algebra tiles, we'll interpret this rod to represent our variable. What will the rod represent?**
Students	A variable.
Teacher	**And the rod has a positive side. That's green. What color is the positive side?**
Students	Green.
Teacher	**The rod also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**If this rod is our variable, then this flat represents the variable squared or x^2. What does the flat represent?**
Students	The variable squared.
Teacher	**This flat represents x^2 because we can multiply x times x** (show multiplication) **to create the area of x^2. Why does the flat represent x^2?**
Students	Because the area created by multiplying x times x equals the area of x^2.
Teacher	**The flat has a positive side. That's blue. What color is the positive side?**
Students	Blue.
Teacher	**The flat also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**Now, let's show an expression with the algebra tiles. Remember, we have pieces that represent the variable squared** (show)**, the variable** (show)**, and the constant** (show)**. Look at this expression.** (Show problem.)
Teacher	**Read the expression.**
Students	__.
Teacher	**How would we show the expression with the algebra tiles? First, do we have any squared variables we need to show?**
Students	Yes/no.
Teacher	*IF YES:* **We need to show a squared variable. Which of the algebra tiles will we use?**
Students	Flat.
Teacher	**Look to see if there's a coefficient with the squared variable. The coefficient tells us how many of the flats we will show. How many flats?**
Students	__.
Teacher	**And is the squared variable positive or negative?**
Students	__.
Teacher	**Let's show __ flats to show the squared variable.** (Show tiles.)
Teacher	**Now, do we have any variables we need to show?**

Students	Yes/no.	
Teacher	*IF YES:*	**We need to show a variable. Which of the algebra tiles will we use?**
Students		Rod.
Teacher		**Look to see if there's a coefficient with the variable. The coefficient tells us how many of the rods we will show. How many rods?**
Students		__.
Teacher		**And is the variable positive or negative?**
Students		__.
Teacher		**Let's show __ rods to show the variable.** (Show tiles.)
Teacher	**Now, do we have any constants we need to show?**	
Students	Yes/no.	
Teacher	*IF YES:*	**We need to show a constant. Which of the algebra tiles will we use?**
Students		Unit.
Teacher		**How many units should we use?**
Students		__.
Teacher		**And is the constant positive or negative?**
Students		__.
Teacher		**Let's show __ units to show the constant.** (Show tiles.)
Teacher	**We used the algebra tiles to show an expression. What expression did we show?**	
Students	__.	
Teacher	**How can you use the algebra tiles to show expressions?**	
Students	Use the flats to show squared variables. Use the rods to show variables. Use the units to show the constant.	

Example

$x^2 - 3x + 4$

EXAMPLE WITH MANIPULATIVES

Teacher	**Let's show different expressions. What's an expression?**
Students	Numbers and operator symbols.
Teacher	**An expression has numbers and operator symbols. An expression does not have an equal sign or inequality symbol. What's not in an expression?**
Students	Equal sign or inequality symbol.
Teacher	**Let's represent different expressions with these algebra tiles.** (Show manipulatives.)
Teacher	**With the algebra tiles, we'll interpret this unit to represent a constant. What's a constant?**
Students	A number or value that does not change.
Teacher	**Yes. A constant is a number or value that does not change.**

Teacher	**We'll use this unit to show the constant. The unit has a positive side. That's brown. What color is the positive side?**
Students	Brown.
Teacher	**The unit also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**With the algebra tiles, we'll interpret this rod to represent our variable. What will the rod represent?**
Students	A variable.
Teacher	**And the rod has a positive side. That's green. What color is the positive side?**
Students	Green.
Teacher	**The rod also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**If this rod is our variable, then this flat represents the variable squared or x^2. What does the flat represent?**
Students	The variable squared.
Teacher	**This flat represents x^2 because we can multiply x times x** (show multiplication) **to create the area of x^2. Why does the flat represent x^2?**
Students	Because the area created by multiplying x times x equals the area of x^2.
Teacher	**The flat has a positive side. That's blue. What color is the positive side?**
Students	Blue.
Teacher	**The flat also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**Now, let's show an expression with the algebra tiles. Remember, we have pieces that represent the variable squared** (show)**, the variable** (show)**, and the constant** (show)**. Look at this expression.** (Show problem.)
Teacher	**Read the expression.**
Students	$x^2 - 3x + 4$.
Teacher	**How would we show the expression with the algebra tiles? First, do we have any squared variables we need to show?**
Students	Yes.
Teacher	**We need to show a squared variable. Which of the algebra tiles will we use?**
Students	Flat.
Teacher	**Look to see if there's a coefficient with the squared variable. The coefficient tells us how many of the flats we will show. How many flats?**
Students	1.
Teacher	**Yes, there's no coefficient so we assume the coefficient is 1. And is the squared variable positive or negative?**
Students	Positive.
Teacher	**Let's show 1 blue flat to show the squared variable.** (Show tiles.)
Teacher	**Now, do we have any variables we need to show?**
Students	Yes.
Teacher	**We need to show a variable. Which of the algebra tiles will we use?**

Students	Rod.
Teacher	**Look to see if there's a coefficient with the variable. The coefficient tells us how many of the rods we will show. How many rods?**
Students	3.
Teacher	**And is the variable positive or negative?**
Students	Negative.
Teacher	**Let's show 3 red rods to show the variable.**
Students	(Show tiles.)
Teacher	**Now, do we have any constants we need to show?**
Students	Yes.
Teacher	**We need to show a constant. Which of the algebra tiles will we use?**
Students	Unit.
Teacher	**How many units should we use?**
Students	4.
Teacher	**And is the constant positive or negative?**
Students	Positive.
Teacher	**Let's show 4 brown units to show the constant.**
	(Show tiles.)
Teacher	**We used the algebra tiles to show an expression. What expression did we show?**
Students	$x^2 - 3x + 4$.
Teacher	**How can you use the algebra tiles to show expressions?**
Students	Use the flats to show squared variables. Use the rods to show variables. Use the units to show the constant.

(3) Representing Equations

Routine

Materials:

- Module 22 Problem Sets
- Module 22 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like two-color counters or multi-colored cubes

ROUTINE WITH MANIPULATIVES

Teacher	**Let's show different equations. What's an equation?**
Students	Two equal expressions with an equal sign.
Teacher	**An equation always has an equal sign. What's always in an equation?**
Students	An equal sign.
Teacher	**Let's represent different equations with these algebra tiles.** (Show manipulatives.)
Teacher	**With the algebra tiles, we'll interpret this unit to represent a constant. What's a constant?**
Students	A number or value that does not change.
Teacher	**Yes. A constant is a number or value that does not change.**
Teacher	**We'll use this unit to show the constant. The unit has a positive side. That's brown. What color is the positive side?**
Students	Brown.
Teacher	**The unit also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**With the algebra tiles, we'll interpret this rod to represent our variable. What will the rod represent?**
Students	A variable.
Teacher	**And the rod has a positive side. That's green. What color is the positive side?**
Students	Green.
Teacher	**The rod also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**If this rod is our variable, then this flat represents the variable squared or x^2. What does the flat represent?**
Students	The variable squared.
Teacher	**The flat has a positive side. That's blue. What color is the positive side?**
Students	Blue.
Teacher	**The flat also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**Now, let's show an equation with the algebra tiles. Remember, we have pieces that represent the variable squared** (show)**, the variable** (show)**, and the constant** (show)**. Look at this equation.** (Show problem.)

Teacher	**Read the equation.**	
Students	___.	
Teacher	**Because we're going to show an equation, let's write an equal sign in the middle of our manipulatives mat.** (Write equal sign.)	
Teacher	**We'll show the left side of the equation on left side of the mat. We'll show the right side of the equation on the right side of the mat. How do we use the mat?**	
Students	Show the left side of the equation on the left side. Show the right side of the equation on the right side.	
Teacher	**Let's show the left side of the equation first. Look at the left side. First, do we have any squared variables we need to show?**	
Students	Yes/no.	
Teacher	*IF YES:*	**We need to show a squared variable. Which of the algebra tiles will we use?**
Students		Flat.
Teacher		**Look to see if there's a coefficient with the squared variable. The coefficient tells us how many of the flats we will show. How many flats?**
Students		___.
Teacher		**And is the squared variable positive or negative?**
Students		___.
Teacher		**Let's show ___ flats to show the squared variable.** (Show tiles.)
Teacher	**Now, do we have any variables we need to show?**	
Students	Yes/no.	
Teacher	*IF YES:*	**We need to show a variable. Which of the algebra tiles will we use?**
Students		Rod.
Teacher		**Look to see if there's a coefficient with the variable. The coefficient tells us how many of the rods we will show. How many rods?**
Students		___.
Teacher		**And is the variable positive or negative?**
Students		___.
Teacher		**Let's show ___ rods to show the variable.** (Show tiles.)
Teacher	**Now, do we have any constants we need to show?**	
Students	Yes/no.	
Teacher	*IF YES:*	**We need to show a constant. Which of the algebra tiles will we use?**
Students		Unit.
Teacher		**How many units should we use?**
Students		___.
Teacher		**And is the constant positive or negative?**
Students		___.
Teacher		**Let's show ___ units to show the constant.**

(Show tiles.)

Teacher **Now, let's focus on the right side of the equation. First, do we have any squared variables we need to show?**

Students Yes/no.

Teacher *IF YES:* **We need to show a squared variable. Which of the algebra tiles will we use?**

Students Flat.

Teacher **Look to see if there's a coefficient with the squared variable. The coefficient tells us how many of the flats we will show. How many flats?**

Students __.

Teacher **And is the squared variable positive or negative?**

Students __.

Teacher **Let's show __ flats to show the squared variable.**

(Show tiles.)

Teacher **Now, do we have any variables we need to show?**

Students Yes/no.

Teacher *IF YES:* **We need to show a variable. Which of the algebra tiles will we use?**

Students Rod.

Teacher **Look to see if there's a coefficient with the variable. The coefficient tells us how many of the rods we will show. How many rods?**

Students __.

Teacher **And is the variable positive or negative?**

Students __.

Teacher **Let's show __ rods to show the variable.**

(Show tiles.)

Teacher **Now, do we have any constants we need to show?**

Students Yes/no.

Teacher *IF YES:* **We need to show a constant. Which of the algebra tiles will we use?**

Students Unit.

Teacher **How many units should we use?**

Students __.

Teacher **And is the constant positive or negative?**

Students __.

Teacher **Let's show __ units to show the constant.**

(Show tiles.)

Teacher **We used the algebra tiles to show this equation. What equation did we show?**

Students __.

Teacher **How can you use the algebra tiles to show equations?**

Students Use the flats to show squared variables. Use the rods to show variables. Use the units to show the constant. Place the algebra tiles for the left side of an equation on the left side of an equal sign. Place the algebra tiles for the right side of an equation on the right side of an equal sign.

Example

$2x^2 - 3x - 7 = x^2 - 3$

EXAMPLE WITH MANIPULATIVES

Teacher	**Let's show different equations. What's an equation?**
Students	Two equal expressions with an equal sign.
Teacher	**An equation always has an equal sign. What's always in an equation?**
Students	An equal sign.
Teacher	**Let's represent different equations with these algebra tiles.** (Show manipulatives.)
Teacher	**With the algebra tiles, we'll interpret this unit to represent a constant. What's a constant?**
Students	A number or value that does not change.
Teacher	**Yes. A constant is a number or value that does not change.**
Teacher	**We'll use this unit to show the constant. The unit has a positive side. That's brown. What color is the positive side?**
Students	Brown.
Teacher	**The unit also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**With the algebra tiles, we'll interpret this rod to represent our variable. What will the rod represent?**
Students	A variable.
Teacher	**And the rod has a positive side. That's green. What color is the positive side?**
Students	Green.
Teacher	**The rod also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**If this rod is our variable, then this flat represents the variable squared or x^2. What does the flat represent?**
Students	The variable squared.
Teacher	**The flat has a positive side. That's blue. What color is the positive side?**
Students	Blue.
Teacher	**The flat also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**Now, let's show an equation with the algebra tiles. Remember, we have pieces that represent the variable squared** (show)**, the variable** (show)**, and the constant** (show)**. Look at this equation.** (Show problem.)
Teacher	**Read the equation.**
Students	$2x^2 - 3x - 7 = x^2 - 3$.
Teacher	**Because we're going to show an equation, let's write an equal sign in the middle of our manipulatives mat.** (Write equal sign.)

Teacher	**We'll show the left side of the equation on left side of the mat. We'll show the right side of the equation on the right side of the mat. How do we use the mat?**
Students	Show the left side of the equation on the left side. Show the right side of the equation on the right side.
Teacher	**Let's show the left side of the equation first. Look at the left side. First, do we have any squared variables we need to show?**
Students	Yes.
Teacher	**We need to show a squared variable. Which of the algebra tiles will we use?**
Students	Flat.
Teacher	**Look to see if there's a coefficient with the squared variable. The coefficient tells us how many of the flats we will show. How many flats?**
Students	2.
Teacher	**And is the squared variable positive or negative?**
Students	Positive.
Teacher	**Let's show 2 blue flats to show the squared variable.** (Show tiles.)
Teacher	**Now, do we have any variables we need to show?**
Students	Yes.
Teacher	**We need to show a variable. Which of the algebra tiles will we use?**
Students	Rod.
Teacher	**Look to see if there's a coefficient with the variable. The coefficient tells us how many of the rods we will show. How many rods?**
Students	3.
Teacher	**And is the variable positive or negative?**
Students	Negative.
Teacher	**Let's show 3 red rods to show the variable.** (Show tiles.)
Teacher	**Now, do we have any constants we need to show?**
Students	Yes.
Teacher	**We need to show a constant. Which of the algebra tiles will we use?**
Students	Unit.
Teacher	**How many units should we use?**
Students	7.
Teacher	**And is the constant positive or negative?**
Students	Negative.
Teacher	**Let's show 7 red units to show the constant.** (Show tiles.)
Teacher	**Now, let's focus on the right side of the equation. First, do we have any squared variables we need to show?**
Students	Yes.
Teacher	**We need to show a squared variable. Which of the algebra tiles will we use?**
Students	Flat.

Teacher	**Look to see if there's a coefficient with the squared variable. The coefficient tells us how many of the flats we will show. How many flats?**
Students	1.
Teacher	**And is the squared variable positive or negative?**
Students	Positive.
Teacher	**Let's show 1 blue flat to show the squared variable.** (Show tiles.)
Teacher	**Now, do we have any variables we need to show?**
Students	No.
Teacher	**Now, do we have any constants we need to show?**
Students	Yes.
Teacher	**We need to show a constant. Which of the algebra tiles will we use?**
Students	Unit.
Teacher	**How many units should we use?**
Students	3.
Teacher	**And is the constant positive or negative?**
Students	Negative.
Teacher	**Let's show 3 red units to show the constant.** (Show tiles.)
Teacher	**We used the algebra tiles to show this equation. What equation did we show?**
Students	$2x^2 – 3x – 7 = x^2 – 3$.
Teacher	**How can you use the algebra tiles to show equations?**
Students	Use the flats to show squared variables. Use the rods to show variables. Use the units to show the constant. Place the algebra tiles for the left side of an equation on the left side of an equal sign. Place the algebra tiles for the right side of an equation on the right side of an equal sign.

D. Problems for Use During Instruction

See Module 22 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 22 Vocabulary Cards.

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Module 22:

Representing Expressions and Equations

Problem Sets

For equations, use Problem Sets from Module 23.

A. $15 - (2 \times 5)$

A. $(8 \times 8) \div 6$

A. $[5 + (9 \div 3)] + 6$

A. $7 \times (2 \div 1) \div 2$

A. $29 - (2 \times 4)$

A. $(6 + 8 - 2)$

A. $(3 + 1) \times 4 \times 5$

A. $(4 \times 6) \div 6$

A. $(6 - 1 + 7)$

A. $$8 + [(9 + 4) - 2]$$

B. $4x$

B. $3r$

B. **11*w***

B.

бу

B.

53*c*

B. **9*u***

B. **14x**

B.

6s

B.

10z

B. **15*t***

c. $4y + 5y$

c. $6r + 8r$

c. $2s \times 5s$

c. $11x - 5x$

c. $12d \div 3d$

c. $6k + 7k$

c. $2f \times 9f$

c. $15v - 6v$

c. $2m \times 8m$

c. $15x \div 5x$

D. $5x + 4x + 1$

D. $8z + 7z - 3$

D. $12 - 3c - 2c$

D. $9b + 6 + 8b$

D. $9w + 7 - 3w$

D. $12n - 2n + 6$

D. $5t + 4t - 10$

D. $2d + 17 - 2d$

D. $3m - 2 + 4m$

D. $10a - 8a + 2$

E. $5y^2 + 3y + 6$

E. $2s^2 + 3s - 1$

E. $x^2 + 2x + 9$

E. $3k^2 + 8k + 2$

E. $5w^2 - 4w - 2$

E. $8a^2 + 2a - 7$

E. $5x^2 + x + 10$

E. $2f^2 + 5f + 7$

E. $7b^2 + 4b + 2$

E. $4y^2 - 3y - 2$

Module 22:

Representing Expressions and Equations

Vocabulary Cards

base
coefficient
constant
equation
exponent
expression

grouping
inequality
like terms
operator
term
variable

base

A number that is multiplied by an exponent.

5^3

5 is the base

---

coefficient

A number that is multiplied by a variable.

$5x + 9 = 24$

5 is a coefficient

constant

A term that does not change; a number on its own.

$$5x + 9 = 24$$

9 and 24 are constants

---

equation

A mathematical statement that two expressions are the same or equal; must have an equal sign.

$$5x + 9 = 24$$

5x + 9 = 24 is an equation

(DOES have an = sign)

exponent

The power to which a number is raised.

$$5^{3}$$

3 is the exponent

---

expression

A combination of variables, numbers, and/or operations that represents a mathematical relationship; does not have an equal sign.

$5x + 9$ 24

5x + 9 and 24 are expressions

(DOES NOT have an = sign)

grouping

A combination of variables, numbers, and/or operations grouped together in parentheses or brackets.

$$(15 + 4) \qquad 2[(6 + 4) \div 2]$$

---

inequality

An algebraic relation showing that a quantity is greater or less than another quantity.

$$5x + 9 > 24$$

The > makes this equation an inequality

like terms

Terms that have the same variable or constant and can be combined.

$$2\underline{y} \qquad 4\underline{y} \qquad 8\underline{y}$$

---

operator

A symbol (+, –, × ÷) that represents a mathematical operation.

$$5x + 9 = 24$$

+ is an operator

term

A single number or a variable, or numbers and variables multiplied together.

$$5x + 9 = 24$$

5x, 9, and 24 are terms

---

variable

A symbol for an unknown value, which is usually represented by a letter.

$$5x + 9 = 24$$

x is a variable

Instructional Routines for Mathematics Intervention

MODULE 23

Solving Equations

Module 23: Solving Equations
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
base	A number that is multiplied by an exponent.
coefficient	A number that is multiplied by a variable.
constant	A term that does not change; a number on its own.
equation	A mathematical statement that two expressions are the same or equal; must have an equal sign.
exponent	The power to which a number is raised.
expression	A combination of variables, numbers, and/or operations that represents a mathematical relationship; does not have an equal sign.
grouping	A combination of variables, numbers, and/or operations grouped together in parentheses or brackets.
inequality	An algebraic relation showing that a quantity is greater or less than another quantity.
like terms	Terms that have the same variable or constant and can be combined.
operator	A symbol ($+, -, \times, \div$) that represents a mathematical operation.
term	A single number or a variable, or numbers or variables multiplied together .
variable	A symbol for an unknown value, which is usually represented by a letter.

B. Background Information

In this module, we focus on early algebraic concepts:

(1) Solving Single-Step Equations with One Variable
(2) Solving Multi-Step Equations with One Variable
(3) Solving Equations with Variables on Both Sides

C. Routines and Examples

(1) Solving Single-Step Equations with One Variable

Routine

Materials:

- Module 23 Problem Sets
- Module 23 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A manipulative like algebra tiles

ROUTINE WITH MANIPULATIVES

Teacher	**Let's solve an equation. What's an equation?**
Students	A mathematical statement with the equal sign.
Teacher	**An equation has numbers and operator symbols. An equation also has an equal sign. What's the symbol that's always in an equation?**
Students	The equal sign.
Teacher	**Let's show different equations and solve them. Let's use these algebra tiles.** (Show manipulatives.)
Teacher	**With the algebra tiles, we'll interpret this unit to represent a constant. What's a constant?**
Students	A number or value that does not change.
Teacher	**Yes. A constant is a number or value that does not change.**
Teacher	**We'll use this unit to show the constant. The unit has a positive side. That's brown. What color is the positive side?**
Students	Brown.
Teacher	**The unit also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**With the algebra tiles, we'll interpret this rod to represent our variable. What will the rod represent?**
Students	A variable.
Teacher	**And the rod has a positive side. That's green. What color is the positive side?**
Students	Green.
Teacher	**The rod also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**If this rod is our variable, then this flat represents the variable squared or x^2. What does the flat represent?**
Students	The variable squared.
Teacher	**This flat represents x^2 because we can multiply x times x** (show multiplication) **to create the area of x^2. Why does the flat represent x^2?**
Students	Because the area created by multiplying x times x equals the area of x^2.

Teacher	**The flat has a positive side. That's blue. What color is the positive side?**
Students	Blue.
Teacher	**The rod also has a negative side. That's red. What color is the negative side?**
Students	Red.
Teacher	**Now, let's solve an equation with the algebra tiles. Remember, we have pieces that represent the variable squared** (show)**, the variable** (show)**, and the constant** (show)**. Look at this equation.** (Show problem.)
Teacher	**Read the equation.**
Students	__.
Teacher	**Because we're going to show an equation, let's write an equal sign in the middle of our manipulatives mat.** (Write equal sign.)
Teacher	**We'll show the left side of the equation on left side of the mat. We'll show the right side of the equation on the right side of the mat. How do we use the mat?**
Students	Show the left side of the equation on the left side. Show the right side of the equation on the right side.
Teacher	**Let's show the left side of the equation first. Look at the left side. How would we show the left side of the equation with algebra tiles?**
Students	(Describe manipulatives.)
Teacher	**Yes, on the left side we show __ flats, __ rods, and __ units.** (Show with manipulatives.)
Teacher	**Let's show the right side of the equation. Look at the right side. How would we show the right side of the equation with algebra tiles?**
Students	(Describe manipulatives.)
Teacher	**Yes, on the right side we show __ flats, __ rods, and __ units.** (Show with manipulatives.)
Teacher	**Now it's time to solve this equation. We'll solve this equation by isolating the variable. What is the variable in this equation?**
Students	*x*.
Teacher	***x* is the variable. We'll isolate the variable by removing the constant from the side of the equal sign with the variable. Where is the variable?**
Students	Left side/right side.
Teacher	**So, we'll remove the constant from the left side/right side of the equation. What's the constant that we should remove?**
Students	__.
Teacher	**We will use the inverse operation and add/subtract __ from the left/right side of the equation.** (Add or subtract with manipulatives.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**
Students	Do the same thing to both sides.

Teacher	**Let's also add/subtract __ from the left/right side of the equation.** (Add or subtract with manipulatives.)
Teacher	**So, did we isolate the variable?**
Students	Yes.
Teacher	**What is equal to *x*?**
Students	__.
Teacher	**Great! *x* equals __. Let's say that together.**
Students	*x* equals __.
Teacher	**We used the algebra tiles to solve an equation. What equation did we solve?**
Students	__.
Teacher	**How can you use the algebra tiles to solve equations?**
Students	Use the algebra tiles to set up the problem. Then, isolate the variable by removing the constant from the variable side. When removing the constant, whatever we do to one side of the equation we also have to do to the other side of the equation.

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's solve an equation. What's an equation?**
Students	A mathematical statement with the equal sign.
Teacher	**An equation has numbers and operator symbols. An equation also has an equal sign. What's the symbol that's always in an equation?**
Students	The equal sign.
Teacher	**Let's show different equations and solve them. Let's use our paper and pencil.** (Show pencil.)
Teacher	**Look at this equation.** (Show problem.)
Teacher	**Read the equation.**
Students	__.
Teacher	**Let's solve this equation. We'll need to focus on the equal sign in this problem. So, let's draw a vertical line down from the equal sign to help us remember to balance both sides of the equation.** (Draw vertical line.)
Teacher	**We'll solve this equation by isolating the variable. What is the variable in this equation?**
Students	*x*.
Teacher	***x* is the variable. We'll isolate the variable by removing the constant from the side of the equal sign with the variable. What's a constant?**
Students	A number that is on its own.
Teacher	**And in this problem, where is the variable?**
Students	Left side/right side.
Teacher	**I like to circle the variable to remember that I'm isolating the variable. Let's circle *x*.**
Students	(Circle *x*.)

Teacher	**So, we'll remove the constant from the left side/right side of the equation. What's the constant that we should remove?**
Students	__.
Teacher	**We will use the inverse operation and add/subtract __ from the left/right side of the equation. What's the inverse operation of the constant?**
Students	Add/subtract.
Teacher	**Let's write plus/minus __ under the constant.** (Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**
Students	Do the same thing to both sides.
Teacher	**Let's also add/subtract __ from the left/right side of the equation. Let's write plus/minus __ under the constant on the other side of the equation.** (Write.)
Teacher	**Let's do the math on the left side of the equation. What's __ plus/minus __** (on left side)**?**
Students	__.
Teacher	**Let's write __.** (Write.)
Teacher	**Let's do the math on the right side of the equation. What's __ plus/minus __** (on right side)**?**
Students	__.
Teacher	**Let's write __.** (Write.)
Teacher	**So, did we isolate the variable?**
Students	Yes.
Teacher	**What is equal to *x*?**
Students	__.
Teacher	**Great! *x* equals __. Let's write that.** (Write.)
Teacher	**Let's read our answer.**
Students	*x* equals __.
Teacher	**What equation did we solve?**
Students	__.
Teacher	**How can solve equations?**
Students	Isolate the variable by removing the constant from the variable side. When removing the constant, whatever we do to one side of the equation we also have to do to the other side of the equation.

Example

$x - 2 = 5$

EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's solve an equation. What's an equation?**
Students	A mathematical statement with the equal sign.
Teacher	**An equation has numbers and operator symbols. An equation also has an equal sign. What's the symbol that's always in an equation?**
Students	The equal sign.
Teacher	**Let's show different equations and solve them. Let's use our paper and pencil.** (Show pencil.)
Teacher	**Look at this equation.** (Show problem.)
Teacher	**Read the equation.**
Students	$x - 2 = 5$.
Teacher	**Let's solve this equation. We'll need to focus on the equal sign in this problem. So, let's draw a vertical line down from the equal sign to help us remember to balance both sides of the equation.** (Draw vertical line.)
Teacher	**We'll solve this equation by isolating the variable. What is the variable in this equation?**
Students	*x*.
Teacher	***x* is the variable. We'll isolate the variable by removing the constant from the side of the equal sign with the variable. What's a constant?**
Students	A number that is on its own.
Teacher	**And in this problem, where is the variable?**
Students	Left side.
Teacher	**I like to circle the variable to remember that I'm isolating the variable. Let's circle *x*.**
Students	(Circle *x*.)
Teacher	**So, we'll remove the constant from the left side of the equation. What's the constant that we should remove?**
Students	-2.
Teacher	**We will use the inverse operation and add or subtract from the left side of the equation. What's the operation of the constant?**
Students	Subtract 2.
Teacher	**What's the inverse operation of subtract 2?**
Students	Add 2.
Teacher	**Let's write plus 2 under the constant.** (Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**

Students	Do the same thing to both sides.
Teacher	**Let's also add 2 to the right side of the equation. Let's write plus 2 under the constant of 5 on the other side of the equation.** (Write.)
Teacher	**Let's do the math on the left side. What's -2 plus 2?**
Students	0.
Teacher	**Let's write 0.** (Write.)
Teacher	**Let's do the math on the right side. What's 5 plus 2?**
Students	7.
Teacher	**Let's write 7.** (Write.)
Teacher	**So, did we isolate the variable?**
Students	Yes.
Teacher	**What is equal to *x*?**
Students	7.
Teacher	**Great! *x* equals 7. Let's write that.** (Write.)
Teacher	**Let's read our answer.**
Students	*x* equals 7.
Teacher	**What equation did we solve?**
Students	$x - 2 = 5$.
Teacher	**How can solve equations?**
Students	Isolate the variable by removing the constant from the variable side. When removing the constant, whatever we do to one side of the equation we also have to do to the other side of the equation.

(2) Solving Multi-Step Equations with One Variable

Routine

Materials:

- Module 23 Problem Sets
- Module 23 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's solve an equation. What's an equation?**
Students	A mathematical statement with the equal sign.
Teacher	**An equation has numbers and operator symbols. An equation also has an equal sign. What's the symbol that's always in an equation?**
Students	The equal sign.
Teacher	**Let's show different equations and solve them. Let's use our paper and pencil.** (Show pencil.)
Teacher	**Look at this equation.** (Show problem.)
Teacher	**Read the equation.**
Students	__.
Teacher	**Let's solve this equation. We'll need to focus on the equal sign in this problem. So, let's draw a vertical line down from the equal sign to help us remember to balance both sides of the equation.** (Draw vertical line.)
Teacher	**We'll solve this equation by isolating the variable. What is the variable in this equation?**
Students	*x*.
Teacher	***x* is the variable. We'll isolate the variable by removing the constant from the side of the equal sign with the variable. Where is the variable?**
Students	Left side/right side.
Teacher	**I like to circle the variable to remember that I'm isolating the variable. Let's circle *x*.**
Students	(Circle *x*.)
Teacher	**So, we'll remove the constant from the left side/right side of the equation. What's the constant that we should remove?**
Students	__.
Teacher	**We will use the inverse operation and add/subtract __ from the left/right side of the equation. Let's write plus/minus __ under the constant.** (Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**

Students	Do the same thing to both sides.
Teacher	**Let's also add/subtract __ from the left/right side of the equation. Let's write plus/minus __ under the constant on the other side of the equation.** (Write.)
Teacher	**Let's do the math. What's __ plus/minus __** (on left side)**?**
Students	__.
Teacher	**Let's write __.** (Write.)
Teacher	**What's __ plus/minus __** (on right side)**?**
Students	__.
Teacher	**Let's write __.** (Write.)
Teacher	**So, did we isolate the variable?**
Students	No.
Teacher	**There's a coefficient with this variable. What's a coefficient?**
Students	A number multiplied by a variable.
Teacher	**To truly isolate the variable, we need to remove the coefficient. We'll remove the coefficient from the left side/right side of the equation. What's the coefficient that we should remove?**
Students	__.
Teacher	**If the coefficient is multiplied by *x*, then we will use the inverse operation and divide __ from the left/right side of the equation. Let's write divide __ under the coefficient.** (Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**
Students	Do the same thing to both sides.
Teacher	**Let's also divide __ from the left/right side of the equation. Let's write divide __ under the constant on the other side of the equation.** (Write.)
Teacher	**Let's do the math. What's __ divided by __** (on left side)**?**
Students	__.
Teacher	**Let's write __.** (Write.)
Teacher	**What's __ divided by __** (on right side)**?**
Students	__.
Teacher	**Let's write __.** (Write.)
Teacher	**Now is the variable isolated?**
Students	Yes.
Teacher	**What is equal to *x*?**
Students	__.
Teacher	**Great! *x* equals __. Let's write that.**

	(Write.)
Teacher	**Let's read our answer.**
Students	*x* equals __.
Teacher	**What equation did we solve?**
Students	__.
Teacher	**How can you solve equations?**
Students	Draw a line vertically down from the equal sign. Circle the variable. Then, isolate the variable by removing the constant. Divide the variable by a coefficient if necessary.

Example

$11 = 2y + 5$

EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's solve an equation. What's an equation?**
Students	A mathematical statement with the equal sign.
Teacher	**An equation has numbers and operator symbols. An equation also has an equal sign. What's the symbol that's always in an equation?**
Students	The equal sign.
Teacher	**Let's show different equations and solve them. Let's use our paper and pencil.** (Show pencil.)
Teacher	**Look at this equation.** (Show problem.)
Teacher	**Read the equation.**
Students	$11 = 2y + 5$.
Teacher	**Let's solve this equation. We'll need to focus on the equal sign in this problem. So, let's draw a vertical line down from the equal sign to help us remember to balance both sides of the equation.** (Draw vertical line.)
Teacher	**We'll solve this equation by isolating the variable. What is the variable in this equation?**
Students	*y*.
Teacher	***y* is the variable. We'll isolate the variable by removing the constant from the side of the equal sign with the variable. Where is the variable?**
Students	Right side.
Teacher	**I like to circle the variable to remember that I'm isolating the variable. Let's circle *y*.**
Students	(Circle *y*.)
Teacher	**So, we'll remove the constant from the right side of the equation. What's the constant that we should remove?**
Students	5.

Teacher	**We will use the inverse operation and add or subtract from the right side of the equation. What's the inverse operation with plus 5?**
Students	Minus 5.
Teacher	**Let's write minus 5 under the constant.** (Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**
Students	Do the same thing to both sides.
Teacher	**Let's also subtract 5 from the left side of the equation. Let's write minus 5 under the constant on the other side of the equation.** (Write.)
Teacher	**Let's do the math on the left side of the equation. What's 11 minus 5?**
Students	6.
Teacher	**Let's write 6.** (Write.)
Teacher	**Let's do the math on the right side of the equation. What's 5 minus 5?**
Students	0.
Teacher	**Let's write 0.** (Write.)
Teacher	**So, did we isolate the variable?**
Students	No.
Teacher	**There's a coefficient with this variable. What's a coefficient?**
Students	A number multiplied by a variable.
Teacher	**To truly isolate the variable, we need to remove the coefficient. We'll remove the coefficient from the right side of the equation. What's the coefficient that we should remove?**
Students	2.
Teacher	**If the coefficient is multiplied by *y*, then we will use the inverse operation and divide 2 from the right side of the equation. Let's write divide 2 under the coefficient.** (Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**
Students	Do the same thing to both sides.
Teacher	**Let's also divide 2 from the left side of the equation. Let's write divide 2 under the constant on the other side of the equation.** (Write.)
Teacher	**Let's do the math on the left side of the equation. What's 6 divided by 2?**
Students	3.
Teacher	**Let's write 3.** (Write.)
Teacher	**What's 2 divided by 2?**

Students	1.
Teacher	**Let's write 1. You could also not write the 1 because it's implied with the *y*.** (Write.)
Teacher	**Now is the variable isolated?**
Students	Yes.
Teacher	**What is equal to *y*?**
Students	3.
Teacher	**Great! *y* equals 3. Let's write that.** (Write.)
Teacher	**Let's read our answer.**
Students	*y* equals 3.
Teacher	**What equation did we solve?**
Students	$11 = 2y + 5$.
Teacher	**How can you solve equations?**
Students	Draw a line vertically down from the equal sign. Circle the variable. Then, isolate the variable by removing the constant. Divide the variable by a coefficient if necessary.

(3) Solving Equations with Variables on Both Sides

Routine

Materials:

- Module 23 Problem Sets
- Module 23 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's solve an equation. What's an equation?**
Students	A mathematical statement with the equal sign.
Teacher	**An equation has numbers and operator symbols. An equation also has an equal sign. What's the symbol that's always in an equation?**
Students	The equal sign.
Teacher	**Let's show different equations and solve them. Let's use our paper and pencil.** (Show pencil.)
Teacher	**Look at this equation.** (Show problem.)
Teacher	**Read the equation.**
Students	__.
Teacher	**Let's solve this equation. We'll need to focus on the equal sign in this problem. So, let's draw a vertical line down from the equal sign to help us remember to balance the sides of an equation.** (Draw vertical line.)
Teacher	**We'll solve this equation by isolating the variable. What is the variable in this equation?**
Students	*x*.
Teacher	***x* is the variable. We'll isolate the variable by removing the constant from the side of the equal sign with the variable. Where is the variable?**
Students	Left side and right side.
Teacher	**I like to circle the variable to remember that I'm isolating the variable. Let's circle *x*.**
Students	(Circle *x*.)
Teacher	**In this equation, *x* is on both sides. So, let's work with the *x* with the greater coefficient by removing the *x* with the coefficient that is less. Which *x* has a greater coefficient?**
Students	Left side/right side.
Teacher	**So, we'll remove the variable with the coefficient that is less from the left side/right side of the equation. Which coefficient and variable should we remove?**
Students	__.
Teacher	**We will use the inverse operation and add/subtract __ from the left/right side of the equation. Let's write plus/minus __ under the coefficient and variable.**

	(Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**
Students	Do the same thing to both sides.
Teacher	**Let's also add/subtract __ from the left/right side of the equation. Let's write plus/minus __ under the coefficient and variable on the other side of the equation.**
	(Write.)
Teacher	**Let's do the math. What's __ plus/minus __** (on left side)**?**
Students	__.
Teacher	**Let's write __.**
	(Write.)
Teacher	**What's __ plus/minus __** (on right side)**?**
Students	__.
Teacher	**Let's write __.**
	(Write.)
Teacher	**We've removed one variable from one side of the equation. So, we'll remove the constant from the left side/right side of the equation. What's the constant that we should remove?**
Students	__.
Teacher	**We will use the inverse operation and add/subtract __ from the left/right side of the equation. Let's write plus/minus __ under the constant.**
	(Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**
Students	Do the same thing to both sides.
Teacher	**Let's also add/subtract __ from the left/right side of the equation. Let's write plus/minus __ under the constant on the other side of the equation.**
	(Write.)
Teacher	**Let's do the math. What's __ plus/minus __** (on left side)**?**
Students	__.
Teacher	**Let's write __.**
	(Write.)
Teacher	**What's __ plus/minus __** (on right side)**?**
Students	__.
Teacher	**Let's write __.**
	(Write.)
Teacher	**So, did we isolate the variable?**
Students	No.
Teacher	**There's a coefficient with this variable. To truly isolate the variable, we need to remove the coefficient.**

Teacher	**We'll remove the coefficient from the left side/right side of the equation. What's the coefficient that we should remove?**
Students	__.
Teacher	**If the coefficient is multiplied by *x*, then we will use the inverse operation and divide __ from the left/right side of the equation. Let's write divide __ under the coefficient.** (Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**
Students	Do the same thing to both sides.
Teacher	**Let's also divide __ from the left/right side of the equation. Let's write divide __ under the constant on the other side of the equation.** (Write.)
Teacher	**Let's do the math. What's __ divided by __** (on left side)**?**
Students	__.
Teacher	**Let's write __.** (Write.)
Teacher	**What's __ divided by __** (on right side)**?**
Students	__.
Teacher	**Let's write __.** (Write.)
Teacher	**Now is the variable isolated?**
Students	Yes.
Teacher	**What is equal to *x*?**
Students	__.
Teacher	**Great! *x* equals __. Let's write that.** (Write.)
Teacher	**Let's read our answer.**
Students	*x* equals __.
Teacher	**What equation did we solve?**
Students	__.
Teacher	**How can you solve this equation?**
Students	Draw a vertical line down from the equal sign. Remove the coefficient and variable of lesser value to the other side of the equal sign. Remove the constant to isolate the variable. Divide by the coefficient.

Example

$4a - 7 = 3a - 3$

EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's solve an equation. What's an equation?**
Students	A mathematical statement with the equal sign.
Teacher	**An equation has numbers and operator symbols. An equation also has an equal sign. What's the symbol that's always in an equation?**
Students	The equal sign.
Teacher	**Let's show different equations and solve them. Let's use our paper and pencil.** (Show pencil.)
Teacher	**Look at this equation.** (Show problem.)
Teacher	**Read the equation.**
Students	$4a - 7 = 3a - 3$.
Teacher	**Let's solve this equation. We'll need to focus on the equal sign in this problem. So, let's draw a vertical line down from the equal sign to help us remember to balance the sides of an equation.** (Draw vertical line.)
Teacher	**We'll solve this equation by isolating the variable. What is the variable in this equation?**
Students	*a*.
Teacher	***a* is the variable. We'll isolate the variable by removing the constant from the side of the equal sign with the variable. Where is the variable?**
Students	Left side and right side.
Teacher	**I like to circle the variable to remember that I'm isolating the variable. Let's circle *a*.**
Students	(Circle a.)
Teacher	**In this equation, *a* is on both sides. So, let's work with the *a* with the greater coefficient by removing the *a* with the coefficient that is less. Which *a* has a greater coefficient?**
Students	Right side.
Teacher	**So, we'll remove the variable with the coefficient that is less from the right side of the equation. Which coefficient and variable should we remove?**
Students	$3a$.
Teacher	**We will use the inverse operation and subtract $3a$ from the right side of the equation. Let's write minus $3a$ under the coefficient and variable.** (Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**
Students	Do the same thing to both sides.

Teacher	**Let's also subtract 3*a* from the left side of the equation. Let's write minus 3*a* under the coefficient and variable on the other side of the equation.** (Write.)
Teacher	**Let's do the math. What's 4*a* minus 3*a*?**
Students	*a*.
Teacher	**Let's write *a*.** (Write.)
Teacher	**What's 3*a* minus 3*a*?**
Students	0.
Teacher	**Let's write 0. We also could not write anything because we have none of the variable on the right side.** (Write.)
Teacher	**We've removed one variable from one side of the equation. So, we'll remove the constant from the left side of the equation. What's the constant that we should remove?**
Students	-7.
Teacher	**We will use the inverse operation and add 7 on the left side of the equation. Let's write plus 7 under the constant.** (Write.)
Teacher	**But, when solving equations, if we do something to one side of the equal sign, we have to do the same thing to the other side of the equal sign. What do we have to do when solving equations?**
Students	Do the same thing to both sides.
Teacher	**Let's also add 7 on the right side of the equation. Let's write plus 7 under the constant on the other side of the equation.** (Write.)
Teacher	**Let's do the math on the left side. What's -7 plus 7?**
Students	0.
Teacher	**Let's write 0. We also don't have to write anything if it's 0.** (Write.)
Teacher	**Let's do the math on the right side. What's -3 plus 7?**
Students	4.
Teacher	**Let's write 4.** (Write.)
Teacher	**So, did we isolate the variable?**
Students	Yes.
Teacher	**What is equal to *a*?**
Students	4.
Teacher	**Great! *a* equals 4. Let's write that.** (Write.)
Teacher	**Let's read our answer.**
Students	*a* equals 4.
Teacher	**What equation did we solve?**
Students	$4a - 7 = 3a - 3$.

Teacher	**How can you solve this equation?**
Students	Draw a vertical line down from the equal sign. Remove the coefficient and variable of lesser value to the other side of the equal sign. Remove the constant to isolate the variable. Divide by the coefficient.

D. Problems for Use During Instruction

See Module 23 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 23 Vocabulary Cards.

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Module 23:
Solving Equations

Problem Sets

B. $4a = 12$

B. $9x = 36$

B. $$15 = 15D$$

B. $56 = 7k$

B. $3r = 21$

B. $32 = 4t$

B. $7b = 49$

B. $5s = 60$

B.

$$132 = 11f$$

B. $25 = 5g$

B. $16 = w - 4$

B. $2 - K = 5$

B. $9 + r = 9$

B. $5 + f = 8$

B. $2 = G - 9$

B. $7 = 11 - M$

B. $c + 4 = 6$

B. $8 + h = 3$

B. $y + 4 = 12$

B. $8 - p = 15$

c. $2 = M \div 6$

c. $10 = E \div 2$

c. $8 = z \div 4$

c. $15 = j \times 5$

c. $5 = R \times 1$

c. $c \times 6 = 24$

c. $n \div 5 = 7$

c. $b \times 9 = 36$

c. $k \times 8 = 72$

c. $4 = h \div 4$

D. $6b - 1 = 11$

D. $30 = 2n \times 3$

D. $21 + 3x = 51$

D. $5a - 2 = 13$

D. $64 - 5x = 14$

D. $6v \times 4 = 72$

D. $7b - 7 = 42$

D. $47 - 3x = 32$

D. $55 = 5c + 5$

D. $3c + 2 = 11$

E. $2y + 5 = 15 - 3y$

E. $2x + 20 = x + 56$

E. $3k + 2 = 50 - k$

E. $4 + p = 3p + 18$

E. $3x + 12 = 72 + 8x$

E. $5x + 10 = x - 14$

E. $6 - 2f = 7f + 1$

E. $b + 2 = 7b + 20$

E. $48 - 5e = 3e + 8$

E. $4e - 7 = 3a - 3$

F. $x^2 + 13x - 7 = 15$

F. $7x^2 + 17x + 10 = 0$

F. $18r^2 + 61r = 50$

F. $k^2 + 9k - 5 = 5$

F. $h^2 - 7h = 0$

F. $y^2 + 5 = 8$

F. $20z^2 - 48z = 6$

F. $6x^2 + 17x - 88 = 0$

F. $g^2 + 18g + 1 = 72$

F. $n^2 + 5 = 11$

Module 23:

Solving Equations

Vocabulary Cards

base
coefficient
constant
equation
exponent
expression
grouping
inequality
like terms
operator
term
variable

base

A number that is multiplied by an exponent.

$$5^3$$

5 is the base

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coefficient

A number that is multiplied by a variable.

$$5x + 9 = 24$$

5 is a coefficient

constant

A term that does not change; a number on its own.

$$5x + 9 = 24$$

9 and 24 are constants

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equation

A mathematical statement that two expressions are the same or equal; must have an equal sign.

$$5x + 9 = 24$$

5x + 9 = 24 is an equation

(DOES have an = sign)

exponent

The power to which a number is raised.

$$5^3$$

3 is the exponent

---

expression

A combination of variables, numbers, and/or operations that represents a mathematical relationship; does not have an equal sign.

$5x + 9$ 24

5x + 9 and 24 are expressions

(DOES NOT have an = sign)

grouping

A combination of variables, numbers, and/or operations grouped together in parentheses or brackets.

$$(15 + 4) \qquad 2[(6 + 4) \div 2]$$

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inequality

An algebraic relation showing that a quantity is greater or less than another quantity.

$$5x + 9 > 24$$

The > makes this equation an inequality

like terms

Terms that have the same variable or constant and can be combined.

$2y$ $4y$ $8y$

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operator

A symbol (+, –, × ÷) that represents a mathematical operation.

$5x + 9 = 24$

+ is an operator

term

A single number or a variable, or numbers and variables multiplied together.

$$5x + 9 = 24$$

5x, 9, and 24 are terms

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variable

A symbol for an unknown value, which is usually represented by a letter.

$$5x + 9 = 24$$

x is a variable